A decomposition of profit inefficiency into price expectation error, preferences towards risk and technical inefficiency

Jean-Philippe Boussemart
LEM-IÉSEG School of Management and University of Lille 3
3, rue de la Digue, 59000 Lille, France
Phone +33/(0)320545892 Fax+33/(0)320574855, jp.boussemart@ieseg.fr

David Crainich[1]
CNRS/LEM and IÉSEG School of Management
3, rue de la Digue, 59000 Lille, France
Phone +33/(0)320545892 Fax+33/(0)320574855, d.crainich@ieseg.fr

Hervé Leleu
CNRS/LEM and IÉSEG School of Management
3, rue de la Digue, 59000 Lille, France
Phone +33/(0)320545892 Fax+33/(0)320574855, h.leleu@ieseg.fr

[1] Corresponding author

Abstract:
The paper addresses the decomposition of firms' profit inefficiency (i.e. the difference between the observed profit and the maximal profit that could have been earned) under output price uncertainty. More precisely, we separate this inefficiency into price expectation error, expected profit loss due to risk preference and technical inefficiency. Within this decomposition, the allocative inefficiency is explicitly defined as the result of price expectation error and risk attitude instead of being a residual (as in the traditional profit inefficiency decomposition). Our theoretical model is then implemented in a Data Envelopment Analysis (DEA) framework which allows the separate estimation of each term of the decomposition. Besides, we offer an operational tool to reveal producers' risk preferences. While the DEA approach is appealing since it imposes very few assumptions on the production set, its main drawback lies in the sensitivity of the measure to outliers. We therefore adapt our model to a robust approach. A 2009 database of French fattening pig farms is used as an illustration. Results indicate that risk and technical components are the main sources of profit inefficiency while price expectation errors do not significantly affect profit losses.

Keywords: Profit Inefficiency, Risk Preference, Technical Inefficiency, Data Envelopment Analysis, Fattening Pig Farms.

JEL Classification: D21, D81, Q12

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1. Introduction

Price uncertainty is considered as one the causes of profit inefficiency. Long production lags imposed by biological processes combined with specific market conditions (inelastic demand, homogenous output, large number of small competitive producers) generating high price volatility are such output prices are unknown when production decisions must be made. As a result, e*x ante* production decisions differ from the choices that would have been made had producers known *ex post* output prices. The difference between the observed profit and the optimal profit is usually defined as the profit inefficiency.

Models dealing with producer behaviour in a context of output price uncertainty are examined in Sandmo (1971) or Chambers (1983) while risk production analysis with stochastic technology have been developed among others by Just and Pope (1978) or Chambers and Quiggin (2002). In the present paper we focus on a framework which associates profit inefficiency and output price uncertainty. More precisely, we decompose the profit inefficiency into three terms: price expectation error, expected profit loss due to risk preference and technical inefficiency. The two first terms define the allocative inefficiency. A main contribution of our model is to consider allocative inefficiency as a consequence of price uncertainty and risk attitude and not as a residual as it is done in traditional profit inefficiency decomposition (Färe et al. 1985, 1994).

A wide range of papers investigated risks preferences. One of the most interesting conclusions of these analyses was that the dispersion of risk preferences is always significant even within relatively homogeneous groups of firms. However, there are fewer empirical studies dealing with the joint estimation of technical or allocative inefficiency and risk preferences in the presence of output price uncertainty. For instance, on a panel of 28 Norwegian salmon farms, Kumbhakar (2002) showed that the degree of risk aversion - which varies substantially across producers and time - might bias parameter estimates on technology (technical change, input elasticity…). Based on the old idea of an inverse relationship between price uncertainty and allocative efficiency, Wu (1979) empirically investigated whether farmers allocate their resources more efficiently when prices are less random. His results based on small scale of

Taiwanese family farms strongly suggest that price and output uncertainty cause profit inefficiency. Our analysis goes beyond the commonly known connection between profit inefficiency and price volatility. We develop a model that formally bridges allocative inefficiency and ex post output price levels to characterize producers' risk aversion.

Our theoretical model is then implemented in a Data Envelopment Analysis (DEA) framework which allows the separate estimation of each term of the decomposition. DEA is particularly relevant to measure inefficiency in production. Since the theoretical background starting with works of Debreu (1951) and Koopmans (1951), going through the seminal paper of Farrell (1957) and its operationalization by Charnes et al. (1978), DEA has proved useful to model the efficient frontier of a technology and to measure the various inefficiencies (profit, allocative, technical, scale…) of observed production plans. As we introduce price expectation error and risk attitudes in the model, DEA allows us to measure the intensity of risk preference. However, while the DEA approach is appealing since it imposes very few assumptions on the production set, its main drawback is the sensitivity of the frontier to outliers (Dervaux et al., 2009). We therefore adapt our model to a robust approach developed by Cazal et al. (2002).

In order to show the applicability of our approach, an empirical illustration is provided using a sample of 149 French pig producers specialized in fattening units followed through the database GTE (Gestion Tecnhico-Economique) from IFIP (Institut de la Filière Porcine). The choice of this industry is motivated by two main reasons. First, the fattening process starts months before pigs are sold and the pig price is quite volatile, which means that output decisions must be made under price uncertainty. Second, comparatively to other agricultural activities or types of pig farms (breeder-fattener or breeder), pigs' fattening is not subject to climatic or technological risk, which is consistent with our model that only includes output price risk.

This paper is structured as follows. The next section first derives the decomposition of the profit inefficiency into price expectation error, expected profit loss due to risk preference and technical inefficiency. Section 3 introduces distance functions representing the production technology and allowing isolating technical and allocative terms from the overall profit inefficiency. Section 4 is devoted to the estimation aspects. It introduces the robust DEA approach to empirically estimate the technology frontier from which the three terms of the profit inefficiency are derived. Section 5 presents the sample and the specification of the empirical technology, tests the model and discusses the different components of the profit decomposition. Conclusions appear in Section 6.

2. Concepts: the profit inefficiency decomposition

In this section, we show how the margin profit inefficiency can be decomposed into three effects: (i) the inefficiency resulting from the output price uncertainty that may lead to inaccurate price anticipation; (ii) the expected profit loss resulting from preferences towards risky situations and; (iii) the technical inefficiency.

To make things simple, suppose that firms produce a single output y using a single input x. The production process is supposed to display variable (decreasing) returns to scale, hence it can be represented by an increasing and concave function: $y = f(x)$ (with $f'(x) > 0$ and $f''(x) < 0$). The input and output markets are both competitive, hence firms take the price of the input (denoted by w) and the price of the output (denoted p) as given. The output price is not known when production decisions are made. If firms knew this price, they would maximize the following (hypothetical) profit:

$$\Pi(p,w) = \max_{(y,x)} \{ py - wx : y \leq f(x) \} \qquad (2.1)$$

The first-order condition related to this program (which is given by $f'(x^*) = p/w$) defines the production plan (y^*, x^*) such that $(y^*, x^*) = \arg\max \Pi(p,w)$ and the associated maximum profit denoted π^*. This occurs at E^* in figure 1. This (optimal) profit is the highest firms can do since it implies: (i) no inefficiency in the technical or allocative process and; (ii) that they know the price at which the output will be sold. We attempt to explain the difference between this hypothetical first-best profit (optimal profit) and the profit resulting from the observed input and output (respectively denoted x^o and y^o). This observed profit (denoted π^o) is given by :

