Small is Beautiful?
Firm's Size, Prevention and Food Safety.

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Abstract

The European General Food Law of 2005 and the newly promulgated FDA Food Safety Modernization Act (FFSMA) of 2010 ask all food operators to implement preventive efforts. In this article, we explore the link between firm's size and preventive efforts. We show two main results. First, when there is no cross-contamination, small firms will provide higher preventive efforts than large firms. When there is cross-contamination, the effort-size curve may have a "inverted-U" shape. From our results we can argue that when implementing or enforcing food safety regulations, public authorities should consider both firms size and food safety hazard.

Keywords: prevention effort, firm size, food safety

JEL codes: Q18; L15; L5.

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1   Introduction

Food safety crisis have led governments to increase the responsibility of food operators in handling and preventing food safety hazards. In Europe, the General Food Law, promulgated in 2002 and enforced in 2005, has highlighted the need for food operators to implement food safety hazard control plans and to better monitor the safety of their inputs and outputs. In other words, European food operators have been encouraged to implement voluntary self-control on their operations. The same trend appears in the US, with the FDA Food Safety Modernization Act (FFSMA) of 2010 that amends the Food, Drug, and Cosmetic Act of 1938. The Act asks all food operators to identify and implement preventive effort to minimize and prevent from food safety hazards. As a consequence in a great part of the world, firms must both implement preventive effort and comply with performance standards (e.g., pathogen counts for products at some stage of the supply chain). With performance standards public authorities require food operators to achieve prescribed product quality standards but the means to reach the standard are not specified by the authorities (Henson and Caswell, 1999).

The scope and enforcement of those two legislations are quite identical on the extent of preventive effort. However, whereas the European General Food Law applies uniformly to all food operators, the FFSMA develops specific considerations concerning food operators' characteristics. The FFSMA takes into account of some characteristics of operators: Processing sector or farm producers, small or large food operators, local or faraway distribution etc. As a result, those considerations mostly result in exempting small businesses from the FFSMA requirements (Ribera and Knutson, 2011). Those exemptions were at the core of a contentious debate during the legislative process (Koenig, 2011). Critics argue that exceptions should better be based on a scientific risk analysis and not only on firm's production or distribution practices. Koenig (2011) reports that opponents underline that those exemptions would leave significant gaps in the food safety system.

In this article, we highlight the link between firm's size and preventive effort. We argue that this link depends on the type of safety hazards. Food safety hazards are contaminants that may cause a food product to be unsafe. There are three common types of hazards. Biological hazards that include bacteria, viruses and parasites. Microbial foodborne illness are caused by eating food contaminated with specific types of microorganisms (e.g. salmonella). Chemical hazards like pesticides, food additives and preservatives that can cause toxic effects, either immediate or long term. Physical hazards are objects that could accidentally get into food (broken glass, hair etc.). We can also consider food safety hazards as regard contamination. Once units are adultared, either there might be cross-contamination between

products (e.g. biological hazard) or cross-contamination is not possible (chemical, physical hazard e.g.). In this framework, we focus on the decision of one firm that chooses its level of preventive effort in order to avoid safety defect. We show two main results. When there is no cross-contamination, small firms provide higher preventive effort than large firms. When there is cross-contamination, results are altered. Large *and* small firms may provide lower preventive effort than medium firms (the effort-size curve may have a "inverted-U" shape).

To our knowledge there is little discussion in the literature on the existing link between firms size and preventive effort as defined in the seminal work by Ehrlich and Becker (1972). Ehrlich and Becker (1972) define that preventive effort have for main characteristic to decrease the probability of a bad event.[1] Cho and Hooker (2007) provide a game theoretic framework that takes into account food operators which might differ in cost. They deal with the efficient regulatory instrument (voluntary or mandatory) to maintain the level of food safety in the market. Similarly, based on empirical evidence Colatore and Caswell (1999) have established that in the seafood industry food safety effort can vary by firm type. Rouvière et al. (2010) show that small firms would implement higher safety effort than large firms in the French fresh produce imports industry. Pouliot (2011) compares a situation with and without an exemption for small firms to comply with FFSMA. He shows that when competing on the same product small firms would benefit from the exemption against large firms that must comply with the law. None of these researches theoretically highlight how industry characteristics influence the link between firms size and preventive effort. A whole strand of the decision theory literature deals with prevention. Recent papers include Dachraoui et al. (2004), Eeckhoudt and Gollier (2005) and Menegatti (2009). This literature focuses on the effect of variations in the utility function of the agent on his prevention effort. None of these articles has considered the role of the number of times the agent faces a risk (the number of units in the context of the present article). In our model, we consider the effect of the firm's size on its preventive effort.

Our results suggest that when implementing or enforcing food safety regulations, public authorities should consider both firms' size and industry characteristics. For instance, our result could support a distinguished definition of small firms in the discussion for the FDA Food Safety Modernization Act (FFSMA).

The paper is organized as follows. Section 1 presents the model. Section 2 sets out our two theoretical propositions. In Section 3, we discuss the exemptions provided by the newly promulgated FDA Food Safety Modernization Act (FFSMA) and conclude.

