Risk-sharing Rules for Water Allocation in Drought Periods: Recommandations for the French Water Policy Reform

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Recommendations for the French Water Policy Reform

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Abstract

This paper addresses the issue of irrigation water allocation when river flows are stochastic, farmers are risk averse and self-assurance strategies exist. Because there is no water market in France, sharing rules are necessary to allocate the water from the river in case of scarcity between all water licence-holders. The objective is to draw recommendations for the French institutions in charge of designing these rules by measuring their efficiency in drought risk management. We model irrigators as agents having access both to a common pool resource of uncertain size (the river), and a costly but guaranteed substitute resource, which constitutes the self-insurance (an on-farm reservoir filled during winter). We first identify the theoretical optimal risk management from the perspective of a regulator than can control both resources (common pool resource and self-insurance) and then for a regulator that only control the sharing of the common ressource in case of scarcity. We then compare the performance of different empirical sharing rules in risk management.

This is a very preliminar draft. Please do not quote.

Keywords: Water allocation, Drought management, Risk sharing, Risk taking, Risk tolerance, Common Pool Resource of uncertain size, Self-insurance, Organisme Unique, Allocation de l'eau, Sécheresse, Partage de risque, Aversion au risque, Auto-assurance

JEL codes*: Q25, Q54, Q58, D91, C91*

1 Motivations

1.1 Empirical Motivations

With climate change, a number of regions in the world are expected to face increasingly frequent and more severe droughts (IPCC [18]), forcing the irrigated agricultural sector often a major consumer in summer periods to face water supply restrictions. There is abundant empirical evidence, backed-up by a large body of theoretical literature, on the relative efficiency of different water-sharing mechanisms, mainly based on pricing and markets. But this literature often overlooks risks associated with the high variability of the resource and rarely provides adequate solutions to the issue of risk-taking and risk-sharing among farmers.

Indeed, farmers indicate that reliability and timing of water delivery are more important than having larger quantities but erratic supplies ([1],[25],[17]). Traditional ways to manage drought risks at farm level are private insurance or self-insurance strategies. The efficiency of the former is limited because of the systemic component of drought risk, which limits the scope for risk diversification. The insurance premia are as a consequence often too high for insurance to be attractive for farmers without state subsidies[1]. Therefore, farmers prefer to adopt risk mitigation strategies. For instance they can build individual reservoirs, change or diversify their cropping patterns, modify input use, diversify their water resources by pumping from groundwater... These strategies reduce the impact of water volume fluctuations but are costly. There are both direct

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[1] France is experimenting a new multi-risk climatic insurance, covering for all risks associated to climatic events (drought but also frost, hail, flood etc.). However, this insurance system remains costly and is heavily subsidized by the state.

costs (reservoir construction...) and indirect costs due to the deviations from optimal production strategies (when input or crops are modified)([23, 31]). Moreover, decentralized individual risk management strategies do not guarantee optimal risk taking at the society level. Too cautious or too risky behaviors can be sub-optimal. In this paper, we study a third alternative that can help to reach efficient drought risk management: the introduction of risk sharing mechanisms in water allocation rules. This opportunity is studied in the French context.

France is currently implementing a reform of its irrigation water allocation rules. We see in the ongoing reform the opportunity to introduce those risk issues in the future water allocation design. The 2006 French law on water requires that decisions on irrigation water allocation be delegated at the scale of each catchment to a single agency (called *organisme unique*). Each newly created agency will be entitled with a unique collective pumping authorization and will be made responsible for sharing efficiently the resource among irrigators. While the agency's main duty is to allocate the available water resource in normal years, it also has to design a scarcity management plan which defines the restrictions for each user in case of a temporary water shortage[2]. Such a plan amounts to the design of contingent sharing rules that define *a priori* the amount of the scarce water resource that will be allocated to each farmer for each possible quantity of the resource. The water manager must thus be able to combine ex-post efficiency (defined by the allocation which creates maximum value from the use of water, when each unit goes to the highest marginal value user) and ex-ante efficiency, which minimizes the costs of risks by allocating risk to those who are most risk tolerant.

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[2] The role of the *organisme unique* is presented in Circulaire of the 30th of June 2008, annexe III

Farmers know that they face the risk of low river flows and that they may have to face restrictions when volumes available are too low to fit the "normal year" allocation plan. They therefore take individual intiatives aimed at preventing or mitigating the adverse consequences of a drought. One solution for them is to build -individually or with a few neighbours - small-scale dams or reservoirs [3] in order to diversify their water resources. The reservoirs are filled during winter, when water is relatively abundant and when the reservoirs' filling activity does not compete with irrigation. According to the definition given by Ehrlich and Becker[12], building storage capacity is self-insurance as it reduces the financial losses by increasing water volumes available in case of drought. Moreover, this resource is perceived as safe by the farmers because the volumes are know in advance (farmers can observe the quantity stored at the end of winter) and administrative restrictions do not apply to this resource ([32]). Reservoir building is assumed to have no effect on the probability of water scarcity occurrence. The farmers agree to reduce their extraction of river water in exchange of public funding of the reservoir. The building of "*réserves de substitution*" in France can benefit from public fundings up to 80% of total cost. The condition is that recipients hand back a share of their pumping licences corresponding to the volume stored in the reservoir. Farmers thus use water stored in reservoirs as a substitute for uncertain river water. They manage a portfolio of resources: a "risk-free" but costly resource, which is the water volumes stored in their individual private reservoirs, and a "risky resource" which is defined by their pumping authorizations on an uncertain common pool resource (CPR). We assume that farmers choose a utility maximizing portfolio, by combining requests from the CPR and storage capacity in reservoirs.

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[3] so called réserves de substitution or retenues collinaires in France

We develop a model that allows us to study the effects of various ex-ante allocation rules on farmers' self-insurance strategies. We start by identifying the socially optimal sharing rule and the optimal level of self-insurance for each agent. It is theoretically equivalent to solving the problem of efficient risk-sharing when risk is endogenous. Then we study the extent to which different water scarcity-sharing rules, which are empirically relevant, can be considered as acceptable second best solutions.

The paper is organized as follows. In the next section, the main findings of the literature on risk sharing and on uncertainty in common pool resources are presented. Section 2 describes a model of risk-sharing with self-insurance when risk is systemic, and calculates first-best and second-best solutions. Section 3 draws recommendations for the French water policy reform and concludes.

1.2 Literature review

The originality of this work is to introduce risk sharing in a context of common pool resource whose size is uncertain. Moreover we deal with the innovative concept of efficient risk sharing of an endogeneous risk. In this section, we briefly recall the main results in these two strands of literature. We both consider theoretical and experimental results. The first sub-section summarizes findings on exploitation of uncertain CPR by risk averse agents. The second sub-section describes how efficient risk-sharing can improve risk management and develops the special case of an endogeneous risk.

1.2.1 Common Pool with uncertain resource size

Surface water management is one of the many real-world social dilemnas where the size of the resource is almost never known with certainty. This issue has been studied both in theory and experimentally. In such CPR dilemmas, there is both strategic uncertainty (due to the imperfect information on others' behaviours, like in CPR dilemna with known size) and environmental uncertainty (the size of the resource is unknown)([22]).