$$\pi^o = p^o y^o - w^o x^o \qquad (2.2)$$

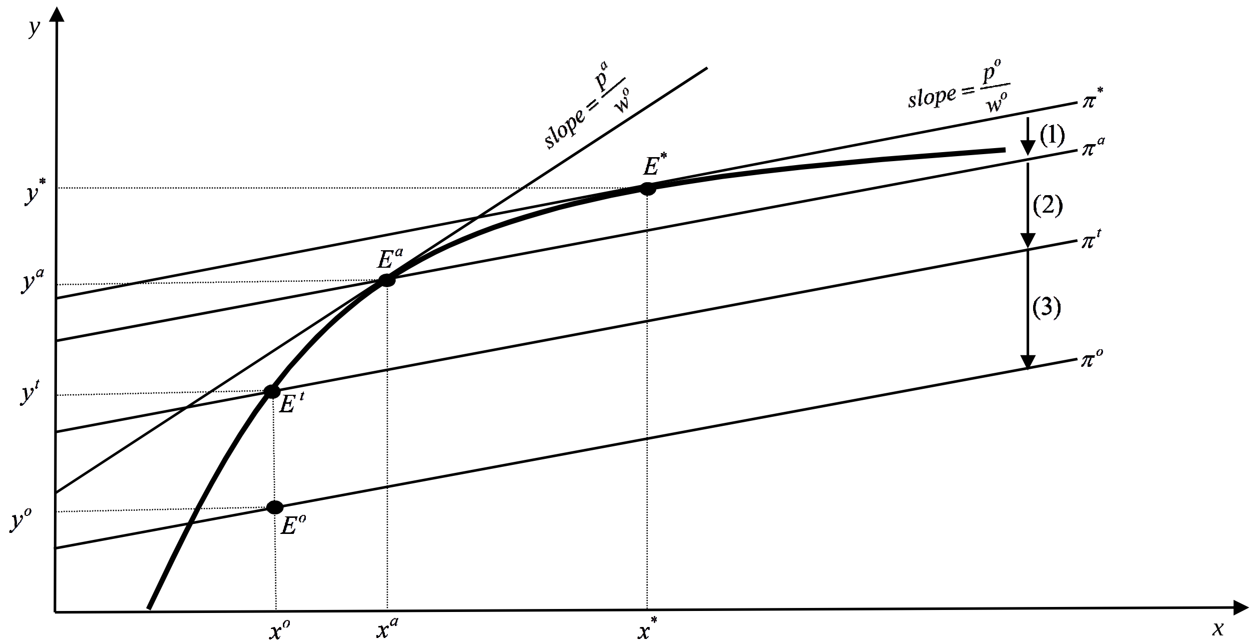
The profit inefficiency PI is defined as the difference between the optimal and the observed profit:

$$PI = \Pi\left(p^o, w^o\right) - \left(p^o y^o - w^o x^o\right) = \pi^* - \pi^o \qquad (2.3)$$

Three sources of profit inefficiency are considered. Besides the technical inefficiency commonly computed, we take into account that firms could misestimate the price at which the output will be sold and that they display risk preferences that may affect their output decisions. The following decomposition determines the three sources of profit inefficiency:

$$\begin{aligned} PI &= \pi^* - \pi^o \\ &= \pi^* - \pi^a \qquad (1) \\ &\quad + \pi^a - \pi^t \qquad (2) \\ &\quad + \pi^t - \pi^o \qquad (3) \end{aligned} \qquad (2.4)$$

Figure 1. Decomposition of profit inefficiency



The different sources of profit inefficiency are given by the three following terms.

Term (1): price anticipation inefficiency

The first term of the decomposition is the profit loss due to inaccurate anticipations of the output price. This inefficiency corresponds to the difference between the profits made at points E^* and E^a in figure 1. To maximize profits, firms must correctly anticipate the ex-post price p^o. Suppose they instead anticipate another price (denoted p^a). The equilibrium is defined through the tangency between the iso-profit curve which slope is p^a/w^o and the production function. The optimal combination of input (denoted by x^a) and output (denoted by y^a $(= f(x^a))$) given that price anticipation occurs at E^a in figure 1. Although the production decision is based on p^a, the output is finally sold at the *ex-post* price p^o. The price anticipation inefficiency is therefore the sole result of the output decision which is based on p^a rather than p^o. The optimal and the expected profits must thus be evaluated at the same price p^o. The latter profit (denoted π^a) is given by:

$$\begin{aligned} &\pi^a = p^o y^a - w^o x^a \\ &\text{where } \left(y^a, x^a\right) = \arg\max \Pi\left(p^a, w^o\right) \end{aligned} \qquad (2.5)$$

The margin profit inefficiency (term (1) in figure 1) resulting from the inaccurate output price anticipation is therefore given by:

$$\pi^* - \pi^a \qquad (2.6)$$

(2.6) is positive (resp. equal to zero) if the expected price p^a differs from (resp. is equal to) the actual price p^o. In figure 1, we have represented a situation where the anticipated price is lower than the ex-post price, leading thus to $x^a < x^*$ and to $y^a < y^*$. The opposite occurs in case $p^a > p^o$. In both situations: $\pi^* \le \pi^*$

Term (2): risk preference

The second cause of profit inefficiency results from firms' risk preferences. Even if firms knew the expected output price (the fact that they possibly do not correctly anticipate it is included in (2.6)), the risk on the output price may lead them to change their production plan. More precisely, Sandmo (1971) and Battra and Ulla (1974) - using respectively a single input-single output technology and a multi-input-single output technology - showed that risky prices give incentives to risk averse firms to reduce - compared to the certainty situation - the output they produce. Since the combination x^a and y^a is the risk neutral choice in case the expected price is p^a, the difference between x^a and the observed input (denoted x^o) indicates firms' risk preference. This production plan $\left(y^t, x^o\right)$ is the result of the following maximization program:

$$\left(y^t, x^o\right) = \arg\max\left\{Eu(\tilde{p}y - wx) : y \leq f(x)\right\} \qquad (2.7)$$

Following Sandmo (1971) and Battra and Ulla (1974), we can state that:

$$\begin{cases} x^o < x^a \Leftrightarrow \text{ risk aversion} \\ x^o = x^a \Leftrightarrow \text{ risk neutrality} \\ x^o > x^a \Leftrightarrow \text{ risk loving} \end{cases} \qquad (2.8)$$

Unless firms are risk-neutral, the output choice is not at the tangency point of an iso-profit curve and the production function since individuals do not maximize the expected profit but the expected utility (that is dictated by individuals preferences) they get from this profit. The point reached (E^t in figure 1) is however still technically efficient ($y^t = f(x^o)$), hence it lies on the frontier. The profit associated to this maximization problem (denoted by π^t) is given by:

