Regulating ambient pollution when social costs are unknown

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June 11, 2012

Abstract

This paper offers a new mechanism in order to Nash-implement a Pareto optimal level of ambient pollution. As usual in the literature on non point source pollution, the proposed scheme is not conditional on individual emissions, since they are not observable; rather it is conditional on aggregate emission. But the novelty here is that we do not assume the regulator knows the agents' preferences, with which he could identify the target level of aggregate emission. Our mechanism dispenses with this information, yet it achieves Pareto optimality provided that the number of agents involved in the problem is known.

Keywords: non point source pollution, mechanism design, Nash-implementation.

JEL Classification: Q0, Q52, H23, C72, D82.

1 Introduction

The ambient tax (Segerson, 1988, Hansen, 1998, Shortle and Horan, 2001 to quote a few) is considered an efficient tool to implement a socially optimum level of emissions as a Nash equilibrium when the regulator cannot observe individual emissions. To achieve a target level of emissions with the ambient tax instrument the regulator only needs to know the level of ambient pollution, *i.e.* the aggregation of all individual emissions. There are several variants of this instrument now, but its basic principle is as follows: each polluter is liable for the ambient pollution in excess of the target, whether or not he is responsible for that outcome. That is, each polluter is required to pay for the total damage to society due to excess pollution. If pollution is below the target level, no tax is paid. The unique Nash equilibrium with the ambient tax

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instrument is such that aggregate emissions are equal to the target level defined by the regulator, so that at equilibrium none of the polluters is required to pay a tax for excess damage.

While the target can be set arbitrarily at any level by the regulator, the most interesting case is where it corresponds to a social optimum. To identify such a target the regulator requires additional information. Besides the ambient level of pollution the regulator also needs to know the social damage function of the ambient pollution. Such knowledge is necessary in order to compute the optimum tax rate which corresponds to the level of emissions for which the marginal social benefit of the ambient pollution is equal to its marginal social damage.

In practice the social damage function is not known by the regulator, and information about it which may be available are questionable. Our purpose in this paper is to design an instrument that does not rest on such pieces of information and nevertheless induces the agents to achieve the socially optimal levels of emissions as a Nash equilibrium. Our proposal applies to the class of non point source regulation problems where emitters are also the recipients of the externality. In order to implement it, the regulator needs to observe the level of ambient pollution and to know the number of agents involved in the problem. But neither the observation of individual emissions nor the knowledge of the social cost are needed.

The mechanism operates as follows: the regulator asks each agent both to choose a level of emission and to provide his expectation about the aggregate level of emission, the ambient pollution. He also announces the following rule: if the actual level of ambient pollution is above the level predicted by the agent, the latter will be liable to a tax proportional to the gap. The key of the mechanism lies in the tax rate which is equal to the proportion of other agents in the population, i.e. $\frac{n-1}{n}$. Under *laissez-faire* each agent chooses a level of emission such that the marginal utility gain of his emissions equals his marginal utility loss from pollution. Once the tax on expectation errors is introduced, each agent takes into account the externality he exerts on the $n-1$ other agents when choosing his own level of emission.

This note is organized as follows. The next section presents a non point source pollution framework. Section 3 explains our mechanism, the regulated game that goes with it, and it demonstrates that its Nash equilibrium exists, is unique under a normality condition and that it corresponds to a Pareto optimal allocation. The last section offers final remarks.

2 A non point source pollution framework

Consider n agents, indexed $i = 1, ..., n$ who emit individual pollutants $e_i \in E_i = \left[0, \overline{e}^i\right] \subset \mathbb{R}_+$, where E_i is a compact and convex subset of the positive real numbers. Individual emissions aggregate into an ambient pollution level

$$X = \sum_{h=1}^{n} e_h.$$

An *allocation* is a $n+1$ dimensional vector $(e_1, ..., e_n, X) \in \times_{i=1}^{n} E_h \times \mathbb{R}_+$. A typical agent i is endowed with a preference ordering over the allocations. His ordering can be represented by a continuous, differentiable utility function, which is strictly increasing with the agent's own emission e_i and strictly decreasing with the ambient pollution X:

$$U^i(e_i, X), \quad U_1^i > 0, U_2^i < 0,$$

where $U_1^i = \partial U^i / \partial e_i$ and $U_2^i = \partial U^i / \partial X$. We also assume that the Hessian matrix $Hess(U^i)$ is semi definite negative (hence utility functions are concave).