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[1]Note that prevention or self-protection must be distinguished of self-insurance. Self-insurance has for main characteristic to reduce damages incurred follownig a bad event.

2   Model Specification

We consider a food processor who has a capacity of production of n homogeneous units he wants to market. The safety of the n units of output is determined by the safety of the inputs, of the production process and by the preventive effort the food operator makes (Starbird and Anomor-Boadou, 2006). Thus, each unit of product has a probability to be alduterated and then may lead to a food safety outbreak. Firms implement preventive effort in order to decrease this probability as defined by Ehrlich and Becker (1972).

In the following, we will consider the two polar cases as regard cross-contamination between units. First, we develop the case of no cross-contamination - for instance all food hazards related to chemical or physical contamination (e.g. pesticides residues) - . In this particular case the occurrence of a product defect (a given unit of product found to be unsafe) is independent across units. Second, we analyze the case of full cross-contamination - for instance all bacterial contaminations (e.g. salmonella contamination) - . In there, the occurrence of a product defect in one unit leads to a product defect in all units.

The food operator can sell unsafe units even he has implemented preventive effort. In such a case, there is a probability that a public authority, one of the downstream operators (thereafter a "third party") detects a product defect. Full traceability is mandatory: when a third party detects an unsafe unit, the unit is destroyed and the food operator will receive no benefit.

Setting

For each of the units the firm produces, the scenario is the following: (1) For one unit, there is some probability, $\alpha \in (0,1)$, that the unit is safe. This probability depends on the preventive effort, $e \geq 0$, the firms makes. The probability $\alpha(e)$ depends positively on the effort e with $\alpha'(e) > 0$. In other words, the greater the effort the higher the probability that the unit is safe. (2) Those units can be inspected by a third party. We define $u \in (0,1)$ as the probability that an unsafe unit is detected as unsafe by a third party. For sake of simplicity, the technology never incorrectly reports a safe unit to be unsafe. However, we consider that Type-II-error disruptions or false positive can occur increasing consumer's risk of illnesses. In other words, the detection technology is such as even when checks reveal no default this does not mean that the product checked is safe. The firm will receive a benefit for each unit without any detected safety failure. That is to say, for each unit the firm receives the unit price, p.[2] It will receive zero for each unit detected as unsafe by a third party. The firm

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[2]In other words, we assume that the firm has no market power.

faces costs of handling the n units, $C(n)$ and marketing costs for the k units he sells, $m(k)$ with $m(0) = 0$, $m' \geq 0$ and $m'' \geq 0$.

Let define b the probability (for each unit) that the firm receives a benefit. Our scenario allows us to write this probability as a function of the firm's preventive effort

$$b(e) = \alpha(e) + (1 - \alpha(e))(1 - u) \tag{1}$$

Consequently, this is the probability that the unit is either safe or unsafe and has not been detected by the third party. Rewrite (1): $b(e) = u \times \alpha(e) - (1 - u)$ and notice $b'(e) = u \times \alpha'(e) > 0$. In other words, the probability that the firm would receive a benefit increases with the preventive effort it makes. The total cost for making preventive effort is linear in the total effort the firm makes (ne) and $c > 0$ is the marginal cost of preventive effort. The total cost is thus cne.

Let define the expected profit of a food processor with a capacity of production of n units in the two polar cases of contamination we have defined above. In both cases, the food processor's objective is to maximize his expected profit with respect to his effort e. We assume that the expected profit is quasi-concave in the effort e and we focus on interior solutions.

Let $\pi(k)$ the (ex-post) profit for the k units he sells with $\pi(k) = pk - m(k)$ and $\pi' > 0$. Notice that $\pi(0) = -m(0) = 0$ and $\pi'' = -m'' \leq 0$.

No cross-contamination: Let k be a random variable for the number of units the food processor produces and which passed through all the steps of the supply chain without any detected safety failure, i.e. k is the number of units without any defect and for which the firm receives a benefit. In other words, k is the number of successes in a random experiment (repeated n times) with a binomial probability of success b. The probability that k units among n pass successfully through all the steps in the supply chain without any detected default is given by $\binom{n}{k}(b(e))^k(1 - b(e))^{n-k}$ where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

The food processor's objective is to maximize his expected profit with respect to his effort e:

$$E\Pi(n, e) = \sum_{k=0}^{n}\left[\binom{n}{k}(b(e))^k(1 - b(e))^{n-k}\pi(k)\right] - cne - C(n), \tag{2}$$

where $b(e) \in (0, 1)$.

To illustrate equation (2), let us calculate the expected profit of a firm who has a capacity of 2 units. With a probability $(1 - b(e))^2$, the two units are detected by the third party as unsafe and the firm obtains $\pi(0) = 0$. With probability $2b(e)(1 - b(e))$, the third party reports either the first unit as unsafe and the second unit as safe, or he reports the first unit

as safe and the second one as unsafe. In this situation, the firm obtains a profit $\pi(1)$ for the safe unit. With probability $(b(e))^2$ none of the two units is reported as unsafe by the third party and the firm obtains $\pi(2)$. Therefore, the expected profit of the firm who has a capacity of 2 units can be written: $E\Pi(2,e) = 2b(e)(1-b(e))\pi(1) + (b(e))^2\pi(2) - 2ce - C(2)$. Equation (2) is the generalization of this formula for a firm who has a capacity of n units.