As soon as we introduce a risky environment, we can expect risk aversion to play a role. Sandler & Sternbenz [28] look at the exploitation of a stock resource, and show that uncertainty on the size of the stock leads risk-averse firms to reduce their exploitation effort compared to the certainty case. Bramoullé & Treich [5] study the contribution to a public bad: CO2 pollution. They find that uncertainty on damages due to pollution can lower the incentives to pollute. Because the variance of damage increases with pollution, risk-averse polluters reduce the risks faced by decreasing their emissions. They trade-off a loss in expected payoff for a reduction in risk. Nevertheless, early experimental observations contradict theoretical results on the impact of environmental risk in social dilemmas. These experimental results show that the well-known overharvesting of the CPR increases with environmental uncertainty ([22, 24, 7, 16]). Messick and al. [22] incorporate a probabilistic destruction of the resource when a threshold is surpassed. They show that even when strategic uncertainty is absent (only one player), the random size of the resource leads to suboptimal outcome. Rapoport & Suleiman [24] show that subjects deal with strategic uncertainty by requesting roughly 1/n of the mean of the resource size. But, as the environmental uncertainty (the variance of the resource distribution) increases, they request more than an equal share from the resource. This result is explained by an “optimism bias” : people tend to perceive the variance and mean of the resource distribution to be positively correlated. Surprisingly, only few experiments measure risk aversion and take it into account when analyzing CPR games with uncertainty. When controlling for risk-aversion, experimental results are in line with theoretical predictions under risk aversion. Chermak & Krause [10] find a significant effect of risk aversion, where more risk-averse agents avoid more the risk of depletion. We find equivalent results in the public good literature [4], with risk-averse players: in Sandler et al. [27], an increase in risk leads to a decrease in expenditure on the public good for a risk-averse contributor. This is confirmed by Gangadharan & Neme [14], who show that risk-averse subjects contribute less to the public good and more to the private good. They divert away from the strategic uncertainty associated with contributing to the public good, and divert away from the environmental risk due to the unguaranteed return from the public good.

Another relevant piece of literature deals with risk management possibilities introduced into CPR experiments. Risk diversification has been introduced in CPR games by Walker & Gardner [33] and Keser & Gardner [19]. In their settings, each appropriator has an endowment of resource which can be invested in the CPR or invested in a safe, outside activity. If appropriators invest all their endowments in the outside alternative, they get a sure payoff, whereas if they invest only a share of their endowments in the CPR, they get a sure payoff plus a payoff from the CPR. The situation where irrigators can have risky but free water from a river or secure but costly water from a reservoir fits typically into this setting.

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[4] The close resemblance between the formal structure of the take-some CPR game and the give-some step level public good game is well documented ([2])

Our model described in section 2 is inspired by this literature but it includes a few significant differences: in order to mimic the existing water sharing policy in France, agents are offered a share of the quantity available when there is a shortage, whereas in most risky CPR experiments, subjects generally receive nothing when the depletion threshold of the resource is overpassed ([30, 6, 33, 7] ...). Our objectie is to design the scarcity sharing rules in the most efficient way. We present in the following part the main results concerning efficient risk management.

1.2.2 Efficient risk management

Where several individuals have joint interest on some risky prospect, the issue of efficient risk-sharing arises. Partners will find it mutually advantageous to agree in advance on an allocation of the prospect, and to do so in a way which reduces the social risk premium [5]. In the literature, this corresponds to the problem of efficient risk sharing of a systemic risk. In order to share a given risk efficiently, two principles must be satisfied ([11]) : the mutuality principle and Borch's condition. The mutuality principle states that all diversifiable risks must be pooled. Most of the experimental literature on risk sharing addresses the question of the extent of risk pooling ([9, 26, 8]). In our setting, the mutuality principle is irrelevant since farmers face an aggregate risk, which is the risk on the total water volume available for agriculture. We therefore focus on the second principle, which is mostly relevant when risks are not independently and identically distributed. This principle states how to share socially undiversifiable risks.

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[5] The risk premium is defined as the maximum amount, beyond the negative of the expected value of the risk itself, which an individual would pay to insure against the risk ([21]). The risk premium depends on the risk aversion and the variance of the risk considered.

Borch's theorem [4] states that the share of the social risk borne by an agent should be proportional to her risk tolerance relative to the group's risk tolerance, measured as the sum of individual risk tolerances. As a result, any allocation rule is Pareto-dominated by some unequal allocation giving to the less risk-averse partner a larger share of the agregate risk. In the special case of mean-variance preferences, it implies that a less risk-averse agent accepts to bear a larger payoff variance, compensated by a larger expected payoff. We retain from this literature that, although the capacity for spreading drought risks at the scale of a water catchment is limited, there is scope to share this systemic risk efficiently by exploiting heterogeneity in water users' risk preferences (provided the regulator can observe individual risk tolerances). It suggests to include risk management in CPR administration by designing contingent allocation rules according to users' risk tolerance. This is an original application of Borch's theorem on drought risk (see Lemaire[20] for other applications).

In Borch's setting, the risk is given. But in the context we have described in section 1.1, risk is endogeneous. As stated by Shogren & Crocker [29], despite the fact that the "view that environmental risk is exogenous, beyond the control of everyday people, dominates the risk assessment/risk management studies, intuition and everyday evidence undeniably show that human actions and reactions help determine the likelihood and the severity of events". Wilson [34] develops a model with endogeneous risk, where the risk level is the consequence of a collective decision of a syndicate. Our model will also endogenize risk by allowing agents to partially "escape" from the systemic risk (the quantity available from the CPR) by individualy investing in a sure resource (the reservoir). The decision of risk taking is here decentralized. The aggregation of individual self-insurance decisions defines the total demand to be drawn from the uncertain CPR and therefore the total risk-taking level.

A requirement for ex-ante efficiency when risk is endogeneous is that collective risk taking be optimal. One of the most obvious signs of inappropriate risk management is when agents face incentives to act against the common interest, for example by engaging in risky farming practices at the expense of social welfare because it increases the social risk premium. The other extreme is seen when, because there is no organised risk-sharing, everyone takes a very cautious and risk-averse approach, leading to over-investment in self- insurance, therefore lowering overall production and welfare. A well-designed risk-sharing system gives each actor an incentive to choose an economically optimal level of drought-vulnerability. But risk taking decisions and risk-sharing opportunities are generally not studied simultaneously. In Wilson [34], the syndicate must both choose a Pareto optimal sharing rule and take a decision. Eeckoudt and al.[11] formulate the question in those terms: “how should the fact that risks can be shared efficiently within the syndicate affect the decision making of the syndicate (in terms of risk-taking)?”. Wilson shows that both problems are independent. The optimal sharing rule does not depend upon the group decision. On the contrary, in Gollier [15], the ability to share risks efficiently has a positive impact on the demand for that risk. In his model, risks are independant and this is why he obtains such a positive result. The only experimental example available, where both a risky decision is taken and an allocation rule of the outcome of this decision is determined, is found in Bone and al. [3]. In their first treatment, subjects only have to choose an allocation of a given prospect. In the second treatment, subjects are required to both choose a prospect (2 choices, one is riskfree and the other risky) and to decide upon the allocation of the prospect. Subjects favor efficient risk sharing when the allocation is the sole issue (first treatment). But in the second treatment, they fail to do so and focus on egalitarian sharing.There are obviously interactions between risk taking and risk-sharing decisions. These interactions need to be further investigated. The next session presents a model where risk-sharing is applied to a systemic endogeneous risk.