Let us assume that corner decisions are dominated strategies, *i.e.* $\forall i$ and whatever the others' strategies $e_{-i} = \sum_{h \neq i} e_h$, $\lim_{e_i \to 0} MRS^i(e_i, e_i + e_{-i}) < 1$ and $\lim_{e_i \to \bar{e}^i} MRS^i(e_i, e_i + e_{-i}) > 1$, where

$$MRS^i(e_i, X) = -U_2^i(e_i, X) / U_1^i(e_i, X)$$

is the absolute value of the *marginal rate of substitution* between agent i's emission and ambient pollution. Any interior Nash equilibria $(\widetilde{e}_1, ..., \widetilde{e}_n)$ solves the system of equations:

$$MRS^i(e_i, X) = 1, \quad i = 1, ..., n. \tag{1}$$

On the other hand, interior Pareto optima $(e_1^*, ..., e_n^*)$ must satisfy the Bowen-Lindhal-Samuelson (BLS) condition:

$$\sum_h MRS^h(e_h, X) = 1. \tag{2}$$

In general (1) and (2) admit different solutions, *i.e.* Nash equillibria are not Pareto optimal allocations. This comes as no surprise since in the absence of cooperation, when choosing his level of emission each agent i is weighing his marginal advantage $U_1^i(e_i, X)$ compared to his individual marginal cost $U_2^i(e_i, X)$, while social efficiency requires to put individual advantages against the marginal *social cost* $\sum_h U_2^h(e_h, X)$. This social dilemma calls for some intervention from a benevolent authority.

It is worth noting that this model can be seen as a reduced form, which can accommodate two interpretations. In a first approach, each agent is a farm household, which consumes from the proceeds of its production, but production is accompanied by individual pollutions. In summary, more individual consumption is associated with more individual pollution. Households have preferences defined over consumption and aggregate pollution. Pareto optima in that case are allocations that maximize the social welfare of the community of farm households producing and suffering from pollution. In a second approach, agents are firms in the usual sense. In that case, functions U^i are better called profit functions. The ambient pollution is then an externality that adversely affects firms' cost and allocations that maximize the *producers' surplus* satisfy the BLS condition (2).

Example 1. *Let the utility functions be:*

$$U^i(e_i, X) = ae_i - \frac{b}{2}(e_i)^2 - cX.$$

At the Nash equilibrium, individual decisions are:

$$\widetilde{e}_i = \frac{a-c}{b} \,,$$

and the symmetric Pareto optimum is:

$$e_i^* = \frac{a-nc}{b} < \widetilde{e}_i.$$

3 The mechanism

3.1 The regulator's information and the regulated game

From the point of view of the regulator, the information structure we consider has two aspects:

1. the regulator cannot observe individual emissions (or only at prohibitive costs). This assumption defines the framework as a *non point source regulation problem*.

2. and he also ignores the agents' utility functions, that are subjective attributes to be elicited. This information asymmetry is at the root of an *implementation problem*.

Those two informational conditions have been studied separately by two (related) strands of literature. The literature on ambient pollution considered only the first aspect. Collective penalties could therefore be defined by targeting a particular pollution level. If it turns out that this level is a Pareto optimal one, collective penalties induce a Pareto optimal outcome. Otherwise this optimality property has no reason to hold. It is conditional on the possibility to elicit agents' utility functions in order to compute the social cost, by some means. The second literature, the theory of mechanisms design, specifically addresses the issue of information asymmetry, about preferences, between agents and the regulatory authority. But it assumes that individual actions are perfectly observable, so that adequately designed transfers can be defined conditionally on the realization of those actions.

Clearly, for a large class of non point source problems, internal consistency requires to merge those two aspects. This is the way we initiate in this paper.

The regulator solicits information from the agents and uses the collected pieces of information to set up a system of incentive taxes, that cannot be based on individual emission because they are not observable, but can be based on ambient pollution.