Full cross-contamination: Because of the contamination between units: if one unit is unsafe, all the units will be unsafe. For the third party, if he detects one unit as unsafe he knows that all units will be unsafe. All units are thus destroyed and the firm's benefit equals to zero. In other words, the firm will obtain $\pi(n)$ with a probability $(b(e))^n$ and $\pi(0) = 0$ with a probability $1 - (b(e))^n$.

The food processor's objective is to maximize his expected profit with respect to his effort e:

$$E\Pi(n,e) = (b(e))^n \pi(n) - cne - C(n) \tag{3}$$

To illustrate equation (3), let us calculate the expected profit of a firm who has a capacity of 2 units. With a probability $(1-b(e))^2$, the two units are detected by the third party as unsafe and the firm obtains $\pi(0) = 0$. With probability $2b(e)(1-b(e))$, the third party reports either the first unit as unsafe and the second unit as safe, or he reports the first unit as safe and the second one as unsafe. In this situation, the firm obtains zero profit. With probability $(b(e))^2$ none of the two units are reported as unsafe by the third party and the firm obtains $\pi(2)$. Therefore, the expected profit of the firm who has a capacity of 2 units can be written: $E\Pi(2,e) = (b(e))^2 \pi(2) - 2ce - C(2)$. Equation (3) is the generalization of this formula for a firm who has a capacity of n units.

3 Food safety hazard and preventive effort

In this section, we develop our main results - under the general assumptions developed the above section - as regard cross-contamination.

Let define e_n^* the optimal effort when capacity is n. Formally,

$$e_n^* = \arg\max_{e \geq 0} [E\Pi(n,e)] \tag{4}$$

3.1 No cross-contamination

Proposition 1: The optimal preventive effort *decreases* with capacity $(e_{n+1}^* \leq e_n^*)$ when there is *no cross-contamination.*

Proposition 1 suggests that an increase in the number of units increases diversification of the risk. A firm who has a small capacity is in a more risky situation than a larger firm. If the profit is concave, then, the higher the level of risk diversification, the lower the preventive effort.

Let us illustrate this insurance effect with a simple example that involves two firms. Firm A has a capacity of one unit. He faces the following situation: either his unit passes successfully through all the steps in the supply chain without any detected default with a probability b and he receives $\pi(1)$ for profit, or, with a probability $(1-b)$ the unit is detected by a third party as being unsafe and he receives $\pi(0) = 0$. Firm B has a capacity of two units. He faces three potential situations: i) the two units pass all the steps in the supply chain and he receives $\pi(2)$ for profit; ii) the two units are detected as being unsafe and there is no profit; iii) only one of the two units is safe, which means he will receive $\pi(1)$. The potential for the third situation to occur provides an insurance effect.

The proof of Proposition 1 - under the general assumptions developed in Section 2 - is provided in Appendix 1.

For expositional ease, we illustrate our reasoning with a particular profit function such as $\pi(k) = pk - \frac{\gamma}{2}k^2$ with $\gamma > 0$ and $p > \gamma\left(n + \frac{1}{2}\right)$.[3] In the following, we write the profit function such as $\pi(k) = pk - \frac{\gamma}{2}k^2 = (p + \frac{\gamma}{2})k - \frac{\gamma}{2}k(k-1)$. We write the expected profit as (see Appendix C):

$$E\Pi(n, e) = \left(p + \frac{\gamma}{2}\right)nb(e) - \frac{\gamma}{2}n(n-1)(b(e))^2 - cne - C(n). \tag{5}$$

For an interior maximum, e_n^*, we must have,

$$\frac{\partial E\Pi}{\partial e}(n, e_n^*) = \left(p + \frac{\gamma}{2}\right)nb'(e_n^*) - \gamma n(n-1)b(e_n^*)b'(e_n^*) - cn = 0 \tag{6}$$

Also, the derivative of $E\Pi(n+1, e)$ with respect to e is,

$$\frac{\partial E\Pi}{\partial e}(n+1, e) = \left(p + \frac{\gamma}{2}\right)(n+1)b'(e) - \gamma(n+1)nb(e)b'(e) - c \times (n+1) \tag{7}$$

And, at e_n^*, this derivative is,

$$\frac{\partial E\Pi}{\partial e}(n+1, e_n^*) = \left(p + \frac{\gamma}{2}\right)(n+1)b'(e_n^*) - \gamma(n+1)nb(e_n^*)b'(e_n^*) - c \times (n+1). \tag{8}$$

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[3]This assumption ensures that π increases with the (integer) number of units n.

And, substituting (6) into (8), we find:

$$\frac{\partial E\Pi}{\partial e}\left(n+1, e_n^*\right) = -\gamma b\left(e_n^*\right) b'\left(e_n^*\right)(n+1) < 0. \tag{9}$$

Since the expected profit is quasi-concave in e, and e_n^* is on the increasing side of expected profit for capacity $n+1$, we can conclude that $e_{n+1}^* < e_n^*$.