2 A model of risk sharing with endogeneous social risk

Assume a pair of agents $i, : j$, each of which requires H units of a resource for her activity. Both agents can obtain the required quantity by combining two possible sources: they can submit costless requests from a common pool resource (CPR) of an uncertain size, or they can draw on a costly, but unlimited resource. We assume that the two resources are perfect substitutes allowing for any combination of the two resources. Agent i chooses to request an amount R_i from the CPR and the complementarity quantity $H - R_i$ from the costly resource. We assume that the quantity of resource needed H is exogeneously determined and equal for all agents. This simplifying assumption allows us to focus on agents' risk tolerance as a criterion for individual risk-taking and collective risk-sharing, by astracting away from other dimensions of the production strategies that would make the problem uncessarily complex.

Agents are exposed to a common risk of shortage for their requests from the CPR. The risk is about the total quantity available from the CPR : $\tilde{x}$. We asume that $\tilde{x}$ takes only two possible values which correspond to the state of nature that obtains : in the *good state*, which occurs with probability p, $0 < p < 1$, the available amount of the resource is large enough to cover the total request of the group of agents (technically we assume that the amount is infinite); in the *bad state* agents have to face a shortage. The quantity available is $\underline{x}$ with $\underline{x} < \sum_{i=1}^{N} R_i$. $\tilde{x}$ is a binary random variable with values $\{\underline{x}, +\infty\}$. The probability of $x = \underline{x}$ is $\rho(\underline{x}) = 1 - p$. We consider the special case where p is exogeneous. The probability of scarcity does not depend upon the the quantity of resource available $\tilde{x}$. This assumption is relevant in the French context where autorized volumes for agriculture (H in the model) are defined such that they can be withdrawn without impacting the environment eight years out of ten. As a result, restrictions are expected only two years out of ten. This corresponds to the probability $\rho(\underline{x}) = 1 - p$. In the good state, agent i thus receives her request R_i from the CPR; in the bad state, agent i receives a share θ_i of $\underline{x}$, the available amount of the resource. This share depends on the sharing rule defined by $\theta = (\theta_i, \theta_j)$. $p, c, \underline{x}$ and $\theta = (\theta_i, \theta_j)$ are known by the agents. The risk $\tilde{x}$ is additive: the random quantity received from the CPR is added to the quantities obtained for sure from the substitute resource.

While the outcome of the variable $\tilde{x}$ is exogeneously determined, the risk borne by the agents depends on their self-insurance choice: their risk taking strategies are determined by their requests from the CPR or their self-insurance choices. The variance of the risk taken by agent i is $p(1-p)\,[R_i - \theta_i]$. We assume that agents are risk averse and choose their request from the CPR in order to maximise their expected utility. We use CARA utility functions. This assumption eliminates the income effect when dealing with decisions to be made about a risk whose size is invariant to changes in wealth (additive risk). For calculation simplicity, we solve the agent's problem as a maximization of the certainty equivalent using the Arrow-Pratt and Taylor approximations[6]. The certainty equivalent equals the mean payoff of the risk minus the absolute risk premium, expressed in moneraty units. The risk premium measures the cost of risk, which is approximately proportional to the variance of the payoffs. The risk premium is also decreasing in risk tolerance, defined as the inverse of absolute risk aversion. We assume complete information of the regulator, especially concerning the risk tolerances of agents. There is no consideration of information revelation.

Since requests from the CPR are not necessarily satisfied in the bas state, the quantity R_i requested from the CPR by agent i will be called the level of risk taken by agent i. The complementary quantity $H - R_i$ requested from the sure resource will be analogously defined as level of self-insurance because relying on this alternative resource decreases the size of the loss in case of shortage. All interpretations will be done relatively to the risk taking level, but the translation in terms of request from the CPR or self-insurance is obvious. To save on terminology, we also call sharing rule or allocation rule the share θ_i received by i in the bad state.

The value of a unit of resource g is constant and the same for all agents. g is normalized to 1. The cost of the secure resource is c per unit of resource. With such an assumption, in the water framing, we assume that the cost of the reservoir is strictly proportional to the volume stored. Moreover, there is only one period in this setting so the cost of the self-insurance is the amortization of the full cost for one period. In the French context, the cost is mainly suported by public autorities. We thus consider that c is the remaining cost for the farmer. All gains and costs are expressed in monetary units.

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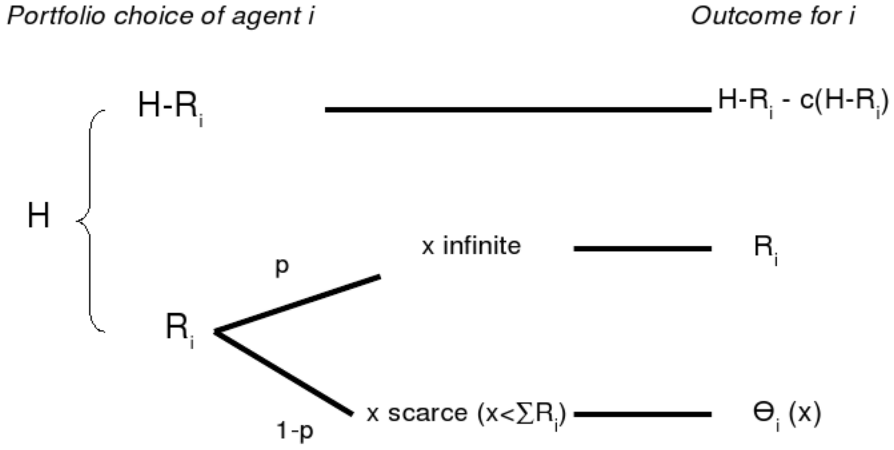
[6] This mean-variance decision criterion is a special case of expected utility theory. The validity of this model depends on the degree of accuracy of the Arrow-Pratt approximation, which can be considered accurate only when the risk is small.

Fig. 1: Representation of the game



The profit of agent i is equal to the number of units she received from both resources minus the cost of the units she ordered from the secure resource $c(H - R_i)$. We assume $p > 1 - c$.

Our aim is to determine both the socially optimum level of risk and the efficient sharing of this risk, i.e. the sharing rule that minimizes the social cost of a drought. The social cost of a drought can be decomposed into three components: first, the risk premium that agents incur due to the variability of the amounts received from the CPR; second, the cost of self-insurance, i.e. the cost $c(H - R_i)$ of buying a risk-free amount of resource; third, the opportunity cost of self insurance due to the foregone quantities from the CPR. Indeed, because H is fixed, any unit obtained from the secure resource is withdrawn from the request into the CPR. The regulator aims at maximizing the weighted sum of the certainty equivalents of all agents. The agents' certainty equivalents take into account these three components of drought's costs.