Each agent is asked to state his expectation of the level of *aggregate pollution*, which will be denoted $\widehat{X}_i \in [0, \sum_h \overline{e}_h]$. Simultaneously he decides upon his level of emission, given that he will have to pay a penalty that depends only on the gap between the actual and his expected level of aggregate emissions:

$$T^i\left(\widehat{X}_i, X\right) = \begin{cases} k(X-\widehat{X}_i) & \text{if } X \geq \widehat{X}_i. \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where $k \in [0,1]$ is a parameter that will play a crucial role in the following. Expression (3) is a tax for agent i only if the realized aggregate pollution is larger than his forecast. This formula contains the original suggestion we make in this article, which departs from existing formulas in two respects: $i)$ the target for the ambient pollution is not exogenously given to the agents. On the contrary, each agent announces his own target $\widehat{X}_i$, $ii)$ the price k for units of pollution above the target is not set equal to the marginal social damage on others, since this information is not available to the regulator. However, we will see below that this parameter can be attributed a value, based on available information, that will induce Pareto optimality.

This mechanism defines a class of games configured by parameter k. In a k - *game* each agent i has two decision variables to be determined simultaneously, $e_i \in E_i \subset \mathbb{R}_+$ and $\widehat{X}_i \in [0, \sum_h \overline{e}_h]$. And faced with this scheme, his payoff (or utility) thus becomes:

$$\Pi^i\left(e_i, e_{-i}, \widehat{X}_i\right) \equiv \begin{cases} U^i\left(e_i - k(e_i + e_{-i} - \widehat{X}_i), e_i + e_{-i}\right) & \text{if } e_i + e_{-i} \geq \widehat{X}_i\ , \\ U^i\left(e_i, e_i + e_{-i}\right) & \text{otherwise.} \end{cases}$$

3.2 The k - interior Nash equilibrium

Under the assumption that the game played by the agents is one of complete information and with common knowledge, the Nash equilibrium is a usual predictor for non cooperative decisions. We will focus on the case where the Nash equilibrium is made of interior decisions.

Definition 1. *A k -* interior Nash equilibrium *(k -INE) is a vector of individual emissions*

$$\left(e_i\left(k\right), ..., e_n\left(k\right)\right) \in \times_{h=1}^{n} E_h\ ,$$

a profile of announcements:

$$\left(\widehat{X}_1\left(k\right), ..., \widehat{X}_n\left(k\right)\right) \in \left[0, \sum_h \overline{e}_h\right]^n\ ,$$

and a profile of transfers

$$\left(\mathbf{T}^1\left(\widehat{X}_1\left(k\right), X\left(k\right)\right), ..., \mathbf{T}^n\left(\widehat{X}_n\left(k\right), X\left(k\right)\right)\right) \in \mathbb{R}_+^n\ ,$$

such that:

1. *announcement decisions $\widehat{X}_i\left(k\right), i = 1, ..., n$ match aggregate emissions:*

$$\widehat{X}_i\left(k\right) = \sum_h e_h\left(k\right), \quad i = 1, ..., n.$$

2. *Emission decisions* $e_i(k), i = 1, ..., n$ *solve the system of necessary conditions:*

$$MRS^i\left(e_i, \sum_h e_h\right) = 1 - k, \quad i = 1, ..., n. \tag{4}$$

To grasp item 1 in the above definition, consider the point of view of agent i. Given his best educated guess about the emissions by the other agents, $\widetilde{e}_{-i}$, and given the own choice of emission $\widetilde{e}_i$ he contemplates, clearly his best announcement is $\widehat{X}_i = \widetilde{e}_i + \widetilde{e}_{-i}$, With this choice he pays no tax. A lower announcement triggers a penalty, and a higher announcement does not increase further his utility. Regarding the equilibrium transfers, an implication of item 1 is:

$$\mathbf{T}^i(k) \equiv T^i\left(\widehat{X}_i(k), \sum_h e_h(k)\right) = 0, \; i = 1, ..., n.$$

Each equilibrium transfer is zero since, at a k - INE, announcements and ambient pollution are identical. This is an interesting advantage from a practical point of view, since mechanisms themselves have implementation costs that are likely to be larger for larger transfers. Here those costs are probably minimized.