Figure 1 illustrates Proposition 1. It represents the expected profit as a function of the firm's preventive effort when the number of units varies between 1 and 17.[4]
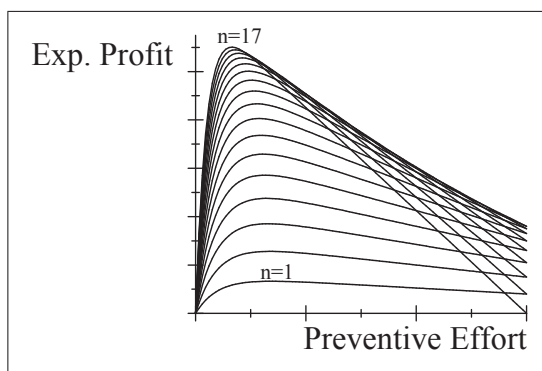

Figure 1: Expected profit and size, decreasing optimal preventive effort ($n = 1, ..., 17$) - no cross-contamination-

3.2 Full cross-contamination

Proposition 2: The optimal preventive effort is a *single-peaked*[5] function of capacity when there is *full cross-contamination*.

When a firm's capacity increases, we can underline two antagonistic effects on the optimal level of the preventive effort. First, his potential benefits π increases and lead the firm to increase his preventive effort. Second, the marginal probability of success - as regard to preventive effort - decreases and lead the firm to decrease his preventive effort (hereafter the contamination effect). For small firms, the first effect prevails on the second one. For large firms, the second effect prevails. Indeed, potential benefits π increase at a decreasing rate, $\pi'' \le 0$ but the marginal probability of success decreases at a constant rate b.

The proof of Proposition 2 - under the general assumptions developed in Section 2 - is provided in Appendix 2. Again, we illustrate our reasoning with the same profit function, that is $\pi(k) = pk - \frac{\gamma}{2}k^2$ with $\gamma > 0$ and $p > \gamma\left(n + \frac{1}{2}\right)$.

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[4]Specification: $b(e) = 1 - \exp(-e)$, $\gamma = 1$, $c = 0.5$, $p = 15$.

[5]i.e. inverted U-form function or always decreasing or always increasing function.

The expected utility of a firm who has a capacity of n can be written as:

$$E\Pi(n, e) = (b(e))^n \left(pn - \frac{\gamma}{2}n^2\right) - cne - C(n). \tag{10}$$

Showing that the preventive effort is a *single-peaked* function of capacity is identical to show that the optimal preventive effort cannot increase with capacity once it has decreased. Formally, if $e_n^* > e_{n+1}^*$ then $e_{n+1}^* \geq e_{n+2}^*$.

For an interior maximum, e_n^*, we must have,

$$\frac{\partial E\Pi}{\partial e}(n, e_n^*) = nb'(e_n^*)(b(e_n^*))^{n-1}\left(pn - \frac{\gamma}{2}n^2\right) - cn = 0. \tag{11}$$

Assume that $e_n^* > e_{n+1}^*$, then

$$\frac{\partial E\Pi(n+1, e_n^*)}{\partial e} = (n+1)b'(e_n^*)(b(e_n^*))^n\left(p(n+1) - \frac{\gamma}{2}(n+1)^2\right) - c(n+1) < 0 \tag{12}$$

Using (11) and (12), we can deduce that $e_n^* > e_{n+1}^*$ is equivalent to:

$$b'(e_n^*)(b(e_n^*))^{n-1}\left[b(e_n^*)\left((n+1)p - \frac{\gamma}{2}(n+1)^2\right) - \left(np - \frac{\gamma}{2}n^2\right)\right] < 0 \tag{13}$$

or,

$$b(e_n^*) < \frac{\pi(n)}{\pi(n+1)}. \tag{14}$$

From (14), we can infer that showing $e_{n+1}^* \geq e_{n+2}^*$ is equivalent to show:

$$b(e_{n+1}^*) < \frac{\pi(n+1)}{\pi(n+2)}. \tag{15}$$

As $b' > 0$ then

$$b(e_{n+1}^*) \leq b(e_n^*), \tag{16}$$

and since $\pi'' \leq 0$ we have

$$\frac{\pi(n)}{\pi(n+1)} \leq \frac{\pi(n+1)}{\pi(n+2)}. \tag{17}$$

As a result,

$$b(e_{n+1}^*) \leq \frac{\pi(n+1)}{\pi(n+2)}, \tag{18}$$

Equation (15) holds and $e_{n+1}^* \geq e_{n+2}^*$.