The model is intended to provide answers to three main questions : 1) What is the socially optimum level of risk for the group of agents ? 2) How should the socially optimum level of risk-taking be shared among the group members ? and 3) Assuming that a given sharing rule is imposed onto the group of agents, what level of risk will be chosen by each member of the group and will this decentralized decisions lead to the socially optimal risk taking level ? The answer to the first two questions will give us the first best solution. This is the solution that would be chosen by a regulator, who is able to impose both the sharing rule to the agents and the amount of risk taken by each of them. Since such a first best solution is usually not feasible, we consider with the third question the more realistic issue when the regulator chooses the sharing rule and agents choose their level of individual risk to maximize their certainty equivalent (second best). In practice, egalitarian and proportional sharing rules have been extensively used as policy tools. How well do these rules perform with respect to the socially optimum rule and risk taking ? The answers to questions 1 and 2 will provide a benchmark with respect to which we can evaluate the performance of various sharing rule that a regulator is or might be implementing. The first-best solutions are indexed by ** and by * for the second-best.

2.1 First best solution: optimal risk taking at the group level and optimal sharing of that risk

The resource manager can both choose the allocation rule that will be implemeted in the bad state and the level of risk taken by each agent. Formally, the regulator chooses both the vector $R = (R_i, R_j)$ and the allocation rule $\theta = (\theta_i, \theta_j)$ in order to maximize the weighted sum of the certainty equivalent of all group members, with λ_i is the weight of agent i in the social utility function. Constraint (1) states that the sharing rule exhausts the total quantity available in the bad state x. According to constraint (2) there is shortage in the bad state since the aggregate request from the CPR is larger than the available amount of resource. Finally, constraint (3) simply reminds that individuals requests cannot exceed the individual demand. This problem writes as:

$$\begin{aligned} \underset{\theta_i,R_i}{Max} \sum_{i,j} \lambda_i CE_i &= \sum \lambda_i \left[(1-c)\,(H-R_i) + p\,R_i \; + (1-p)\theta_i - \frac{1}{2T_i} p\,(1-p)[R_i-\theta_i]^2 \right] \\ &(1) \quad x = \sum \theta_i \\ &(2) \quad \sum \theta_i \leq \sum R_i \\ &(3) \quad R_i \leq H \end{aligned}$$

We are interested in the case where constant (2) and (3) ar not binding. This is the case where there is a shortage in the bad state and agent diversify their resource by demanding some units from the secure resource.

The solution of the regulator maximization program (proof in appendix A) is such that:

- The aggregate risk taking level is $(R_j + R_i)^{**} = x + (T_j + T_i)\frac{p-(1-c)}{p\,(1-p)}$ (10).
- A optimum risk-sharing arrangement satisfies :

$$T_i \frac{p-(1-c)}{p\,(1-p)} < R_i^{**} \leq H \; \forall \; i,j$$

Interior solutions, i.e. $R_i^{**} < H$, $\forall \; i,j$, require equal weights : $\lambda_i \; = \lambda_j$. If $\lambda_i \neq \lambda_j$, the optimal solution requires that the favoured agent takes the maximum level of risk H.

- The quantity received by an agent in the bad state is such that (11)

$$\theta_i^{**} = R_i - T_i \frac{p - (1 - c)}{p\,(1 - p)}$$

  Any pair $(\theta_i^{**}, R_i^{**})$ $\forall\, i, j$ that respects (11) is pareto optimal.

We easily verify that the objective function of the regulator only depends upon $(R_j + R_i)$ (see (15) in Appendix A). The regulator is only interested in the total risl taking level. The higher the group risk tolerance, the resource size x, the probability of good state, and the cost of self-insurance, the higher the optimal level of risk. The sharing of this risk taking level between the two agents is undefined, as soon as the sharing of the shortage risk is made according to (11). At optimum, the quantity of resource received in case of shortage is increasing in the risk taking level and decreasing in risk tolerance. From (19), we verify that the variance of the risk born by agent i is increasing in relative risk tolerance and thus respects efficient risk sharing principle (Borch [4]). We obtain a different result from Gollier [15], because we have a systemic risk. In our case, the sharing rule has no impact on the optimal social risk taking level.

Back to the water management context, this first best solution corresponds to the situation where the water manager is both in charge of sharing the river flows and managing the self-insurance strategies of the irrigants. This is the case when the irrigants need an administrative autorization in order to build on-farm storage or when they ask for public fundings. Self-insurance levels can thus be controled by the regulator. The next section presents the case of a regulator only responsible for the water allocation. Because he has no control over the risk taking level and he is constrained by the individual decentralized decisions, this solution constitutes a second-best.

2.2 Second best: optimal risk sharing when risk taking decisions are decentralized

Let us assume now that the resource manager is only responsible for the resource allocation in case of scarcity (θ_i, θ_j) . Agents' self-insurance decisions are assumed to be decentralized, so (R_i, R_j) is not controled by the regulator. In this case, we are interested in what would be an optimal sharing rule for the resource manager, knowing this rule will impact the risk-taking decisions of the agents.

We consider a vast array of rules $\theta_i(x, \{R_j\})$ where the individual quantity received in the bad state may depend on the amount of resource available x, and eventually on the vector of requests (R_i, R_j). The agents and the regulator play a Stackelberg game where the regulator is the leader and the agent the follower. The regulator anounces the sharing rule. The agent thus knows perfectly the amount of resource that she will receive in each state. The regulator choose the optimal sharing rule such that the decentralized risk taking strategies of the agents lead to the first best level of risk determined in (10).

The condition for the announced sharing rule to implement the first best risk taking level is that the allocation is independent of the risk taking decisions. Complete proof is given in Appendix B. The following lines propose the resolution of the Stackelberg game.

The equilibrium risk taking level $\bar{R}_i$ is the solution of the maximization of the agent's certainty equivalent (see (18)). At equilibrium, each agent requests at least what she will receive in the bad state θ_i. The additional request depends positively on the probability of the good state p, the cost of self insurance c, the effect of her request upon her share $\frac{\partial \theta_i}{\partial R_i}$ and her risk tolerance T_i.

$$\bar{R}_i = T_i \, \frac{p - (1-c) + (1-p)\frac{\partial \theta_i(x,\{R_j\})}{\partial R_i}}{p(1-p)\left[1 - \frac{\partial \theta_i(x,\{R_j\})}{\partial R_i}\right]} + \theta_i(x,\{R_j\})$$

The regulator chooses the optimal sharing rule such that the decentralized risk taking strategies of the agents lead to the first best level of risk determined in (10). A sufficient condition for $\bar{R}_i + \bar{R}_j = (R_i + R_j)^{**}$ is $\frac{\partial \theta_i^*}{\partial R_i} = 0 \; \forall i$ and $\sum \theta_i^* = x$.

Any announced sharing rule, perfectly known by the agents, and independent of the risk taking decisions, implements optimal risk taking level. Such a rule creates no interactions with the preferences of EU maximizer risk averse agents. When agents can choose the level of risk they bear, the regulator does not need to take into account risk taking decisions or risk tolerance. First-best risk taking level is obtained by letting agents choose the level of risk corresponding to their risk aversion level without need of taking it into account in the resource allocation rule.

Despite this solution is the second best, the social welfare is not reduced compare to the first best. Indeed, the social welfare at optimum only depends on $(R_i + R_j)$ from (15). Because θ_i^* is defined such that the first best risk taking level is reached, social welfare under second best is equal to social welfare under first-best. Moreover, as for the first best, the variance of the risk born by agent i is increasing in its risk tolerance.