The second item in the definition is the usual first order condition for interior optimal individual emissions. Consistently with this second item, it is easy to single out economic environments that discard corner decisions as dominated strategies:

Assumption 1. *Assume,* $\forall i, \forall e_{-i}, \forall \widehat{X}_i$:

$$\lim_{e_i \to 0} MRS^i\left(e_i - T^i\left(\widehat{X}_i, e_i + e_{-i}\right), e_i + e_{-i}\right) < (1 - k)$$

and

$$\lim_{e_i \to \overline{e}^i} MRS^i\left(e_i - T^i\left(\widehat{X}_i, e_i + e_{-i}\right), e_i + e_{-i}\right) > (1 - k).$$

Theorem 1 (Existence). *The* k *- game admits at least one Nash equilibrium. Under Assumption 1, any Nash equilibrium is interior (*k *- INE).*

Proof. See Appendix A.1. ■

For reasons that will become clear below, let us now focus on the case where $k = 1 - 1/n$. With this value for parameter k, each agent is taxed up to the share $(n-1)/n$ of the aggregate externality generated. And the first order conditions (4) rewrite:

$$MRS^i(e_i, X) = \frac{1}{n}, \quad i = 1, ..., n.$$

It is as if the agent's marginal utility of polluting that goes through the first argument in his payoff function was cut down to only $(1/n)^{\text{th}}$ of the original level, making individual emissions less advantageous. We are now in position to make a crucial observation. By summing-up the above equations over the agents:

$$\sum_{h=1}^{n} MRS^h\left(e_h, X\right) = 1,$$

one finds the BLS conditions (2). At the same time:

$$\mathbf{T}^i\left(\widehat{X}_i\left(1-1/n\right), X\left(1-1/n\right)\right) = 0,\ \forall i.$$

Therefore a k - INE has the following desirable property:

Theorem 2 (Pareto optimality). *A k - INE with $k = 1 - 1/n$, is a Pareto optimal allocation.*

In practice the exact number n of agents involved in the pollution problem might not be known exactly. In this case, a k - INE only approaches a Pareto optimal allocation. Intuitively, as parameter k gets larger the individual incentives to pollute at a k - INE are getting weaker, and when the number of agents used to construct the mechanism increases to get closer to its true value n, the k - INE eventually gets closer to a Pareto optimal allocation.

Example 2 (continued). *With the mechanism, and when the forecast is correct $\widehat{X}_i = \sum_h \widetilde{e}_h$, the Nash equilibrium is now configured by parameter k:*

$$\widetilde{e}_i\left(k\right) = \frac{a - \frac{c}{1-k}}{b}\ .$$

And when $k = 1 - 1/n$, the Nash equilibrium is Pareto optimal:

$$\widetilde{e}_i\left(\frac{n-1}{n}\right) = \frac{a - nc}{b} = e_i^*\ .$$

In the above example, the k - INE exists and is unique. Uniqueness is an interesting property since the Nash implementation approach is often plagued with a serious problem of multiplicity of equilibria. To put it differently, Nash-implementation mechanisms often solve the social dilemma but at the cost of a coordination problem. Can we expect the uniqueness property to hold beyond our illustration? Under what condition can we guarantee this property?

To analyze this question, it will prove useful to reformulate each agent's problem. Define

$$\begin{aligned} c_i &= e_i - k\left(X - \widehat{X}_i\right), \\ &= X - X_{-i} + k\left(\widehat{X}_i - X\right)\ . \end{aligned}$$

Or upon rewriting:

$$c_i - \left(1-k\right)X = k\widehat{X}_i - X_{-i}\ . \tag{5}$$

With this rewriting, each agent's problem can be seen as one where the goal is to choose X in order to maximize:

$$U^i(c_i, X),$$

subject to (5) and $X \geq X_{-i}$. It looks like a consumer's problem, who is endowed with an exogenous "income" $y_i = k\widehat{X}_i - X_{-i}$, and has to allocate it optimally between his consumption of a private good c_i and of cleanup $-X$, whose price relative to c_i is $p = 1 - k$. Denote $c_i = C^i(p, y_i)$ and $X = D^i(p, y_i)$ the two demand functions derived from this problem. And assume that c_i and $-X$ are normal goods, *i.e.* the demands $C^i(p, y_i)$ and $X = D^i(p, y_i)$ are respectively strictly increasing and strictly decreasing functions of income:

Assumption 2 (Normality). $C_2^i(p, y_i) > 0,\ D_2^i(p, y_i) < 0.$

As far as X is concerned, the normality assumption is akin to a form of strategic complementarity: the larger the aggregate contributions of others, X_{-i}, the lower y_i and the larger X.