Figure 2 illustrates Proposition 2. It represents the expected profit as a function of the

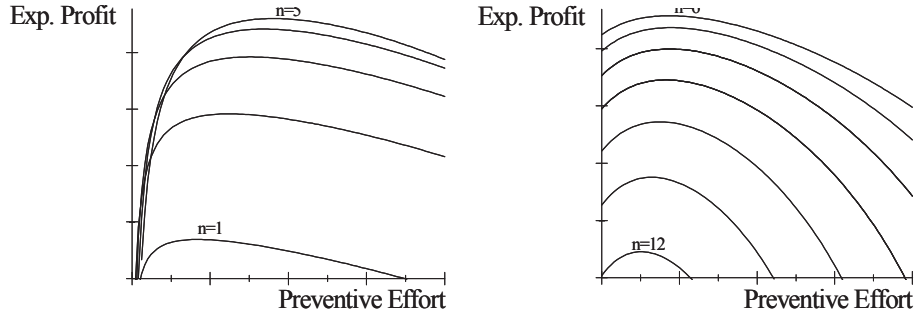firm's preventive effort when the number of units varies between 1 and 12.[6]

Figure 2: Expected profit and size, increasing optimal preventive effort ($n = 1, ..., 5$) and decreasing optimal preventive ($n = 6, ..., 12$)- full cross-contamination -

4   Discussion and conclusion

The 2002 European General Food Law and the 2010 Food Safety Modernization Act have similar food safety requirements for food operators. Food operators must implement preventive effort in order to avoid food safety defect. However, these two bills also differ on the way to consider food operators. Indeed, the European General Food Law applies uniformely to all food operators whereas the FFSMA includes exemptions according to the food operators' size. In this perspective, our results support the way the US have defined the scope of their bill. Food safety legislation must consider firms' scale of production because, as we have shown, preventive effort differ in firm's size.

However, it is not obvious from our result how this bill have to consider firm's size. Our result establishes that small firms should not be always exempted and that the bill needs to consider the food hazard small firms might face. In this article, we turn out to consider firm's decision according to the food safety hazard they might face and the individual economic consequences of a safety defect. We show that when cross-contamination (pathogenic hazard) could occur, small firms could be more at risk than medium or large firms whereas when there is no cross-contamination (chemical hazard) small firms would implement higher safety effort than large firms.

While they would need more empirical studies to be reinforced, our qualitative results imply that the usual way the FFSMA define exemptions needs to take into account of the hazard the food operators face and thus needs to consider the link between firms' size and their preventive effort. In other words, our results support that it should be better to base exceptions both on an industry risk analysis and firms'size.

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[6]Specification: $b(e) = \frac{\sqrt{e}}{0.2 + \sqrt{e}}$, $\gamma = 1$, $c = 0.5$, $p = 15$.

In this context, whereas the European General Food Law might not be the first best to provide food safety legislation, public authorities can benefit to remain general. Indeed, it might be a simplest and costless to avoid transactions costs linked to the definition of exemptions for each potential food safety hazard situations.

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Appendix A

Proof of Proposition 1: We will first prove a useful lemma. Let $\Delta_k = [\pi(k) - \pi(k-1)] - [\pi(k-1) - \pi(k-2)]$ for $k = 2, ..., n+1$, the differences of difference of utility. Let $\Lambda(e) \equiv \frac{1}{n+1} E\Pi(n+1, e) - \frac{1}{n} E\Pi(n, e)$. We will prove the following result:

Lemma: The derivative of the average payoff decreases with capacity. Formally, $\Lambda'(e) \leq 0$.

Proof of the Lemma: To prove this lemma, we prove two intermediary results.

We first show that the difference of expected utilities can be written as a function of the Δ_k:

Result 1: $nE\Pi(n+1, e) - (n+1) E\Pi(n, e) = \sum\limits_{k=2}^{n+1} \Psi_k(b) \Delta_k$ where:

$$\Psi_{k+2}(p) = \sum_{t=0}^{k} (1+t) \left[n \binom{n+1}{k-t} (1-b) - (n+1) \binom{n}{k-t} \right] b^{k-t} (1-b)^{n+t-k} + k + 1, \qquad (19)$$

for $k \in \{0, ..., n-2\}$, and

$$\Psi_{n+1}(b) = nb^{n+1}. \qquad (20)$$

Proof of Result 1:

$$nE\Pi(n+1, e) - (n+1) E\Pi(n, e)$$

$$= \sum_{k=0}^{n+1} \left[n \binom{n+1}{k} b^k (1-b)^{n+1-k} \pi(k) \right] - \sum_{k=0}^{n} \left[(n+1) \binom{n}{k} b^k (1-b)^{n-k} \pi(k) \right]$$

$$= np^{n+1} \pi(n+1) + \sum_{k=0}^{n} \left[\left[n \binom{n+1}{k} (1-b) - (n+1) \binom{n}{k} \right] b^k (1-b)^{n-k} \pi(k) \right] \qquad (21)$$

or,

$$nE\Pi\left(n+1,e\right)-\left(n+1\right)E\Pi\left(n,e\right)=\sum_{k=2}^{n+1}\left[\Psi_k\left[\pi\left(k\right)-2\pi\left(k-1\right)+\pi\left(k-2\right)\right]\right]$$

$$=\sum_{k=2}^{n+1}\Psi_k\pi\left(k\right)-2\sum_{k=2}^{n+1}\Psi_k\pi\left(k-1\right)+\sum_{k=2}^{n+1}\Psi_k\pi\left(k-2\right) \tag{22}$$