$$\pi^t = p^o y^t - w^o x^o \qquad (2.9)$$

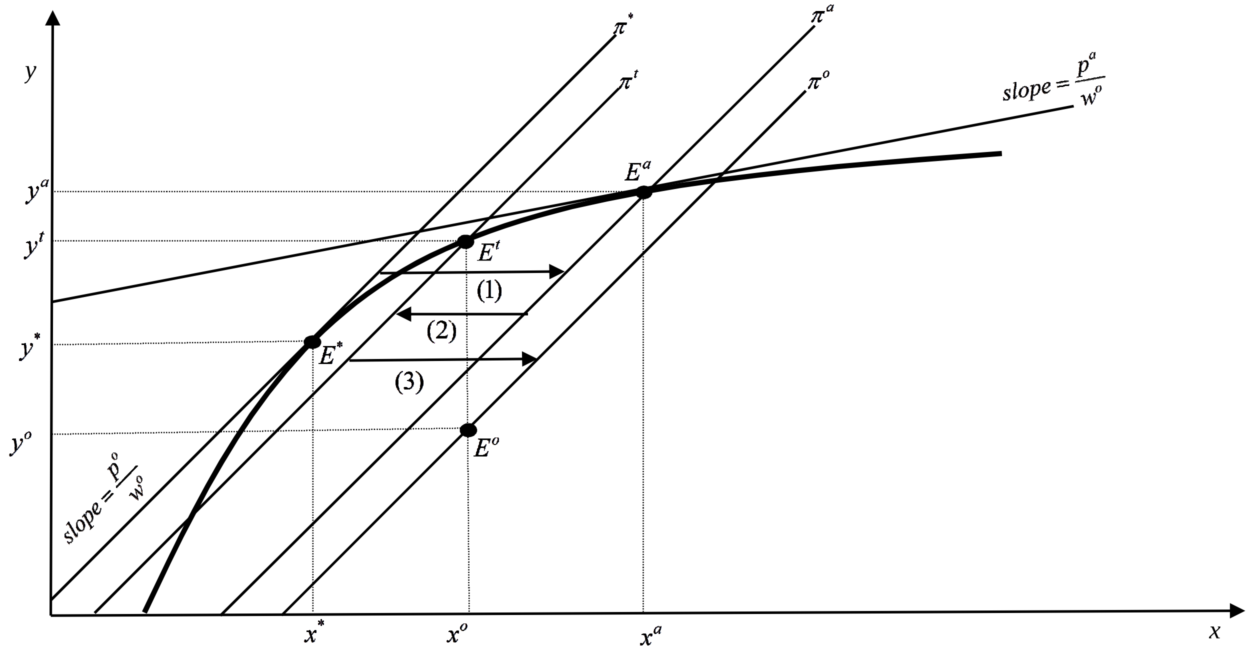
Once again, even if the output choice is based on the available information (*i.e.* the anticipated average price) the impact of this choice on the profit is evaluated at the ex-post price.

The risk preference inefficiency (term (2) in figure 1) corresponds to the difference between the profits made at points E^a and E^t in figure 1 and is given by:

$$\pi^a - \pi^t \qquad (2.10)$$

The situation represented in figure 1 is such that firms are risk averse and underestimate future output prices. Both effects lead firms to produce less output (compared to the certainty case) and therefore add to each other in the profit inefficiency decomposition. This happens not to be the case if (as represented in figure 2) risk averse preferences partially compensate overestimations of future output prices. In the same way, risk loving preferences can compensate profit losses due to underestimations of future output prices. Finally, profit inefficiency resulting from risk loving behaviour and future output prices overestimations add up since they both lead firms to produce more than in the certainty case. One can conclude that term (2) is positive if firms are risk averse or risk loving and nil in case they are risk neutral.

Figure 2. Effect of risk preferences on allocative inefficiency



Term (3): technical inefficiency

The third term in the margin profit inefficiency decomposition refers to the technical inefficiency that occurs when the output (denoted y^o) is lower than it should be given the input used (denoted x^o) and the production function. Technical inefficiency (which is related to elements such as non-optimal combinations of inputs, waste…) thus occurs in case $y^o < f(x^o)$. This inefficiency corresponds to the difference between the profits made at points E^t and E^o (that does not lie on the production function since it is not technically efficient) in figure 1. This inefficiency is given by:

$$\pi^t - \pi^o \qquad (2.11)$$

A recent paper from Fandel and Lorth (2009) has analysed profit maximisation under technical inefficiency by establishing mathematical and economic conditions for the profit-maximization approach to yield a technically efficient or inefficient solution. However they use a nonlinear inequality-constrained optimization framework to demonstrate that a profit-maximizing production may imply technical inefficiency. As we keep a traditional framework, we exclude this case and all production plans that maximize profit are necessarily technically efficient.

3. Methods: Shephard's distance function as a tool to measure profit inefficiencies terms

We first discuss how we implement the concepts defined in the preceding section into a general framework. We follow Shephard (1953, 1970) who developed a comprehensive framework to model production technology and all its dual representations (production function, cost function, revenue function, profit function…). Within Shephard's approach we can define the production function, its frontier and a measure of technical inefficiency such as the distance to the frontier. We can also model the profit function and find the profit-maximizing production plans relative to the observed and expected output prices.

Our model decomposes the profit inefficiency PI into three terms, namely the technical inefficiency TI, the price expectation inefficiency PEI and the risk preference RP. The former is a quantity effect which can be defined from a production frontier approach while the last two terms are price effects which can be derived from a profit function approach.

In order to assess the technical inefficiency, we begin by considering production technology. Following Shephard (1953, 1970), the technology is modelled with a production set in which a set of r resources (or inputs) $x \in \Re_{+}^{r}$ is used to produce a set of q outputs $y \in \Re_{+}^{q}$. The production set is defined as the set of physically attainable production plans (y,x):

$$\Psi = \left\{ (y,x) \in \Re_{+}^{q+r} : x \text{ can produce } y \right\} \qquad (3.1)$$

The boundary of this set (denoted by $\partial\Psi$) defines the efficient frontier where maximal output is obtained from a fixed input vector or equivalently where minimal inputs are used to reach a fixed output level. For any point (y,x) of the production set, the distance to the frontier can be defined by the output distance function:

$$\theta(y,x) = \min \left\{ \theta : (\frac{y}{\theta}, x) \in \Psi \right\} \qquad (3.2)$$

Following Farrell (1957) the inverse of the Shephard output distance function can be interpreted as the technical inefficiency defined as the maximal feasible output increase while maintaining a constant level of input. From this definition, the boundary of the production set can be defined by:

$$\partial\Psi = \left\{ (y,x) : \theta(y,x) = 1 \right\} \qquad (3.3)$$

We interpret the boundary of the production set as the efficient frontier. Production plans on the frontier are efficient with a technical inefficiency $\theta^{-1}(y,x) = 1$ while inefficient production plans are below the frontier with a technical inefficiency $\theta^{-1}(y,x) > 1$.

In order to assess the profit efficiency, we also consider input prices $w \in \Re_{++}^{r}$ and output prices $p \in \Re_{++}^{q}$. Let $(p,w) \in \Re_{++}^{q+r}$ denote a given output-input price vector, the profit function is defined by:

$$\Pi(p,w) = \max_{(y,x)} \left\{ py - wx : (y,x) \in \Psi \right\} \qquad (3.4)$$

Therefore, for any observed production plan $\left(y^{o}, x^{o}, p^{o}, w^{o}\right)$ the profit inefficiency PI is the difference between maximal and observed profits:

$$PI = \Pi\left(p^o, w^o\right) - \left(p^o y^o - w^o x^o\right) \qquad (3.5)$$

In order to decompose the profit inefficiency (3.5) into its terms, we compute a value-based measure of the technical inefficiency TI :

$$TI = \left(p^o \theta^{-1} y^o - w^o x^o\right) - \left(p^o y^o - w^o x^o\right) = p^o\left(\theta^{-1} - 1\right) y^o \qquad (3.6)$$

Considering now the expected output prices $p^a \in \Re_+^q$, we can define the optimal expected profit $\Pi\left(p^a, w^o\right)$ and the associated production plan $\left(y^a, x^a\right)$:

$$\Pi\left(p^a, w^o\right) = \max_{(y,x)}\left\{p^a y - w^o x : (x, y) \in \Psi\right\} \qquad (3.7)$$

$$\left(y^a, x^a\right) = \arg\max \Pi\left(p^a, w^o\right) \qquad (3.8)$$

The price expectation inefficiency PEI is defined as the difference between the maximum profit and the maximum anticipated profit, both computed at the observed prices:

$$PEI = \Pi\left(p^o, w^o\right) - \Pi\left(p^a, w^o\right) = \Pi\left(p^o, w^o\right) - \left(p^o y^a - w^o x^a\right) \quad (3.9)$$

where $\left(y^a, x^a\right)$ is defined in (3.8). As shown in the preceding section, the optimal production plan related to the expected output price can differ from the observed production plan due to the risk preference. A measure of the risk preference (RP) is given by the difference of profit between these two production plans:

$$RP = \left(p^o y^a - w^o x^a\right) - \left(p^o \theta^{-1} y^o - w^o x^o\right) \qquad (3.10)$$