2.3 Distance from efficiency of empirical rules

The efficiency of different empirical rules is adressed in this section. >From our observation of water sharing rules, we retain three broad categories: egalitarian rule, shortage sharing rule and proportional rule. We also would like to test the efficiency of the first-best sharing rule when played in a Stackelberg game. This rule appears to be a special case of the shortage sharing rule.

2.3.1 Egalitarian rule

Under egalitarian sharing rule, each agent will receive an equal share of the resource available from the CPR. In the bad state, each subject receives $min(R_i;\theta_i)$ where $\theta_i = \frac{x}{n}$. In practice, we observe egalitarian sharing when left bank irrigators can use water odd-day and right bank farmers even-day. Ceteris paribus, everyone receives the same quantity of resource from the river. Because $\frac{\partial \theta_i^{egalitarian}}{\partial R_i} = 0$, this rule will implement first-best risk taking level. The equilibrium request under egalitarian rule is $\bar{R}_i^{egaliratian} = T_i \, \frac{p-(1-c)}{p(1-p)} + a_i x$. We easily verify that this leads to the first best risk taking level at the group scale. The maximum social welfare is reached.

2.3.2 Proportional rule

Under proportional rule, an irrigator receives a share of the available resource proportional to its request from the CPR. We observe proportional sharing when volumes distributed are proportions of the subscribed quotas or irrigated land. In Spain for example, the proportional allocation doctrine prevails, guaranting all irrigators the same volume per irrigated hectare [7]. In the bad state, the agent that requests R_i will get $\theta_i = x\frac{Ri}{\Sigma Ri}$. Because the total volume available is limited, the choice of one agent influences the outcomes for the others. There are strategic interactions.

This rule does not implement first-best risk taking level $\left(\frac{\partial\theta_i^{proportional}}{\partial R_i} \neq 0 \; \forall i, j\right)$. The Nash equilibrium solution can't be explicitely defined. As a result we do not measure the distance from efficiency of this rule. In any case, this result of inefficiency of the propotional rule is striking. Because the proportional rule gives more to the one who faces more risks, we could think it respects efficient risk sharing principle. Faysse [13] shows that any rule that allocates water ex-post according to needs, as does the proportional rule, both maximizes total value of water use and shares risk efficiently. But here, the introduction of self-insurance gives the agents the opportunity to manage their vulnerability to scarcity risk. This modifies the efficient risk management principles and leads to the inefficiency of the proportional rule.

2.3.3 Shortage sharing rule

When the resource available is scarce, restrictions are set up. The allocation of the resource can be such that the missing quantities are shared between agents, according to a coefficient. The shortage sharing rule translates this idea: $\theta_i^{shortage} = R_i - a_i(R_j + R_i - x)$, with $a_i + a_j = 1$. Each agent receives less than her request, the difference being a proportion of the total missing quantities. The total missing quantities are defined by the total demand minus the total quantity available. In the fields, we observe this rule when irrigation's bans are applied indifferently to all farmers, but farmers differ

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[7] There are records of districts and basins prioritising certain crops and certain farmers, but this is the exception rather than the rule.

in their pumping equipment. As a result, the farmers with the biggest equipments can pump more water when it's authorized. They will thus receive more water even if the restriction is the same for all the farmers. Here, we assume that the farmers that invest the more in pumping equipment are the ones that rely more on the river. In this case, R_i is also a measure of the dimension of the pumping equipment, and $a_i(R_j + R_i - x)$ is the restriction applied to each farmer, with $\sum a_i = 1$. If restrictions are homogeneous, $a_i = \frac{1}{n}$.

This rule does not implement first-best risk taking level $\left(\frac{\partial \theta_i^{shortage}}{\partial R_i} \neq 0 \; \forall i, j\right)$. Nevertheless, for a particular specification of $\{a_i\}$, this rule is the first best sharing rule.

Special case: First best sharing rule

The first-best sharing rule is a special case of the shortage sharing rule where $a_i = \frac{T_i}{T_j + T_i}$ and $\theta_i^{FB} = R_i - \frac{T_i}{T_j + T_i}(R_j + R_i - x)$ (see Appendix C (20)).

Because $\frac{\partial \theta_i^{FB}}{\partial R_i} \neq 0 \; \forall i, j$, the first-best sharing rule does not implement optimal risk taking. When risk taking decisions are decentralized, the strategic interactions amount in non optimal risk taking level.

The distance from the first best risk taking level is measured by $\overline{R_j + R_i}^{FB} - (R_i + R_j)^{**}$. Complete calculations are presented in Appendix C. We cannot compute individual Nash equilibrium but obtain an aggregate nash equilibrium. For $T_i \neq T_j$ we have corner solutions with $R_i = H$ or $R_j = H$. But the aggregate nash equilibrium $\overline{R_j + R_i}^{FB}$ does not depend upon the heterogeneity between T_i and T_j. If we define agent i as the more risk tolerant $T_i \geq T_j$,

$$\overline{R_j + R_i}^{FB} \quad -(R_i + R_j)^{**} = \quad \frac{c}{p\,(1-p)}\frac{T_j}{T_i}(T_i + T_j) > 0$$

The first-best sharing rule, when introduced in a Stackelberg game, leads to higher than optimal risk taking at the group level. It creates substitution effects and self-insurance is decreased compare to the optimal level. This inefficiency decreases the heterogeneity of risk tolerance $\frac{T_j}{T_i}$ (if i is the more risk tolerant). On the contrary, it increases with the group risk tolerance $(T_j + T_i)$, and with the cost of self-insurance c, and with the probability of the good state p. Of course, a non optimal risk taking level decreases social welfare.

This result means that if the regulator is not able to control risk taking strategies of the agent, he should not use the first best sharing rule. This rule does not maximize the social welfare under risk aversion. This rule will be efficient only if the conditions defined by the parameters tend to restrain the risk taking behaviors: cost of self-insurance, probability of good state and total group risk tolerance are very low.

3 Discussion

The main contributions of this paper are presented. Then, some recommendations for the French water policy reform are drawn.

3.1 Main contributions

We defined the socially efficient level of risk and the efficient sharing of this risk in order to minimize the social cost of a drought, when vulnerability to drought risk is endogeneous. The instrument in the hands of the regulator to share risks is the administrative allocation of the resource in case of scarcity; in this case, the water restrictions. We study more precisely the interactions between this instrument and the self-insurance strategies of risk-averse agents. Because the social cost of drought has several components (risk premium, direct cost of self-insurance and opportunity cost), we have adapted the efficient risk management principles to this setting.

The first contribution of this work is the originality of the proposed drought management scheme. In most of the existing literature in agricultural and development economics, the solution against the negative impacts of drought are either private strategies or insurance strategies. Introducing risk sharing mechanisms at the basin level, through water allocation schemes is an interesting, complementary solution. But because mechanisms generate interactions with self-insurance strategies, the efficiency of such a mechanism is not strait-forward. We have shown that individual risk taking strategies depend on the allocation rule. We have also demonstrated that first-best sharing rules are no longer optimal when the regulator has no control on individual risk taking strategies.