$$=\sum_{k=2}^{n+1}\Psi_k\pi\left(k\right)-2\sum_{k=1}^{n}\Psi_{k+1}\pi\left(k\right)+\sum_{k=0}^{n-1}\Psi_{k+2}\pi\left(k\right) \tag{23}$$

$$=\sum_{k=2}^{n-1}\left[\Psi_{k+2}-2\Psi_{k+1}+\Psi_k\right]\pi\left(k\right)+\Psi_{n+1}\pi\left(n+1\right)+\left(\Psi_n-2\Psi_{n+1}\right)\pi\left(n\right)$$

$$+\left(\Psi_3-2\Psi_2\right)\pi\left(1\right) \tag{24}$$

We now define:

$$\Psi_2 = \left(1-b\right)^n\left[n\left(1-b\right)-\left(n+1\right)\right]+1, \tag{25}$$

$$\Psi_3 = 2\Psi_2-\left(n+1\right)nb^2\left(1-b\right)^{n-1}, \tag{26}$$

$$\Psi_{n+1} = nb^{n+1}. \tag{27}$$

And, for $k\in\left\{2,...,n-2\right\}$,

$$\Psi_{k+2}-2\Psi_{k+1}+\Psi_k=\left[n\binom{n+1}{k}\left(1-b\right)-\left(n+1\right)\binom{n}{k}\right]b^k\left(1-b\right)^{n-k}. \tag{28}$$

This expression can be rewritten as a recursive formula for the differences of Ψs:

$$\Psi_{k+2}-\Psi_{k+1}=\left[n\binom{n+1}{k}\left(1-b\right)-\left(n+1\right)\binom{n}{k}\right]b^k\left(1-b\right)^{n-k}+\Psi_{k+1}-\Psi_k, \tag{29}$$

It can be easily deduced that:

$$\Psi_{k+2}-\Psi_{k+1}=\sum_{t=0}^{k-2}\left[n\binom{n+1}{k-t}\left(1-b\right)-\left(n+1\right)\binom{n}{k-t}\right]b^{k-t}\left(1-b\right)^{n+t-k}+\Psi_3-\Psi_2. \tag{30}$$

We have now a recursive formula for the Ψs:

$$\Psi_{k+2}=\sum_{t=0}^{k-2}\left[n\binom{n+1}{k-t}\left(1-b\right)-\left(n+1\right)\binom{n}{k-t}\right]b^{k-t}\left(1-b\right)^{n+t-k}+\Psi_3-\Psi_2+\Psi_{k+1}. \tag{31}$$

And then,

$$\Psi_{k+2} = \sum_{h=0}^{k-2}\sum_{t=0}^{k-2-h}\left[n\,\tbinom{n+1}{k-t-h}(1-b) - (n+1)\tbinom{n}{k-t-h}\right]b^{k-h-t}(1-b)^{n+t+h-k} + k\Psi_3 - (k-1)\Psi_2$$

$$(32)$$

The terms in the summation can be grouped such that:

$$\Psi_{k+2} = \sum_{t=0}^{k-2}(1+t)\left[n\,\tbinom{n+1}{k-t}(1-b) - (n+1)\tbinom{n}{k-t}\right]b^{k-t}(1-b)^{n+t-k} + k\Psi_3 - (k-1)\Psi_2 \quad (33)$$

for $k \in \{2, ..., n-2\}$.

Using the expressions of Ψ_2 and Ψ_3 in (25) and (26), we have, for $k \in \{2, ..., n-2\}$:

$$
\begin{aligned}
\Psi_{k+2} &= \sum_{t=0}^{k-2}(1+t)\left[n\,\tbinom{n+1}{k-t}(1-b) - (n+1)\tbinom{n}{k-t}\right]b^{k-t}(1-b)^{n+t-k}\\
&\quad -k(n+1)nb^2(1-b)^{n-1} + (k+1)\left[(1-b)^n\left[n(1-b)-(n+1)\right]+1\right] \quad (34)\\
&= \sum_{t=0}^{k}(1+t)\left[n\,\tbinom{n+1}{k-t}(1-b) - (n+1)\tbinom{n}{k-t}\right]b^{k-t}(1-b)^{n+t-k} + k + 1 \quad (35)
\end{aligned}
$$

It can easily be checked that this formula is also true for $k = 0$ and $k = 1$. This concludes the proof of Result 1.

We now prove the following second result:

Result 2: For all $k \in \{0, ..., n-1\}$, $\Psi_{k+2}(b)$ increases in b.

Proof of Result 2: We first show that the Ψ's are increasing in b. The last term, $\Psi_{n+1} = nb^{n+1}$ increases with b. Now we compute the derivative of the generic term for $k \in \{0, ..., n-2\}$:

$$
\begin{aligned}
\Psi'_{k+2} &= n\sum_{u=0}^{k}(1+k-u)(u(1-b)-(n+1-u)b)\tbinom{n+1}{u}b^{u-1}(1-b)^{n-u}\\
&\quad -(n+1)\sum_{u=0}^{k}(1+k-u)(u(1-b)-(n-u)b)\tbinom{n}{u}b^{u-1}(1-b)^{n-u-1} \quad (36)
\end{aligned}
$$