Finally we obtain the following decomposition of the profit inefficiency:

$$PI = TI + PEI + RP \qquad (3.11)$$

4. Estimation: A robust DEA approach

As inefficiency measures involves comparison between actual and optimal performances located on the relevant production frontier, the production set Ψ as well the distance function $\theta\left(y^o, x^o\right)$ and the profit function $\Pi\left(p^o, w^o\right)$ defined in (3.1), (3.2) and (3.3) respectively need to operationalized. As the true frontier is unknown, an empirical best practice frontier has to be estimated. In that perspective the DEA framework, developed in Charnes et al.'s seminal paper (1978), is commonly considered as one of the relevant models for analysing such technical and allocative inefficiencies in a general multi-output multi-input context. Compared to econometric techniques, the non-parametric nature of this linear programming approach enables to avoid confounding the misspecification effects due to an arbitrary choice of functional forms of the technology and the inefficiency components. Nevertheless, as mathematical programming techniques are inherently enveloping techniques, the main practical inconvenient of a basic DEA approach is the difficulty to incorporate a statistical error component as usual econometrical approaches. Therefore its results are considered to be very sensitive to extreme observations of the reference production set which can be considered as potential outliers. To avoid this main drawback, Cazals et al (2002) and Darairo and Simar (2005) have recently developed robust alternatives to the traditional DEA inefficiency's estimator. These alternatives lie on the concept of partial frontier in contrast to the usual full frontier. In that line, this section is devoted to the estimation of the robust production frontier and finally the technical and profit inefficiencies from a sample of observed firms. Notice that throughout the presentation of the theoretical model we have always assumed a well-defined technology frontier. However in the empirical work, in order to take into account heterogeneity and exogenous factors in firms' production, we allow for the presence of outliers (producing above the frontier). We therefore need to compute the expected maximal profit and the various expected inefficiency sources in a robust way. We consider explicitly the presence of possible outlier observations by applying a variant of an approach devised by Cazals, Florens, and Simar (CFS) (2002).

Consider a sample of N observed firms for which input and output vectors for firm i $(i=1,\dots N)$ are respectively denoted by $x_i \in \Re_+^r$ and $y_i \in \Re_+^q$. The input and output price vectors are respectively denoted by $w_i \in \Re_{++}^r$ and $p_i \in \Re_{++}^q$. Let Ψ be the production set satisfying the core Shephard axioms (Shephard, 1953):

- A1: $(0,0) \in \Psi$, $(y,0) \in \Psi \Rightarrow y = 0$ i.e., no free lunch;

- A2: the set $B(x)=\{(y,u)\in\Psi : u\leq x\}$ of dominating observations is bounded $\forall x\in R_{+}^{r}$, i.e., infinite outputs cannot be obtained from a finite input vector;
- A3: Ψ is closed;
- A4: For all $(y,x)\in\Psi$, and all $(v,u)\in R_{+}^{q+r}$, we have $(-y,x)\leq(-v,u)\Rightarrow(v,u)\in\Psi$ (free disposability of inputs and outputs).
- A5: Ψ is convex;

We adopt the standard assumption that all observed firms face the same technology ψ and that all observed production plans are feasible: $(y_i,x_i)\in\Psi, i=1,\ldots,N$. From the observed sample and the set of axioms (A1-A5) the estimated technology can be represented by:

$$\hat{\Psi}=\Big\{(y,x):\ x\in R_{+}^{r},\ y\in R_{+}^{q},\ \sum_{i=1}^{N} z_i y_{ik}\geq y_k,\ k=1,\ldots,q,\\ \sum_{i=1}^{N} z_i x_{il}\geq x_l,\ l=1,\ldots,r, \sum_{i=1}^{N} z_i=1, z_i\geq 0,\ i=1,\ldots,N\Big\} \tag{4.1}$$

$\hat{\Psi}$ is an estimator of Ψ defined in (3.1). The estimated production set only relies on all the observed firms' production plans and the maintained axioms.

The output distance function (3.2) is easily derived from (4.1) and can be estimated with a linear program (LP):

$$\begin{aligned}
&\hat{\theta}\left(x^o, y^o\right)=\min_{\mathbf{z},\theta}\ \theta\\
&s.t.\\
&\sum_{i=1}^{N} z_i y_{ik}\geq\theta^{-1}y_k^o\quad\forall k=1,\ldots,q\\
&\sum_{i=1}^{N} z_i x_{il}\leq x_l^o\quad\forall l=1,\ldots,r\\
&\sum_{i=1}^{N} z_i=1\\
&z_i\geq 0\quad\forall i=1,\ldots,N
\end{aligned} \tag{4.2}$$

From (4.2), the estimated production frontier is defined as the boundary of the estimated production set:

$$\partial\hat{\Psi} = \left\{ (x,y) : \hat{\theta}(x,y) = 1 \right\} \qquad (4.3)$$

In the same way, the profit function is derived from (3.4) and (4.1) as an LP:

$$\begin{aligned}
&\hat{\Pi}\left(p^o, w^o\right) = \max_{\mathbf{z},\tilde{\mathbf{y}},\tilde{\mathbf{x}}} \ \sum_{k=1}^{q} p_k^o \tilde{y}_k - \sum_{l=1}^{r} w_l^o \tilde{x}_l \\
&s.t. \\
&\quad \sum_{i=1}^{N} z_i y_{ik} \geq \tilde{y}_k \quad \forall k = 1,\ldots,q \\
&\quad \sum_{i=1}^{N} z_i x_{il} \leq \tilde{x}_l \quad \forall l = 1,\ldots,r \\
&\quad \sum_{i=1}^{N} z_i = 1 \\
&\quad z_i \geq 0 \ \ \forall i = 1,\ldots,N
\end{aligned} \qquad (4.4)$$

From (4.2) and (4.4), estimates of the profit inefficiency and each term of its decomposition can be obtained. This is the traditional deterministic approach. However, while this approach is appealing since it imposes very few assumptions on the production set, its main drawback is the sensitivity of the frontier to outliers. To circumvent this problem, we use a variant of the CFS (2002) approach. We refer interested readers to Cazals, Florens, and Simar (CFS) (2002) for all theoretical and methodological developments. For our purposes, we only provide an intuitive presentation of the approach.

Estimations from (4.2) and (4.4) can be biased if outliers exist defining the estimated production set and the associated frontier. To avoid this problem, we select a large number of sub-samples, of a predetermined size, from the initial observed firms, and compute the final estimate as the average over the sub-samples. Therefore, since the estimated production set varies over the samples, the evaluated production plan is not always compared with potential outliers, but the outlier is not totally ignored either. The final result can be interpreted as a robust measure of the inefficiency.