The second contribution of this paper is to state the importance of considering both optimal risk taking and efficient risk sharing when dealing with risk management. In our setting with a risky CPR, efficient risk management requires the two criteria be satisfied. First, the level of risk taken by the group must be optimal, i.e. must depend on the agregate risk tolerance and the quantity of resource available from the CPR in the bad state (10). Second, any sharing of this total risk between agents is optimal as soon as the quantity received by an agent, when restrictions are organized, depends on the individual risk taking level (11).

3.2 Recommendations for the French Water Policy Reform

We draw from the theoretical results some recommendations for the French water policy ongoing reform.

The organizational problem of the new agency (Organisme Unique) is to determine sharing rules that define a priori how much of the scarce water is allocated to each irrigator, for each possible state of nature. In order to reduce the social cost of drought, we have shown that the water manager must first determine if he controls self-insurance strategies or not. We demonstrate that the regulator can easily reach the maximal social welfare without controling (R_i, R_j). Any simple rule, like the egalitarian rule, is optimal as soon as irrigants are totally free and able to manage their risk taking level (by constructing on-farm storage), and are able to anticipate the volumes they will receive in case of restrictions. The main advantage is that the regulator does not need information on individual self-insurance strategies and risk tolerance. The French water manager is actually doing well when irrigators are equally restricted, and can irrigate either on odd-days or even-days. The main failure of the actual system is that irrigators have no exact anticipation on the volumes they will receive in case of drought. The main recommendation is thus to improve the communication of the water manager toward the irrigators, such that they can know the exact contingent allocation of water.

We have shown that the simpler the allocation, the better. Of course, this results holds only under the assumption that farmers are both able to determine the best level of self-insurance for them and finance it.

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Appendix A

From the Arrow-Pratt and Taylor approximations, we know that the expected utility writes as the utility of the certainty equivalent. As a result, the maximization of the expected utility is equivalent to the maximization of the certainty equivalent when U_i is increasing in CE_i.

$$\begin{aligned} & EU_i\left((1-c)(H-R_i)+\theta_i(\tilde{x})\right) \\ = \; & U_i\left(CE_i\right) \\ = \; & U_i\left((1-c)(H-R_i)+pR_i+(1-p)\theta_i(\underline{x})-\frac{1}{2T_i}p\,(1-p)[R_i-\theta_i(\underline{x})]^2\right) \end{aligned}$$

The first best is the solution of the maximization of the weighted sum of the certainty equivalents, which leads to the Lagrangian under constraints (1), (2) and (3). The lagrangian of the program writes as follow where μ is the lagrangian multiplier associated to the sharing rule constraint, η the lagrangian multiplier associated to the shortage constraint. The support of the solutions is $0 \leq R_i \leq H$ and $\theta_i > 0$ for i and j.

$$\begin{aligned}
\mathcal{L} &= \sum_{i,j} \lambda_i \left[(1-c)\,(H-R_i) + p\,R_i \ + (1-p)\theta_i - \frac{1}{2T_i} p\,(1-p)[R_i-\theta_i]^2 \right] \\
&- \mu \left[x - \sum \theta_i \right] \\
&- \eta \left[\sum \theta_i - \sum R_i \right]
\end{aligned}$$

Suppose that the constrained maximum for this problem is obtained for $\theta_i = \theta_i^{**}$ and $R_i = R_i^{**}\ \forall i, j$. Then there are different cases to consider. At $(\theta_i^{**}, R_i^{**})$ the shortage constraint (2) is either inactive ($\sum \theta_i^{**} < \sum R_i^{**}$) or active ($\sum \theta_i^{**} = \sum R_i^{**}$). The interesting case for drought management is when constraint (2) is not binding, thus $\eta = 0$.

Interior solutions $R_i < H,\ R_j < H$

The first order conditions (FOC) are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial R_i} &= \lambda_i \left[(p-(1-c)) - \frac{1}{T_i} p (1-p) [R_i - \theta_i] \right] = 0 && (1) \\
\frac{\partial \mathcal{L}}{\partial R_j} &= \lambda_j \left[(p-(1-c)) - \frac{1}{T_j} p (1-p) [R_j - \theta_j] \right] = 0 && (2) \\
\frac{\partial \mathcal{L}}{\partial \theta_i} &= \lambda_i \left[(1-p) + \frac{1}{T_i} p (1-p) [R_i - \theta_i] \right] + \mu = 0 && (3) \\
\frac{\partial \mathcal{L}}{\partial \theta_j} &= \lambda_j \left[(1-p) + \frac{1}{T_j} p (1-p) [R_j - \theta_j] \right] + \mu = 0 && (4) \\
\frac{\partial \mathcal{L}}{\partial \mu} &= x - \theta_i - \theta_j = 0 && (5)
\end{aligned}$$

>From (1):

$$(p-(1-c)) = \frac{1}{T_i} p (1-p) [R_i - \theta_i]$$

$$R_i - \theta_i = T_i \frac{(p-(1-c))}{p(1-p)}$$

>From (2):

$$R_j - \theta_j = T_j \frac{(p-(1-c))}{p(1-p)}$$

>From (4):

$$(1-p) + \frac{1}{T_j} p (1-p) [R_j - \theta_j] = -\frac{\mu}{\lambda_j}$$

$$R_j - \theta_j = \frac{T_j}{p}\left[-\frac{\mu}{\lambda_j(1-p)} - 1\right] = -\frac{T_j}{p}\left[\frac{\mu + \lambda_j(1-p)}{\lambda_j(1-p)}\right]$$

>From (3):

$$\begin{aligned}
\mu &= -\lambda_i\left[(1-p) + \frac{1}{T_i}p\,(1-p)\,[R_i - \theta_i]\right] && (6)\\
&= -\lambda_i\left[(1-p) + \frac{1}{T_i}p\,(1-p)\left[T_i\frac{(p-(1-c))}{p\,(1-p)}\right]\right] && (7)\\
&= -\lambda_i\,[(1-p) + p - \,(1-c)] && (8)\\
&= -\lambda_i c && (9)
\end{aligned}$$

$$R_j - \theta_j = T_j\frac{(p-1+c)}{p(1-p)} = T_j\left[\frac{\lambda_i c - \lambda_j(1-p)}{\lambda_j p(1-p)}\right] = T_j\left[\frac{(p-1) + \frac{\lambda_i}{\lambda_j}c}{p(1-p)}\right]$$

We have an interior solution only if $\lambda_i = \lambda_j$

Since $\theta_i + \theta_j = x$ from constraint (1),

$$R_j - \theta_j + R_i - \theta_i = (T_j + T_i)\frac{(p-1+c)}{p(1-p)}$$

The interior first best solution is defined such that these three conditions hold:

$$\begin{aligned}
(R_j + R_i)^{**} &= x + (T_j + T_i)\frac{p - (1 - c)}{p\,(1 - p)} && (10)\\
\theta_i^{**} &= R_i^{**} - T_i\frac{(p - (1 - c))}{p\,(1 - p)}\ \forall i, j && (11)\\
\lambda_i &= \lambda_j && (12)
\end{aligned}$$

The problem admits an interior solution only for equal weights.

Asymetric Corner solutions

$R_j = H$ is a solution if $\lambda_i < \lambda_j$.