or,

$$
\begin{aligned}
\Psi'_{k+2} &= n\sum_{u=0}^{k}(1+k-u)(u-(n+1)b)\tbinom{n+1}{u}b^{u-1}(1-b)^{n-u}\\
&\quad -(n+1)\sum_{u=0}^{k}(1+k-u)(u-nb)\tbinom{n}{u}b^{u-1}(1-b)^{n-u-1} \quad (37)
\end{aligned}
$$

and, expanding this expression, we have:

$$
\begin{aligned}
\Psi'_{k+2} &= n(n+1)\sum_{u=1}^{k}(1+k-u)\binom{n}{u-1}b^{u-1}(1-b)^{n-u}\\
&\quad -n(n+1)\sum_{u=0}^{k}(1+k-u)\binom{n+1}{u}b^{u}(1-b)^{n-u}-n(n+1)\sum_{u=1}^{k}(1+k-u)\binom{n-1}{u-1}b^{u-1}(1-b)^{n-u-}\\
&\quad +n(n+1)\sum_{u=0}^{k}(1+k-u)\binom{n}{u}b^{u}(1-b)^{n-u-1}
\end{aligned}
\tag{3\text{?}}
$$

This expression can be rearranged as follows:

$$
\begin{aligned}
\frac{1}{n(n+1)}\Psi'_{k+2} &= \sum_{u=1}^{k}(1+k-u)\binom{n}{u-1}b^{u-1}(1-b)^{n+1-u}-\sum_{u=1}^{k}(1+k-u)\binom{n-1}{u-1}b^{u-1}(1-b)^{n-u-1}\\
&\quad +\sum_{u=0}^{k}(1+k-u)\binom{n}{u}b^{u+1}(1-b)^{n-u-1}
\end{aligned}
\tag{39}
$$

Or,

$$
\begin{aligned}
\frac{1}{n(n+1)}\Psi'_{k+2} &= \sum_{u=1}^{k}(1+k-u)\binom{n-1}{u-1}b^{u-1}(1-b)^{n+1-u}+\sum_{u=2}^{k}(1+k-u)\binom{n-1}{u-2}b^{u-1}(1-b)^{n+1-u}\\
&\quad -\sum_{u=1}^{k}(1+k-u)\binom{n-1}{u-1}b^{u-1}(1-b)^{n-u-1}+\sum_{u=0}^{k}(1+k-u)\binom{n-1}{u}b^{u+1}(1-b)^{n-u-1}\\
&\quad +\sum_{u=1}^{k}(1+k-u)\binom{n-1}{u-1}b^{u+1}(1-b)^{n-u-1}
\end{aligned}
\tag{40}
$$

Or,

$$
\begin{aligned}
\frac{1}{n(n+1)}\Psi'_{k+2} &= \sum_{u=1}^{k}(1+k-u)\binom{n-1}{u-1}b^{u-1}(1-b)^{n+1-u}+\sum_{u=2}^{k}(1+k-u)\binom{n-1}{u-2}b^{u-1}(1-b)^{n+1-u}\\
&\quad +\left((1-b)^{2}+b^{2}-1\right)\sum_{u=1}^{k}(1+k-u)\binom{n-1}{u-1}b^{u-1}(1-b)^{n-u-1}\\
&\quad +\sum_{u=0}^{k}(1+k-u)\binom{n-1}{u}b^{u+1}(1-b)^{n-u-1}
\end{aligned}
\tag{41}
$$

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Thus, we can get an homogeneous expression:

$$\frac{1}{n(n+1)}\Psi'_{k+2} = \sum_{u=1}^{k}(1+k-u)\binom{n-1}{u-1}b^{u-1}(1-b)^{n+1-u}$$

$$+\sum_{u=2}^{k}(1+k-u)\binom{n-1}{u-2}b^{u-1}(1-b)^{n+1-u} - 2\sum_{u=1}^{k}(1+k-u)\binom{n-1}{u-1}b^{u}(1-b)^{n-u}$$

$$+\sum_{u=0}^{k}(1+k-u)\binom{n-1}{u}b^{u+1}(1-b)^{n-u-1} \tag{42}$$

Now, we make changes in the indices of the two last summations:

$$\frac{1}{n(n+1)}\Psi'_{k+2} = \sum_{u=1}^{k}(1+k-u)\binom{n-1}{u-1}b^{u-1}(1-b)^{n+1-u}$$

$$+\sum_{u=2}^{k}(1+k-u)\binom{n-1}{u-2}b^{u-1}(1-b)^{n+1-u} - 2\sum_{h=2}^{k+1}(2+k-h)\binom{n-1}{h-2}b^{h-1}(1-b)^{n+1-h}$$

$$+\sum_{u=2}^{k+2}(3+k-h)\binom{n-1}{h-2}b^{h-1}(1-b)^{n+1-h} \tag{43}$$

And the terms can be grouped such that:

$$\frac{1}{n(n+1)}\Psi'_{k+2} = \sum_{u=2}^{k}\left[\begin{array}{c}(1+k-u)\binom{n-1}{u-1}+(1+k-u)\binom{n-1}{u-2}\\-2(2+k-u)\binom{n-1}{u-2}+(3+k-u)\binom{n-1}{u-2}\end{array}\right]b^{u-1}(1-b)^{n+1-u}$$