We now describe how the computational algorithm works. First, for a given evaluated production plan $\left(y^o, x^o\right)$, we repeat several Monte-Carlo replications to compute a robust output distance function. For each replication $(b = 1, \dots, B)$ we draw a random sample of size M with replacement from the initial sample of observed firms. The associated production set is denoted by $\hat{\Psi}_M^b$:

$$\hat{\Psi}_M^b = \Big\{ (x, y):\ x \in R_+^r,\ y \in R_+^q,\ \sum_{i=1}^{M} z_i y_{ik} \geq y_k,\ k = 1, \dots, q, \\ \sum_{i=1}^{M} z_i x_{il} \geq x_l,\ l = 1, \dots, r, \sum_{i=1}^{M} z_i = 1, z_i \geq 0,\ i = 1, \dots, M \Big\} \tag{4.5}$$

Next, we compute the output distance function (3.2) relative to this sample:

$$\hat{\theta}_M^b\left(y^o, x^o\right) = \min\left\{\theta : \left(\theta^{-1} y^o, x^o\right) \in \hat{\Psi}_M^b\right\} \tag{4.6}$$

Finally, we repeat this for $(b = 1, \dots, B)$, where B is the number of Monte-Carlo replications and we compute the final robust output distance function as:

$$\hat{\theta}_M\left(y^o, x^o\right) = \frac{1}{B}\sum_{b=1}^{B} \hat{\theta}_M^b\left(y^o, x^o\right) \tag{4.7}$$

In the same way, we compute the profit function relative to the subsamples:

$$\hat{\Pi}_M^b\left(p^o, w^o\right) = \max_{(y,x)}\left\{p^o y - w^o x : (x, y) \in \hat{\Psi}_M^b\right\} \tag{4.8}$$

And the final robust maximal profit:

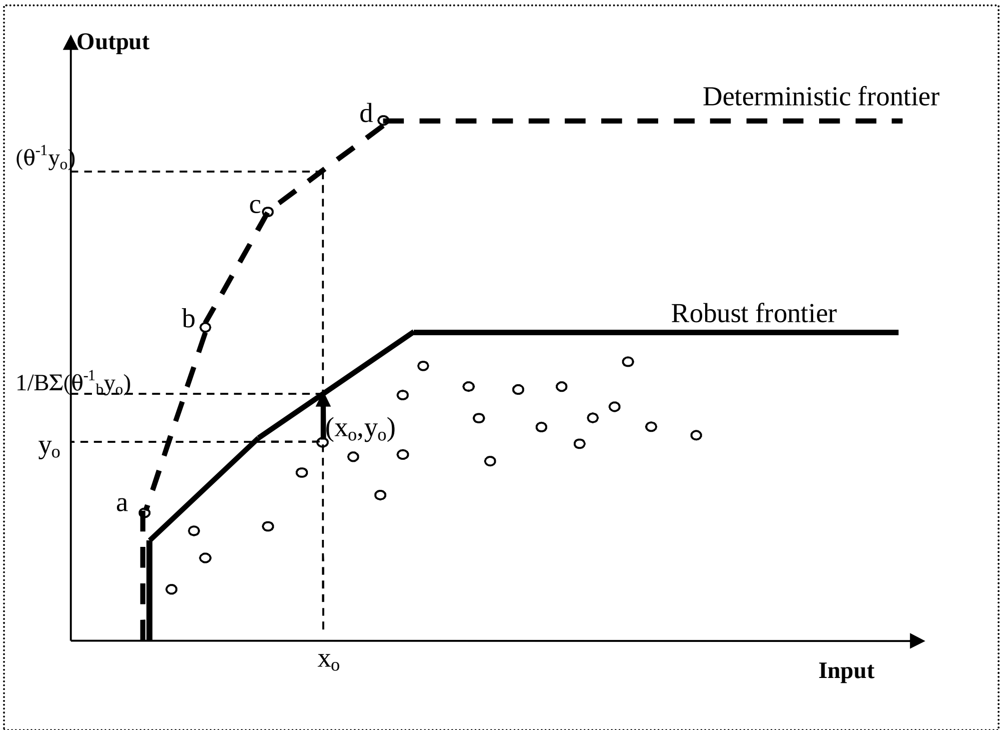
$$\hat{\Pi}_M\left(p^o, w^o\right) = \frac{1}{B}\sum_{b=1}^{B} \hat{\Pi}_M^b\left(p^o, w^o\right) \tag{4.9}$$

The difference between the CFS approach (4.5) and the traditional deterministic model (4.1) lies in the observations selected in the production set. While the entire N observed firms are in (4.1), only a subsample of M can be found in (4.5). A direct consequence is that, in a deterministic approach, the evaluated production plan always belongs to the production set and therefore the output distance function is always less than or equal to 1. However, in the CFS approach, the sampling process used on the production set does not imply that the evaluated production plan is always in the sample. Therefore, the output distance function can be strictly greater than 1 if the evaluated production plan does not belong to the subsample.

In order to develop the measure of efficiency under the CFS approach, two parameters are introduced – the number of replications B and the size of the sub-samples M. The number of B Monte-Carlo replications is not crucial, since as in all Monte-Carlo processes, we can control the sensitivity of the results by making B sufficiently large. (The number chosen for the B replications is just a question of tractable computational time.) The size of the samples denoted as M is a more central issue in applied analyses. From a theoretical point of view, by using M approaching infinity, we retrieve the traditional estimator (4.2) since the probability for each production plan in the initial observed sample belonging to each subsample approaches one. In using the robust version, we have to choose the value of M between zero and infinity (or at least large enough values for M). Again we can control the sensitivity of the results by making M vary in a sufficiently large range.

To illustrate the intuition behind the robust approach, Figure 3 summarizes the CFS model compared to a deterministic model. For the sake of simplicity, we restrict our example to technology with one output and one input. The distance function and the related measure of inefficiency is output-oriented and is measured along a direction given by the output vector. The dashed line frontier represents the deterministic frontier while the frontier in bold represents the robust approach. It is clear from the illustration that four production plans (*a*, *b*, *c*, and *d*) determine the whole deterministic frontier while the points *b*, *c*, and *d* can be considered as outliers. However, the robust approach operates differently. First, we draw B random samples of size M with replacement. If $B=500$ and $M=10$, we will draw 500 samples of 10 observations. Now the robust output distance function is computed for each subsample and the final result is simply the average. Note that, observations *b*, *c*, and *d* will be in some but probably not in all the samples. If none of these observations are in a sample, the production plan $\left(y^o, x^o\right)$ will be efficient since no other points dominate it. As a result, the evaluated production plan is not always compared to the outliers and we obtain a more robust measure of the estimated output distance function and the related technical inefficiency.

Figure 3. The robust vs. the deterministic production frontier



We are now in position to give the robust computation of each term of the decomposition of the profit inefficiency:

$$PI = TI + PEI + RP \qquad (4.10)$$

where:

$$PI = \hat{\Pi}_M\left(p^o, w^o\right) - \left(p^o y^o - w^o x^o\right)$$

$$TI = p^o(\hat{\theta}_M^{-1}(x^o, y^o) - 1)y^o$$

$$PEI = \hat{\Pi}_M\left(p^o, w^o\right) - \hat{\Pi}_M\left(p^a, w^o\right)$$

$$RP = \left(p^o y^a - w^o x^a\right) - \left(p^o \hat{\theta}_M^{-1}(x^o, y^o) y^o - w^o x^o\right)$$

where $(y^a, x^a) = \arg\max \hat{\Pi}_M\left(p^a, w^a\right)$

5. Empirical analysis of pig fattening producers' attitudes to risk

This section describes the data used, the technology specification and the farmers' adaptive expectation process for output price. The profit decomposition and the inefficiency scores are then estimated, analysed and discussed.

5.1. Sample description

This study uses technical and accountancy figures of the year 2009 from a sample related to 149 French pig producers specialized in fattening units. These producers are followed by the IFIP (Institut de la Filière Porcine) organization which is a French research institute for pig industry. The database of technical and economic indicators enables us to analyse and compare the performance of pig producers. The farms are distributed among nine main different regions of France. Forty percent of them are located in Brittany which is the most concentrated area for pig production. Compared to the national average of fattening pigs per farm which is around 260 pigs, the mean size of our sample is significantly higher (1859 animals). This is explained by the specialization of producers in the sample in fattening units. The most frequent system of French pig farming is the "breeder-fattener" structure which is usually smaller.