Theorem 3 (Uniqueness). *If Assumption 2 holds, the k - INE is unique.*

Proof. See Appendix A.2. ■

This result applies also in the particular case where $k = 1-1/n$,, which ensures that a Pareto optimal allocation can be implemented as a unique interior Nash equilibrium. Assumption 2 can be relaxed somewhat in order to hold only for the particular value $k = 1-1/n$ that is consistent with Pareto optimality.

Example 3 (continued). *Applying the reformulation (5) to the quadratic example, each agent's problem now reads as:*

$$\max_X U^i(c_i, X) = a[pX + y_i] - \frac{b}{2}[pX + y_i]^2 - cX.$$

subject to the constraint $X \geq k\widehat{X}_i - y_i$. The two demand functions are:

$$\begin{aligned} c_i &= C^i(p, y_i) = \frac{a - c/p}{b} + (1 - p/b)\, y_i\ , \\ X &= D^i(p, y_i) = \frac{a - c/p}{bp} - \frac{y_i}{b}. \end{aligned}$$

As one can readily check, consumption and cleanup ($-X$) are normal goods.

4 Conclusion

Three last remarks are in order.

Firstly, the mechanism proposed here has a similarity to Falkinger's mechanism (1996) that Nash implements a Pareto outcome in the public good model. It also uses a price for the externality which is calibrated by using the number of agents involved in the problem as the only piece of information. Actually, his formula for k is exactly the same as ours ($k = 1 - 1/n$). But Falkinger's mechanism differs in that it penalizes or subsidizes deviations between own decisions and average decisions by others, whereas our mechanism introduces a new decision variable for each agent (his forecast about ambient pollution) and it only penalizes deviations between actual aggregate decisions and individual forecasts. A second difference lies in the number of Nash equilibria that can be unique with our mechanism (Theorem 3), whereas Falkinger's mechanism (1996) has a continuum of Nash equilibria when $k = 1-1/n$, though the Nash equilibrium can be unique for arbitrarily lower values of k. For this reason, Falkinger suggests to use his mechanism with $k \simeq 1-1/n$, in order to implement near-optimal contributions to public goods.

Secondly, at a k - INE the budget of the regulatory authority is balanced. But this property need not hold outside equilibrium decisions. More precisely, as soon as one agent makes a wrong prediction, a penalty will be paid and this will produce a budget surplus. Many mechanisms try, by construction, to avoid the imbalance of the budget. The reason comes from an implicit general equilibrium perspective, from which one deduces that any budget surplus necessarily returns to some economic agents (see for instance the discussion in Green and Laffont, 1979, Chap. 9). This redistribution, if correctly anticipated by agents, may compromise the good incentive properties of the mechanism under consideration. In defence of unbalanced budgets, one finds at least three possible reactions in the literature. The first one is that the regulator could announce that any redistribution of surplus will be lump-sum, hence without effects on incentives. The second one is to assume a form of myopia from the agents regarding the redistribution of the public budget. The last reaction does not rest on an assumption of bounded rationality. It assumes a partial equilibrium point of view, which is more in line with the problem studied in this paper. The economy has many agents and only a small subset of them is involved in the non point source pollution problem. A budget surplus, if any, can be redistributed to the rest of society, hence without interfering with the incentives given to the regulated agents. The budget surplus may even produce a double dividend, by financing interventions to promote other social goals.

Lastly, this paper is the first of its kind at the intersection of the literature on non point source pollution and on the design of mechanisms and may provide inspiration to sketch a research program. Just repeat the steps that each of the two literatures has already travelled by separately, while taking now into account both the constraints of non-observability and information asymmetry. For example, one could search for general possibility results of implementation in Nash equilibrium when only aggregate decisions can serve as support for incitative transfers.

Aknowledgements

We thank all the participants at the workshop on "Conflicts", organized in Rennes, May 2012, 23rd - 25th, for their comments on a first version of this work. All errors and flaws are of course ours.

Appendix

A Existence and uniqueness of a k - INE

A.1 Existence

A standard set of sufficient conditions for the existence of pure strategy Nash equilibria is: $i)$ strategy sets are non empty compact and convex sets, $ii)$ for each player, his payoffs are continuous and quasi-concave with respect to his own decision variables.