$$+k(1-b)^{n}+\binom{n-1}{k}b^{k+1}(1-b)^{n-1-k} \tag{44}$$

Moreover, the term in brackets can be simplified:

$$\left[\begin{array}{c}(1+k-u)\binom{n-1}{u-1}+(1+k-u)\binom{n-1}{u-2}\\-2(2+k-u)\binom{n-1}{u-2}+(3+k-u)\binom{n-1}{u-2}\end{array}\right]$$

$$= (1+k-u)\binom{n-1}{u-1}+(1+k-u-4-2k+2u+3+k-u)\binom{n-1}{u-2}$$

$$= (1+k-u)\binom{n-1}{u-1} > 0 \tag{45}$$

Hence, $\Psi'_{k+2} > 0$. This concludes the proof of Result 2.

Now, since π has increasing differences, we have $\Delta_k \leq 0$ for all k and using Result 1 and Result 2, we know that $nE\Pi(n+1,e)-(n+1)E\Pi(n,e)$ decreases in e and then $\Lambda'(e) \leq 0$. At point e_n^*, we have $\Lambda'(e_n^*) = \frac{1}{n+1}E\Pi(n+1,e_n^*) \leq 0$ and then $e_{n+1}^* \leq e_n^*$.

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Appendix B

Proof of Proposition 2:

Preventive effort is a *single-peaked* function in n is equivalent to: if $e^*_{n+1} \leq e^*_n$ then $e^*_{n+2} \leq e^*_{n+1}$.

Let introduce $\Lambda(n, e)$ such as

$$\begin{aligned}\Lambda(n, e) &= \frac{1}{n+1} \frac{\partial E\Pi}{\partial e}(n+1, e) - \frac{1}{n} \frac{\partial E\Pi}{\partial e}(n, e) \\ &= b'(e) [b(e)]^{n-1} [b(e)\pi(n+1) - \pi(n)]\end{aligned} \tag{46}$$

for all $e \geq 0$ and for all n as an integer.

Assume that $e^*_{n+1} \leq e^*_n$. First, $E\Pi$ is quasi-concave in e, then

$$\frac{\partial E\Pi(n+1, e^*_n)}{\partial e} \leq 0. \tag{47}$$

Second, e^*_n is an interior solution, then

$$\frac{\partial E\Pi}{\partial e}(n, e^*_n) = 0 \tag{48}$$

Consequently,

$$\Lambda(n, e^*_n) \leq 0 \tag{49}$$

which is equivalent to

$$b(e^*_n) \leq \frac{\pi(n)}{\pi(n+1)} \tag{50}$$

as $b' > 0$ then

$$b(e^*_{n+1}) \leq b(e^*_n) \tag{51}$$

and since $\pi'' \leq 0$ we have

$$\frac{\pi(n)}{\pi(n+1)} \leq \frac{\pi(n+1)}{\pi(n+2)} \tag{52}$$

As a result,

$$b(e^*_{n+1}) \leq \frac{\pi(n+1)}{\pi(n+2)} \tag{53}$$

which is equivalent to

$$\Lambda(n+1, e^*_{n+1}) \leq 0 \tag{54}$$

Or,

$$\frac{1}{n+2} \frac{\partial E\Pi}{\partial e}(n+2, e^*_{n+1}) \leq 0 \tag{55}$$

and as $E\Pi$ is quasi-concave in e, this results in $e^*_{n+2} \leq e^*_{n+1}$.

Appendix C

We illustrate our reasoning with a particular profit function such as $\pi(k) = pk - \frac{\gamma}{2}k^2$ with $\gamma > 0$ and $p > \gamma n$. First notice that we can write the profit function such as $\pi(k) = (p + \frac{\gamma}{2})k - \frac{\gamma}{2}k(k-1)$. Now, in the case of no cross-contamination, the expected profit becomes:

$$E\Pi(n,e) = \sum_{k=1}^{n} \left[\binom{n}{k} (b(e))^k (1 - b(e))^{n-k} \left((p + \frac{\gamma}{2})k - \frac{\gamma}{2}k(k-1) \right) \right] - cne - C(n). \quad (56)$$

Using the Newton binomial theorem[7] and $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$, we have:

$$E\Pi(n,e) = (p + \frac{\gamma}{2}) \underbrace{\sum_{k=0}^{n} \left[\binom{n}{k} (b(e))^k (1 - b(e))^{n-k} k \right]}_{=nb(e)} \quad (57)$$

$$-\frac{\gamma}{2} \underbrace{\sum_{k=0}^{n} \left[\binom{n}{k} (b(e))^k (1 - b(e))^{n-k} k(k-1) \right]}_{=n(n-1)(b(e))^2} - cne - C(n),$$

and then,

$$E\Pi(n,e) = (p + \frac{\gamma}{2})nb(e) - \frac{\gamma}{2}n(n-1)(b(e))^2 - cne - C(n). \quad (58)$$

---

[7]The Newton binomial theorem states that: $\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = (x+y)^n$ for all real number x, y and for all positive interger n.