Table 1: General Descriptive Statistics (year 2009)

	Mean	std	coef var	min	max
Total entries of piglets	1 939	1 384	71.4%	239	7 235
Total fattening pigs	1 859	1 337	71.9%	237	6 962
Total kilos of meat	216 762	153 692	70.9%	26 700	802 410
Sales (€)	284 269	200 520	70.5%	34 844	1 055 169
Feeding cost (€)	118 962	78 611	66.1%	16 408	392 378
Piglet cost (€)	119 476	91 019	76.2%	16 679	522 234
Gross profit (€)	45 831	42 306	92.3%	-4 959	207 022

General descriptive statistics of the sample are detailed in Table 1. Farms fatten on the average 1 939 piglets, which represent 1 859 final produced pigs, 217 tonnes of meat and a total revenue of 284 264€. The two main variable costs (feed and piglets) have nearly the same level: 119 000 euros each, leading to a gross margin of 45 831€. The sample contains some heterogeneity in production with a standard deviation higher than 1 300 pigs and an interval of variation of nearly 7 000 units. Since the gross profit takes into account both the technical and the managerial capabilities, its coefficient of variation is significantly higher than other indicators that are only related to the technical dimension.

5.2. Technology specification

First, in order to compare farms independently of their respective heterogeneous sizes (which can be considered as fixed in the short run period), the retained variables in the technology and the associated gross profit function are expressed by pig. Second, given the fact that their respective capacity constraints are fully employed, producers are not able to significantly adapt their production to the output price by changing the number of pig entries. Output can however be adapted through the duration in fattening days of each batch and the total number of batch rotations per year. This results in changes in the number of pigs produced annually. This is confirmed by noticing that there are few differences between the sizes of different batches within the same pig farm. In figure 4 we notice that the range of pig entries in each batch is between 155 and 172. However, pig producers substantially differ in the number of days during which they fatten pigs. On average the duration of fattening period is 118 days in our sample but varies from 93 to 140 days (see figure 5).

Figure 4. Average number of pig entries per batch

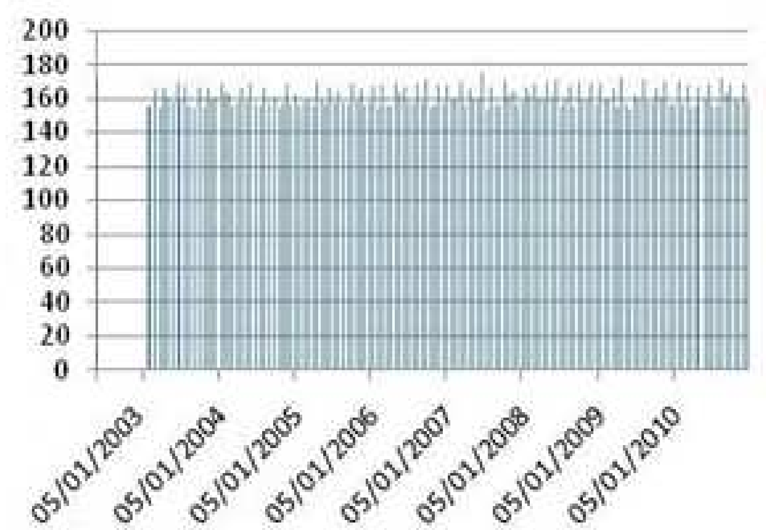
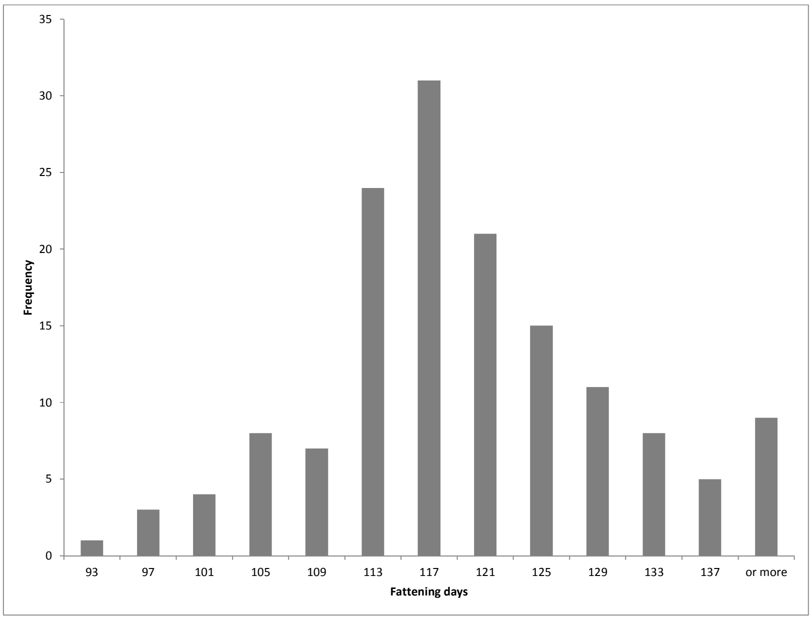


Figure 5. Distribution of fattening period



Taking into consideration these elements, the fattening performance for pig farms can be analysed through usual criteria: Average Daily Gain (ADG), Feed Conversion (FC), and Feed Efficiency (FE). ADG is expressed as the ratio of the pig growth weight to the fattening. Feed conversion (FC) is defined as feed consumption divided by the weight growth. Feed efficiency (FE) is the corresponding productivity index defined as the ratio of ADG to FC. For a certain carcass quality, an efficient feeding strategy depends both on a rapid growth rate and a low feed conversion ratio. As shown by Quiniou et al. (2004), this latter index is also influenced by the piglet weight entry. Starting from this standard benchmark approach for pig producers, the retained technology and its associated gross profit function are therefore defined by the following variables: the daily average pig weight growth is the output, the kilos of feed used per pig per day and the piglet entry weight divided by the number of fattening days are the two inputs. Finally, the daily gross profit in euros is defined by the difference between the value of meat and the two input costs. Compared to the ADG and FC indices commonly used by zoo-technicians, our model offers a more general specification. First we extend the FC partial productivity index to a global productivity index by taking into account both feeding and piglet weight entry. Second, while the ADG index relies on a linear relationship between entry and exit weights, our specification include increasing, constant or decreasing marginal growth rate of pig weight.

Table 2 presents the descriptive statistics related to our model. For an average fattening period of 118.3 days, producers obtain a pig weight of 118.3 kilos for a cost of 99.6 € (feed + piglet). As the initial piglet weight is 28.5 kilos, the net fattening weight is around 90 kilos which means an average daily gain of 760 grams. Considering the observed sale price per kilo of meat, the daily gross margin per fattening pig is 0.48 €. Table 2 presents all the data expressed per day.

Table 2: Descriptive statistics of the production technology and profit function variables

	Mean	Coef var
Average Daily Gain (kg)	0.76	0.08
Pig weight / fattening days (kg)	1.00	0.10
Feed per pig per day(kg)	2.26	0.08
Piglet weight / fattening days (kg)	0.24	0.22
Observed sale price (€ per kg)	1.32	0.10
Expected sale price (€ per kg)	1.54	0.10
Feeding cost (€ per kg)	0.19	0.06
Piglet cost (€ per kg)	1.72	0.19
Gross margin per pig per day (€)	0.48	0.23