Indeed, the following condition holds only if $\lambda_i < \lambda_j$:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial R_j} &= \lambda_j\left[(p - (1 - c)) - \frac{1}{T_j}p\,(1 - p)\,[R_j - \theta_j]\right] > 0 && (13)\\
&\frac{T_j}{p}\frac{(p - (1 - c))}{(1 - p)} > \frac{T_j}{p}\left[\frac{(p - 1) + \frac{\lambda_i}{\lambda_j}c}{(1 - p)}\right] && (14)
\end{aligned}$$

In this case,

$$
\begin{aligned}
R_j^{**} &= H \\
R_i^{**} &= T_i\frac{(p-1+c)}{p\,(1-p)} + T_j\left[\frac{(p-1)+\frac{\lambda_i}{\lambda_j}c}{p(1-p)}\right] + x - H \\
\theta_j^{**} &= H - T_j\left[\frac{(p-1)+\frac{\lambda_i}{\lambda_j}c}{p(1-p)}\right] \\
\theta_i^{**} &= R_i^{**} - T_i\frac{(p-1+c)}{p\,(1-p)} = T_j\left[\frac{(p-1)+\frac{\lambda_i}{\lambda_j}c}{p(1-p)}\right] + x - H
\end{aligned}
$$

The other possible solution when $\lambda_i < \lambda_j$ is :

$$R_i = 0 \text{ and } R_j^{**} = T_j\tfrac{(p-1+c)}{p\,(1-p)} + T_i\left[\tfrac{(p-1)+\frac{\lambda_i}{\lambda_j}c}{p(1-p)}\right] + x$$

This is not an equilibrium because θ_i^{**} would be negative.

.

When $\lambda_i > \lambda_j$, the equilibrium is :

$$
\begin{aligned}
R_i^{**} &= H \\
R_j^{**} &= T_j\frac{(p-1+c)}{p\,(1-p)} + T_i\left[\frac{(p-1)+\frac{\lambda_i}{\lambda_j}c}{p(1-p)}\right] + x - H \\
\theta_i^{**} &= H - T_i\left[\frac{(p-1)+\frac{\lambda_i}{\lambda_j}c}{p(1-p)}\right] \\
\theta_j^{**} &= R_j^{**} - T_j\frac{(p-1+c)}{p\,(1-p)} = T_i\left[\frac{(p-1)+\frac{\lambda_i}{\lambda_j}c}{p(1-p)}\right] + x - H
\end{aligned}
$$

For the same reasons than in the previous case, the following solution is not an solution :

$$R_j = 0 \text{ and } R_i^{**} = T_j \tfrac{(p-1+c)}{p\,(1-p)} + T_i \left[\tfrac{(p-1)+\frac{\lambda_i}{\lambda_j}c}{p(1-p)} \right] + x$$

When the regulator chooses to favor one agent, the optimal solution is that this agent takes the maximum level of risk. The optimal level of risk for the less favoured agent is such that the total risk level reaches the first-best.

Symetric corner solutions

High equilibrium >From the case above, we know that both FOC according to R_i and R_j will be positive if $\lambda_i > \lambda_j$ and $\lambda_i < \lambda_j$. Hence, the upper bound solution $R_i = R_j = H$ is an equilibrium only if :

$$\begin{aligned} \lambda_i &= \lambda_j \\ 2H - x &= (T_j + T_i)\frac{(p-1+c)}{p(1-p)} \end{aligned}$$

We will not consider this set of parameter to avoid corner solutions.

Low equilibrium Both FOC according to R_i and R_j will be negative if:

$$\begin{aligned} \lambda_i &= \lambda_j \\ -x &= (T_j + T_i)\frac{(p-1+c)}{p(1-p)} \end{aligned}$$

This is not possible under the conditions $p > 1 - c$.

Summary: first-best solutions

To sum-up, if $\lambda_i = \lambda_j$ we have an interior solution defined by (10) and (11). If $\lambda_i \neq \lambda_j$, the optimal solution is such that the favoured agent takes the maximum level of risk H. The optimal level of risk for the less favoured agent is such that the first best total risk level (10) is reached.

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In the case of interior solution where $\lambda_i = \lambda_j = \lambda$, we easily verify in that $(R_j + R_i)$ is the only variable at the optimal point when weights are equal.

$$\begin{aligned}\lambda_i CE_i(R_i, \theta_i^{**}) + \lambda_j CE_j(R_j, \theta_j^{**}) &= \lambda \left[\left(p - (1-c)\right) \left(R_i + R_j\right) + (1-p)x + 2(1-c)H \right] \\ &- \frac{1}{2}\lambda \left[\left(T_i + T_j\right) \frac{\left(p - (1-c)\right)^2}{p\,(1-p)} \right] \qquad (15)\end{aligned}$$

Under such optimal conditions, the variance of the risk born by agent i is:

$$\sigma^2 = p\,(1-p) \left[T_i \frac{p - (1-c)}{p(1-p)} \right]^2 = p\,(1-p) \left[R_i - \frac{T_i}{T_j + T_i} (R_i + R_j - x) \right]^2 \qquad (16)$$

Appendix B

The Stackelberg game has two steps. In the first step, the agent chooses her request $\bar{R}_i$, to maximize her certainty equivalent. We search the solutions such that $0 \bar{\leq} R_i \leq H$

$$CE_i = (1-c)\,(H-R_i)+p\,R_i\ +(1-p)\theta_i(x,\{R_j\})-\frac{1}{2T_i}p\,(1-p)\left[R_i-\theta_i(x,\{R_j\})\right]^2 \quad (17)$$

The FOC leads to the equilibrium :

$$\bar{R}_i = T_i\,\frac{p-(1-c)+(1-p)\frac{\partial\theta_i(x,\{R_j\})}{\partial R_i}}{p(1-p)\left[1-\frac{\partial\theta_i(x,\{R_j\})}{\partial R_i}\right]} + \theta_i(x,\{R_j\}) \quad (18)$$

In step 2, the regulator determines θ_i^* such that the equilibrium risk taking is optimal.

$$\bar{R}_i + \bar{R}_j \;=\; T_i\,\frac{p-(1-c)+(1-p)\frac{\partial\theta_i}{\partial R_i}}{p(1-p)\left[1-\frac{\partial\theta_i}{\partial R_i}\right]} + T_j\,\frac{p-(1-c)+(1-p)\frac{\partial\theta_j}{\partial R_j}}{p(1-p)\left[1-\frac{\partial\theta_j}{\partial R_j}\right]} + \theta_i + \theta_j$$

We recall that $(R_j+R_i)^{**} = x + (T_j+T_i)\frac{p-(1-c)}{p\,(1-p)}$

A sufficient condition for $\bar{R}_i + \bar{R}_j = (R_i+R_j)^{**}$ is $\frac{\partial\theta_i^*}{\partial R_i} = 0\ \forall i$ and $\theta_i^* + \theta_i^* = x$.

We verify that the variance of the risk born by agent i is increasing in risk tolerance:

$$\sigma^2 = p(1-p)\left[\bar{R}_i - \theta_i^*\right]^2 = p\,(1-p)\left[T_i\frac{p-(1-c)}{p(1-p)}\right]^2 \quad (19)$$

Appendix C

We show that the first best sharing rule is a special case of the shortage sharing rule.