The first condition is satisfied in the k - game. Regarding the second condition, payoff functions are continuous (though they are not differentiable everywhere because of the transfers), but quasi-concavity is an open question. For each agent i, taking e_{-i} and $\widehat{X}_i$ as given parameters we must ascertain the quasi-concavity of the function:

$$\Pi^i\left(e_i\right) \equiv \begin{cases} f^i\left(e_i\right) & \text{if } e_i < \widehat{X}_i - e_{-i}\ , \\ g^i\left(e_i\right) & \text{if } e_i \geq \widehat{X}_i - e_{-i}. \end{cases} \tag{6}$$

where

$$\begin{aligned} f^i\left(e_i\right) &\equiv U^i\left(e_i, e_i + e_{-i}\right), \\ g^i\left(e_i\right) &\equiv U^i\left(e_i - k\left(e_i + e_{-i} - \widehat{X}_i\right), e_i + e_{-i}\right)\ . \end{aligned}$$

Actually it can be shown that $\Pi^i\left(.\right)$ is concave (hence it is quasi-concave). The architecture of the proof is to show that the functions $f^i(.)$ and $g^i(.)$ are concave and that combination of both in (6) has a "min" property (see below). The latter property is important to establish the concavity of function $\Pi^i\left(.\right)$. The concavity of both functions $f^i(.)$ and $g^i(.)$ is not enough to guarantee that $\Pi^i\left(.\right)$ is also concave. It is easy to exhibit examples where $f^i(.)$ and $g^i(.)$ are concave, whereas $\Pi^i\left(.\right)$ is not.

Note that $f^i\left(.\right)$ is concave by definition. And it can be shown that $g^i\left(.\right)$ is concave as well. Indeed it second order derivative is:

$$g^{i\prime\prime}\left(e_i\right) = \left(1-k\right)^2 U_{11}^i + 2\left(1-k\right) U_{12}^i + U_{22}^i.$$

And from the assumptions we made about utility functions:

$$v * Hess\left(U^i\right) * v^T = [v_1, v_2] * \left[\begin{array}{cc} U^i_{11} & U^i_{12} \\ U^i_{21} & U^i_{22} \end{array}\right] * \left[\begin{array}{c} v_1 \\ v_2 \end{array}\right] \leq 0, \quad \forall v = [v_1, v_2] \in \mathbb{R}^2.$$

To obtain the sign $g^{i\prime\prime}(e_i) \leq 0$, showing that g^i is concave, it suffices to substitute the vector $[v_1, v_2] = [1-k, 1]$ in the above inequality.

The important "min" property mentioned above is the following: because $U^i_1 > 0$, function $\Pi^i(.)$ can also be written as:

$$\Pi^i(e_i) \equiv \min\left\{f^i(e_i), g^i(e_i)\right\} \ .$$

This is easy to check. For all $e_i < \widehat{X}_i - e_{-i}$, under $g^i(.)$ agent i would perceive a subsidy, hence $f^i(.) \leq g^i(.)$. And as soon as $e_i \geq \widehat{X}_i - e_{-i}$, under $g^i(.)$ agent i would be liable to a tax, thus $g^i(.) \leq f^i(.)$.

Recall that by definition a concave function is such that the set of points lying on or below its graph, *i.e.* its *hypograph*, is convex. Since $f^i(e_i)$ and $g^i(e_i)$ are concave, their hypographs:

$$\begin{array}{rcl} hypo\left(f^i\right) & = & \left\{(e_i, \pi_i) \in E_i \times \mathbb{R} : f^i(e_i) \geq \pi_i\right\} \ , \\ hypo\left(g^i\right) & = & \left\{(e_i, \pi_i) \in E_i \times \mathbb{R} : g^i(e_i) \geq \pi_i\right\} \ , \end{array}$$

are convex. And the hypograph of $\Pi^i(.)$ is:

$$\begin{array}{rcl} hypo\left(\Pi^i\right) & = & \left\{(e_i, \pi_i) \in E_i \times \mathbb{R} : \Pi^i(e_i) \geq \pi_i\right\} \ , \\ & = & hypo\left(f^i\right) \cap hypo\left(g^i\right) \ , \end{array}$$

where the second line is obtained because $\Pi^i(.)$ is constructed as the min of $f^i(.)$ and $g^i(.)$. So $hypo(\Pi^i)$ is necessarily convex, as the intersection of two convex sets. Therefore $\Pi^i(.)$ is concave and the set of Nash equilibria for the k-game is not empty.