5.3 Price expectations

This sub-section describes the way we model the output price expectations made by farmers. We assume that farmers' output decisions are based on the expected prices extracted from the quarterly letter published by the IFIP. The IFIP letter gives output price forecasts for the following months and farmers in the sample are all affiliated to the IFIP (the extract of the forecast for the year 2009 is given in Appendix). It thus seems plausible to assume that farmers have these forecasts in mind when making decisions. Therefore, we are not constrained to make assumptions about the way previous output prices enters into the individual forecast made by farmers about future output prices (as in Chavas and Holt (1990) for example). More precisely, the production plan decided in the m^{th} months takes into account the most recent output price forecast (denoted $p_{m,m+4}$) made for the $(m+4)^{th}$ month (piglets are sold on average about 4 months after their entry). In case a more recent output price forecast (denoted by $p_{m+n,m+4}$) is available in the $(m+n)^{th}$ month ($n<4$), farmers adjust their feeding process and, at the end of the $(m+4)^{th}$ month, the overall production plan has been made according to an expected output price (denoted by p^a_{m+4}) which is a weighted average of the two forecasts, *i.e.* $p^a_{m+4}=\frac{n}{4}p_{m,m+4}+(\frac{4-n}{4})p_{m+n,m+4}$. In order to illustrate this technique, we explain how we computed the price at which farmers, during the feeding process, expected to sell their output in April 2009. Pigs sold in April 2009 entered in the farm in January 2009 (*i.e.* 4 months earlier). At that time, farmers expected to sell their output in April at 1.56€/Kg (according to the forecast made by the IFIP in November 2008, cf. Appendix). In February 2009, farmers still expected to sell their output at the same price (according to the same forecast). A new IFIP newsletter was published in March 2009, indicating that the price of the output in April 2009 should be 1.37€/Kg. The same price is taken into account in April 2009, date at which the output is sold. Therefore, during the feeding process, farmers expected for two months the output price of 1.56€/Kg and the output price 1.37€/Kg for the two other months. The expected price taken into account in our calculation is therefore given by: $\frac{1.56+1.56+1.37+1.37}{4}=1.465$

Besides, we observe divergence among farmers' prices explained by elements such as the quality of meat, the local market price or the farmer's reputation,… As a result, this gap between the individual price and the average price is included in farmers' price expectations. According to these assumptions, we evaluate an expected price of 1.54 € which is 17% above the observed price on average.

5.4. Profit decomposition and inefficiency results

In order to soften the potential outlier influences on the final results, we develop a robust approach. Therefore the production frontier and its associated profit function are estimated for

B=100 replications of each simulated production plan with 3 different values of the M parameter: 33%, 50% and 75% of the initial sample size respectively.

The daily average maximum gross margin per pig evaluated with the observed prices can reach around 61 euro cents. Therefore producers could improve this margin by about 13 euro cents. Table 3 indicates how this profit gap can be split between expectation, risk and technical components according different M values. For instance, with a M value of 75%, the profit gap decomposition is established as follow: 0.4, 3.7 and 9.3 euro cents respectively. It is worth noting that the magnitudes of profit gap decomposition remain constant independently of the different scenarios of M values. As a consistent result, risk and technical components are the main sources of overall inefficiency while price expectation errors do not significantly affect profit losses.

Table 3: Average inefficiency results according different simulation scenarios

	Overall	Expectation	Risk	Technical
M = 75%				
Potential profit growth in euro cents	13.4	0.4	3.7	9.3
Potential profit growth in %	27.7	0.8	7.7	19.3
Inefficiency decomposition in %	100.0	2.8	27.6	69.6
M = 50%				
Potential profit growth in euro cents	12.7	0.3	3.8	8.6
Potential profit growth in %	26.3	0.6	7.9	17.8
Inefficiency decomposition in %	100.0	2.2	30.0	67.8
M = 33%				
Potential profit growth in euro cents	12.0	0.3	4.0	7.8
Potential profit growth in %	24.9	0.6	8.2	16.0
Inefficiency decomposition in %	100.0	2.4	33.1	64.5

The technical component includes various factors. Some of them are directly linked to inefficiencies such as technical and managerial farmers' skills. However some of them are due to specificities not included in the analysis as the pig type, its genetic quality, the feeding composition or observed differences in the state of the housing units. As our framework aims at comparing the relative shares of total profit losses due to price expectation errors and risk inefficiency, it is important to explicitly consider the profit gap explained by technical inefficiency and to project farms on the production frontier on which the profit function is defined.

Beyond this profit decomposition, a by-product of our analysis is the assessment of farmer's risk attitude. The risk attitude is determined by the comparison of the efficient and the expected outputs. As indicated in section 2, if the former is greater than the latter, the farmer appears as risk lover. Since a robust analysis has been developed, each farmer is evaluated in 100 different subsamples. Therefore, we can establish the frequency with which he/she appears as risk lover, risk neutral or risk averse. For example, on average, farmers are risk averse, risk neutrals and risk lovers in 58.1%, 1.2% and 40.7% of the replications respectively. As shown in table 4, this conclusion holds steady independently of the 3 different simulation scenarios estimating the robust production frontier.

Table 4: average frequencies of risk attitudes (%)

M value	Risk averse	Risk neutral	Risk lover	Total
75%	58.1	1.2	40.7	100.0
50%	62.8	1.7	35.5	100.0
33%	66.9	2.4	30.7	100.0

5.5. Discussion

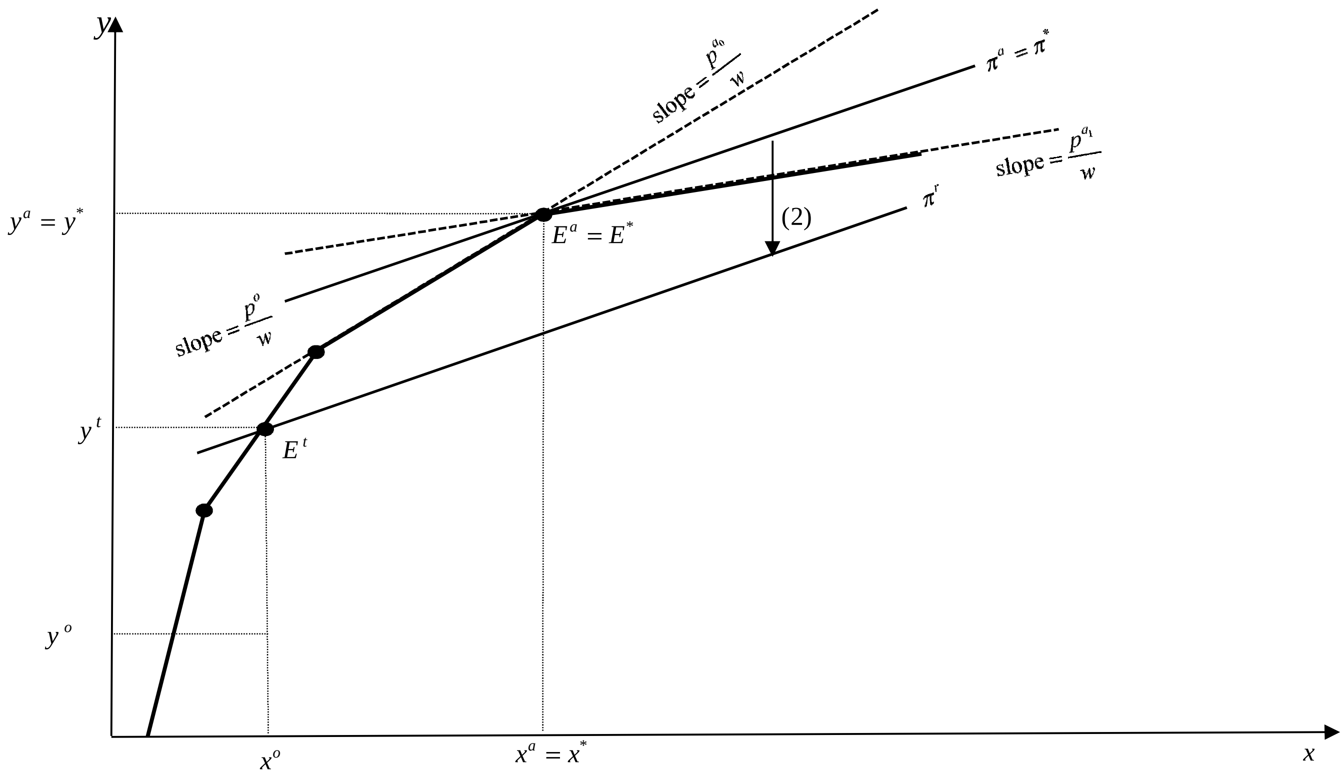
Our empirical estimation indicates that inaccurate price anticipations play a minor role in the farmers' expected profit loss while risk preferences have a substantial effect on this loss. In this section, we provide the main reasons driving this result.