>From (1), (2) and constraint (1):

$$\theta_i = R_i - \frac{T_i}{T_j}\left[R_j - \theta_j\right] = x - \theta_j$$

Thus,

$$\theta_i = R_i - \frac{T_i}{T_i + T_j}\left[R_i + R_j - x\right] \tag{20}$$

This is the shortage sharing rule for $a_i = \frac{T_i}{T_j + T_i}$.

In order to measure the distance from efficiency of the first best sharing rule, we first determine the reaction of the follower to this rule ($R_i^{\bar{F}B}$). Because the rule creates strategic interactions, we compute the Nash equilibrium. Each agent maximizes her certainty equivalent with respect to R_i. We solve for 2 agents under the constraint that $0 \leq R_i \leq H$ for i,j .

$$\begin{aligned} CE_i &= (1-c)\,(H - R_i) + p\,R_i \; + (1-p)\left(R_i - \frac{T_i}{T_j + T_i}(R_j + R_i - x)\right) \\ &- \frac{1}{2T_i}p\,(1-p)\left[\frac{T_i}{T_j + T_i}(R_j + R_i - x)\right]^2 \end{aligned}$$

$$\frac{\partial CE_i}{\partial R_i} = p - (1-c)\; + (1-p)\frac{T_j}{T_j + T_i} - \frac{1}{T_i}p\,(1-p)\left(\frac{T_i}{T_j + T_i}\right)^2\left[R_j + R_i - x\right] \tag{21}$$

Interior solutions

The interior solutions are such that $\frac{\partial CE_i}{\partial R_i} = 0$ for i,j. We obtain an aggregate Nash equilibrium. It tells us nothing about how this risk taking is shared between the two agents.

$$\begin{aligned} \overline{R_j + R_i}^{FB} &= x + T_i \frac{p - (1-c) + (1-p)\frac{T_j}{T_j+T_i}}{p\,(1-p)\left(\frac{T_i}{T_j+T_i}\right)^2} && (22) \\ &= x + T_j \frac{p - (1-c) + (1-p)\frac{T_i}{T_j+T_i}}{p\,(1-p)\left(\frac{T_j}{T_j+T_i}\right)^2} && (23) \end{aligned}$$

This constitutes a Nash equilibrium only if both terms (22) and (23) are equals. The necessary condition to have equality is:

$$[p - 1 + c]\left[T_i^2 - T_j^2\right] = [p-1]\left[T_i^2 - T_j^2\right]$$

The necessary condition to have the equality between both terms is $T_i = T_j$ or $c = 0$. Under the assumption $c > 0$, and $T_i = T_j = T$, the equilibrium is :

$$\overline{R_j + R_i}^{FB} = x + T\frac{4(p-1+c) + 2(1-p)}{p\,(1-p)} \tag{24}$$

Asymetric corner solutions $R_i = H$ or $R_j = H$

$\bar{R}_j = H$ is a corner solution if $\frac{\partial CE_i}{\partial R_i} = 0$ and $\frac{\partial CE_j}{\partial R_j} > 0$. It requires:

$$\begin{aligned}
x+T_j\frac{p-(1-c)\,+(1-p)\frac{T_i}{T_j+T_i}}{p\,(1-p)\left(\frac{T_j}{T_j+T_i}\right)^2} &> x+T_i\frac{p-(1-c)\,+(1-p)\frac{T_j}{T_j+T_i}}{p\,(1-p)\left(\frac{T_i}{T_j+T_i}\right)^2}\\
[p-(1-c)]\left[T_i^2-T_j^2\right] &> [p-1]\left[T_i^2-T_j^2\right]\\
T_i &> T_j\ for\ c>0\ and\ p>1-c
\end{aligned}$$

In this case, $\bar{R}_i = x - H + T_i\frac{p-(1-c)\,+(1-p)\frac{T_j}{T_j+T_i}}{p\,(1-p)\left(\frac{T_i}{T_j+T_i}\right)^2}$

If $T_i < T_j$, we obtain the opposite solution $\bar{R}_i = H$ and $\bar{R}_j = x - H + T_j\frac{p-(1-c)\,+(1-p)\frac{T_i}{T_j+T_i}}{p\,(1-p)\left(\frac{T_j}{T_j+T_i}\right)^2}$

The corner solutions $\bar{R}_i = 0$ and $\bar{R}_j = 0$ are not equilibrium solutions because we would have $\theta_i = \bar{R}_i - \frac{T_i}{T_j+T_i}(\bar{R}_j + \bar{R}_i - x) < 0$ and $\theta_j = \bar{R}_j - \frac{T_j}{T_j+T_i}(\bar{R}_j + \bar{R}_i - x) < 0$.

Symetric corner solutions

High equilibrium $\bar{R}_i = \bar{R}_j = H$ if

$$\begin{aligned}
p-(1-c)\,+(1-p)\left(1-\frac{T_i}{T_i+T_j}\right)-\frac{1}{T_i}p\,(1-p)\left(\frac{T_i}{T_i+T_j}\right)^2(2H-x) &> 0\\
p-(1-c)\,+(1-p)\left(1-\frac{T_j}{T_i+T_j}\right)-\frac{1}{T_j}p\,(1-p)\left(\frac{T_j}{T_i+T_j}\right)^2(2H-x) &> 0
\end{aligned}$$

We assume the parmeters are not such that these two conditions holds. Hence, the upper corner solution where both agents ask H units into the CPR is not an equilibrium.

Low equilibrium $\bar{R}_i = \bar{R}_j = 0$ if

$$\begin{aligned} p-(1-c)\ +(1-p)\left(1-\frac{T_i}{T_i+T_j}\right)+x\frac{1}{T_i}p\,(1-p)\left(\frac{T_i}{T_i+T_j}\right)^2 &< 0 \\ p-(1-c)\ +(1-p)\left(1-\frac{T_j}{T_i+T_j}\right)+x\frac{1}{T_j}p\,(1-p)\left(\frac{T_j}{T_i+T_j}\right)^2 &< 0 \end{aligned}$$

For $p > 1-c$, this is not an equilibrium because both conditions does not hold.

Summary: equilibriums with the first-best sharing rule

To sum-up, if $T_i = T_j$, we have an interior solution defined by (**??**). If $T_i > T_j$, $0 < R_i < H$ and $R_j = H$. If $T_i < T_j$, $0 < R_j < H$ and $R_i = H$. In any case, the more risk tolerant takes less risk.

Distance from first best risk taking level

Because the first best total risk taking level does not depend upon (λ_i, λ_j), the distance from this first best does not depends on the solution we consider (interior or corner). Equivalently, the aggregate equilibrium when the sharing rule is the first best sharing rule does not depend on (T_i, T_j), so any equilibrium solution will have the same distance from efficiency.

$$\begin{aligned}\overline{R_j + R_i}^{FB} \quad -(R_i + R_j)^{**} = &\quad T_i \frac{p - (1-c) \; + (1-p)\frac{T_j}{T_j + T_i}}{p\,(1-p)\left(\frac{T_i}{T_j+T_i}\right)^2} - \frac{p-(1-c)}{p(1-p)}\,(T_i + T_j) \\ &= \frac{c}{p\,(1-p)}\frac{T_j}{T_i}(T_i + T_j) > 0 \qquad (25)\end{aligned}$$