Of course, this does not mean that (some) Nash equilibria are interior. However, Assumption 1 rules out corner decisions, which ensures in this case the existence of at least one interior Nash equilibrium.

QED.

A.2 Uniqueness

The strategy of proof given here follows the elegant one offered by Andreoni and Bergstrom (1996) for the case of Nash contributions to a public good, though some modifications are required in our context, because action spaces also incorporate the announcements $\widehat{X}_i$, $i = 1, ...n$.

To prove uniqueness in Theorem 3 we first need the following lemma:

Lemma 1. *Suppose there exists two distinct k - INE, denoted $\left(e_1', ..., e_n', \widehat{X}_1', ..., \widehat{X}_n'\right)$ and $\left(e_1'', ..., e_n'', \widehat{X}_1'', ..., \widehat{X}_n''\right)$, and suppose without loss of generality that $X'' \geq X'$. Then for any agent i,*

i) $k\widehat{X}_i'' - X_{-i}'' \leq k\widehat{X}_i' - X_{-i}'$,

ii) $X_{-i}'' = \sum_{h \neq i} e_h'' \geq X_{-i}' = \sum_{h \neq i} e_h'$.

Proof. Since $\left(e_1'', ..., e_n'', \widehat{X}_1'', ..., \widehat{X}_n''\right)$ is a k - INE, it must be the case that $X'' = \sum_{h=1}^{n} e_h'' = D^i\left(p, k\widehat{X}_i'' - X_{-i}''\right)$. By the same argument, it must also be the case that $X' = \sum_{h=1}^{n} e_h' = D^i\left(p, k\widehat{X}_i' - X_{-i}'\right)$. And since by assumption $X'' \geq X'$, one must deduce that $D^i\left(p, k\widehat{X}_i'' - X_{-i}''\right) \geq D^i\left(p, k\widehat{X}_i' - X_{-i}'\right)$. And, if ambient clean-up is a normal good, one obtains item i) of the Lemma, *i.e.* $k\widehat{X}_i'' - X_{-i}'' \leq k\widehat{X}_i' - X_{-i}'$. To demonstrate item ii), rewrite item i) as $k\left(\widehat{X}_i'' - \widehat{X}_i'\right) \leq X_{-i}'' - X_{-i}'$. Finally, using the properties that $\widehat{X}_i'' = X'', \widehat{X}_i' = X'$ at any k - INE, and the assumption $X'' \geq X'$ one arrives at item ii) of the Lemma, $X_{-i}'' \geq X_{-i}'$. ■

Equipped with this Lemma, assume by way of contradiction that there exists (at least) two distinct k - INE

$$\left(e_1', ..., e_n', \widehat{X}_1', ..., \widehat{X}_n'\right) \ ,$$

and

$$\left(e_1'', ..., e_n'', \widehat{X}_1'', ..., \widehat{X}_n''\right) \ .$$

Suppose, without loss of generality, that $X'' = \sum_h e_h'' \geq X' = \sum_h e_h'$. Then according to item ii) in Lemma 1, it must be the case that, for any agent i, $X_{-i}'' = \sum_{h \neq i} e_h'' \geq X_{-i}' = \sum_{h \neq i} e_h'$. And, since the two equilibria are distinct, for some agent i the previous inequality is strict, $X_{-i}'' > X_{-i}'$, which implies:

$$X'' > X'. \tag{7}$$

Since c_i is a normal good and, from item i) in Lemma 1, $k\widehat{X}_i' - X_{-i}' \geq k\widehat{X}_i'' - X_{-i}''$, then $c_i' = C^i\left(p, k\widehat{X}_i' - X_{-i}'\right) \geq c_i'' = C^i\left(p, k\widehat{X}_i'' - X_{-i}''\right)$ for all i and with a strict inequality for at least one agent. Therefore $\sum_h c_h' > \sum_h c_h''$. But in equilibrium it must also be:

$$\sum_{h=1}^{n} c_h' = \sum_{h=1}^{n} e_h' = X'$$

Applying the same observation to the second k - INE, one finds:

$$\sum_{h} c_h'' = \sum_{h=1}^{n} e_h'' = X''.$$

Necessarily, if $\sum_h c_h' > \sum_h c_h''$ then $X'' < X'$, in contradiction with (7). So there cannot be two distinct k - INE. QED.

References and Notes

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