It may seem quite unexpected that risk aversion (when combined with price overestimation) produces substantial profit losses. As explained in section 2 (see fig. 2), risk averse behavior (that most farmers of our sample display) generates expected profit gains when the future prices are overestimated (as it was the case in 2009). Actually, the impacts that inaccurate price anticipations and risk preferences have on the expected profit loss are both the consequence of the fact that, for most farmers of our sample, the profit based on the anticipated price (π^a) and the optimal profit (π^*) are equal. This can be explained for two reasons:

1. The production function is piecewise linear

When the estimated production function is piecewise linear, price anticipations in the neighborhood of the actual price lead to the matching of the production plans at the optimum and at the anticipated prices ($x^a = x^*$ and $y^a = y^*$). Consequently, the optimal profit and the profit at the anticipated prices ($\pi^a = \pi^*$) are equal. This can be explained graphically using figure 6.

Figure 6. Effect of piecewise linear frontier on optimal and anticipated profit maximization



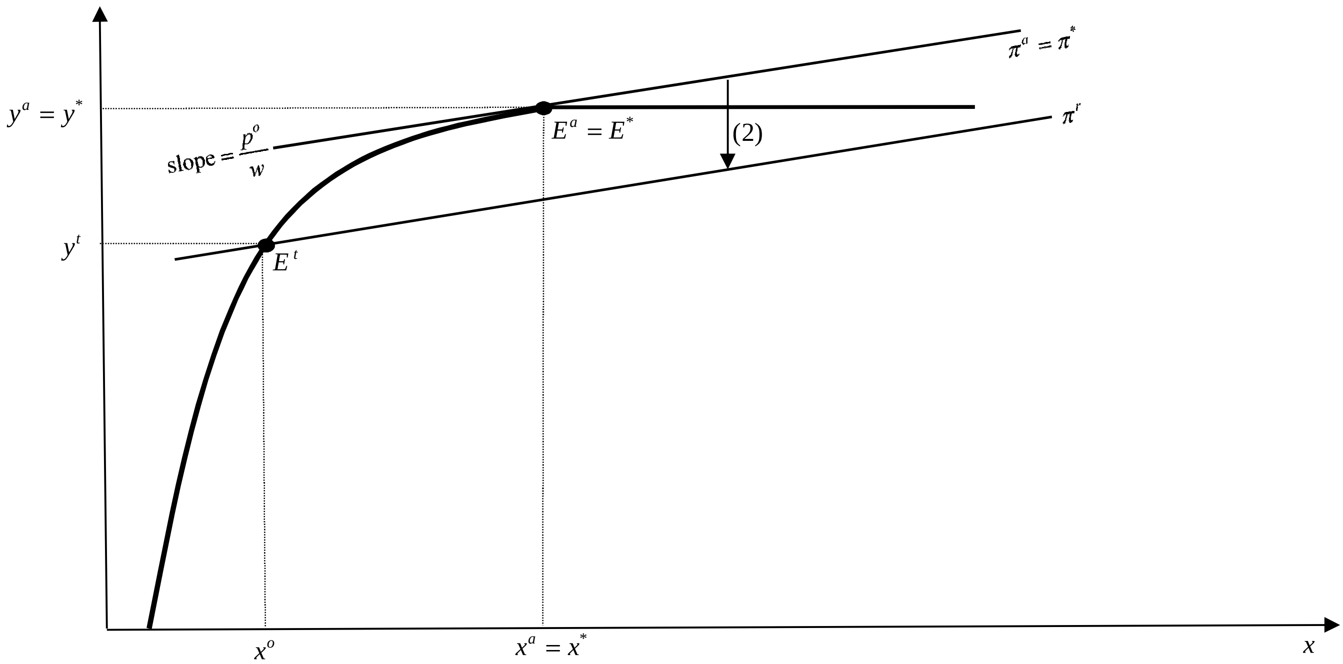
In this example, the current output price p_y is such that the input x^* and the output y^* maximize the expected profit. However, any anticipated output price included between p^{a_0} and

p^{a_1} also leads to the same production plan. Therefore, errors in price anticipations (as long as the anticipated price is not too far from the current price) do not result in differences between the optimal profit $(\pi^* = p^o y^* - w^o x^*)$ and the profit that farmers make at the anticipated price $(\pi^a = p^o y^a - w^o x^a)$. As a result, lower output due to risk aversion does not compensate the profit loss due to the overestimation of the future output price but is a source of expected profit loss.

2. The production function is strongly concave (almost horizontal)

To understand the effect of low slopes of the production function on the profit decomposition, let us first show that the matching of the production plans at the optimum and at the anticipated prices occurs when the production function is horizontal.

Figure 7. Effect of strong concavity on optimal and anticipated profit maximization



In figure 7, the input x^* and the output y^* maximize the expected profit at the current output price p^o. It is also straightforward to notice that any anticipated price higher than p^o leads to the same production plan ($x^a = x^*$ and $y^a = y^*$). Thus, here again, risk aversion lowers the expected profit $(\pi^r < \pi^*)$ even if the anticipated price is higher than the actual price. Since the IFIP overestimated output prices in 2009, the low slope of the production function in the area considered explains that the impact of the anticipated price of the profit loss is low while risk aversion substantially lowers the expected profit.

The two above explanations are more likely to appear in the presence of a small set of very efficient firms which define the shape of the production function. It is the reason why we have attached a great importance to propose a robust approach in order to control for the presence of outliers.

6. Conclusion

It has long been suspected that price uncertainty may cause profit inefficiency by inducing wrong decisions in output/input firm choices. So far, most empirical studies have investigated whether firms allocate their resources more efficiently when prices are less random. Beyond this commonly known connection between profit inefficiency and price volatility, our analysis goes one step further by making the bridge between allocative inefficiency and *ex post* output price levels. Instead of measuring allocative inefficiency as a residual term as in traditional DEA framework, we explicitly model allocative inefficiency due to error in price expectation and inefficiency due to risk preference. A contribution of our model is to typify producers regarding their risk attitudes and measure the intensity of their risk preference. We also investigate the way to estimate a robust production frontier to take into account the fact that, in empirical works, outliers often arise from heterogeneity and exogenous factors in firms' production.

An illustration for the French pig industry has been presented. Our results reveal that the main sources of profit inefficiency are the risk and technical components while price anticipations do not significantly affect profit losses. Further, the frequencies of farmers' risk attitudes have been determined. On average, farmers appear risk averse, risk neutrals and risk lovers in 58.1%, 1.2% and 40.7% of the cases respectively. We hope that this methodological paper can open the door to other empirical works which will aim at measuring the impact of risk preference on profit.

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Appendix

French price of finishing and fattening pigs
(quotation Class E, slaughter entry, €/kg of carcass)

		Forecast date			
		November 2008	**March 2009**	**June 2009**	**September 2009**
	January 2009	1,35			
	Fabruary 2009	1,40			
	March 2009	1,50			
	April 2009	1,56	1,37		
Anticipated	**May 2009**	1,55	1,46		
output	**June 2009**	1,76	1,67	1,40	
price	**July 2009**	1,78	1,70	1,55	
	August 2009	1,84	1,76	1,59	
	September 2009	1,80	1,73	1,55	
	October 2009	1,70	1,64	1,45	1,25
	November 2009	1,66	1,58	1,48	1,20
	December 2009	1,64	1,56	1,45	1,15

Source: IFIP