Direct-selling farming under urban pollution: what impact on competition, variety and goods quality?

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Abstract

In this paper, we study how the proximity to cities affects the decision of farmers to enter the direct-selling market. We develop a spatial economic model which takes into account the externality of urban pollution on agricultural yields. We find that urbanization may foster direct-selling farming development provided that the market size effect dominates the net income effect. A corollary of this result is that regions hosting an intermediate-size city are more likely to supply a wider range of varieties. Additionally, we highlight that the greater the gap in productivity between the urban fringe and the right-hand side direct-selling boundary, the smaller the opportunities for farmers to engage in direct-selling, and the larger the quality differentiation between varieties. We finally show that the market equilibrium always leads to a number of direct-selling farmers which is too low to fully satisfy urban households, but too much high from the farmers standpoint.

Keywords: Urban Pollution, Direct-selling Farming, Spatial Externality.

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1 Introduction

In the present context of rapid worldwide urbanization, feeding the cities in the "Global North" is drawing a substantial public awareness [Morgan, 2014]. Evidence of this trend is found in the growing policy support for sustainable food supply chains, combining geographical proximity, reduced-reliance on synthetic inputs, and food quality and traceability. In the US as in several European countries, national programs for sustainable development now often address urban food supply, with a strong emphasis on building local alternatives [see notably USDA [2014] for the US, Kneafsey et al. [2013] for the EU, or DGAL [2011] for France]. Initiatives of cities such as New York, Montreal, London, or Paris are among the many examples illustrating that urban agriculture is gradually gaining ground.

When considering the impact of pollution stemming from urban activities on agricultural yields however, the benefits of local food production can be seriously questioned. As now shown by numerous research, urban pollution adversely affect agriculture in many complex ways, causing reduced yield and quality in crops exposed to pollutants [see e.g., Adams et al. [1986], Kuik et al. [2000], Emberson et al. [2009], or Feng et al. [2015]]. Avnery et al. [2011] notably estimate that reductions of global yields due to ozone exposition could reach 2% for maize, 3.9 to 15% for wheat, and 8.5 to 14% for soybean. Still focusing on ozone pollution, Holland et al. [2006] show that the directly-induced economic consequences are far from being negligible, establishing the losses for Europe in 2000 to 6.7 billion euros. Moreover, in contrast with GHG emissions and global warming, the issue of pollution is fundamentally local and strengthens therefore the necessity to adopt a local focus.

In this paper, we investigate whether alternative farming can develop in the neighboring of highly-crowded cities. Current contributions on farming development in areas under urban influence commonly emphasize that local food outlets such as farmers' markets are likely to meet a significant and fast-increasing new kind of demand. In affluent cities notably, consumers, seeking both for quality and safety, have more and more demanding expectations with respect to the social and ecological implications of the food they purchase [Deutsch et al., 2013]. Since products available for sale at farmers' markets are viewed to be of higher quality compared to products available elsewhere [Smithers et al., 2008], it seems reasonable to think that direct-selling farming have strong opportunities to develop.

Although the literature on periurban agriculture is quite extensive, covering various topics such as the impacts of urbanization on agriculture [Berry, 1978], land value [Plantinga et al. [2002]; Anderson and West [2006]], or neighboring conflicts [Lisansky [1986]; Irwin and Bockstael [2004]], issues seem to have been mainly analyzed from the amenities standpoint, most of the works focusing on the impacts of agriculture and farmland on cities, but rarely the reverse [Cavailhès et al. [2004]; Bento et al. [2006]; Coisnon et al. [2014]]. In fact, there are to our knowledge, only few theoretical formalizations of the issue we propose to handle. Among these papers are Lopez et al. [1988] , Lockeretz [1989], and Wu et al. [2011].

The first ones have developed a framework to estimate the effects of suburbanization on agricultural production choices, prices, and profits, and found that, although vegetable production may benefit from urbanization, other agricultural sub-sectors such as grain crops or livestock are adversely affected. Focusing on the agricultural trends in mid-western counties at varying distances from the urban centers, Lockeretz [1989] has emphasized that metropolitan counties experienced the most rapid loss of farmland, a loss that was however partially offset by increasing intensity. Finally, Wu et al. [2011] offer one of the most complete work from a theoretical standpoint. Investigating the effects of urbanization on the viability of farm-supporting sectors, they have built a model where opportunities lie on the benefits offered from being part of a large farming community and have highlighted that the effect of urbanization on the agricultural infrastructure, inputs costs, and farmers' profit can be either positive or negative.

The purpose of this paper is to develop a theoretical framework enabling to investigate *how the proximity to cities affects the decision of farmers to enter the direct-selling market* and therefore, *food diversity*, as well as the *quality of the agricultural goods* supplied to consumers. Formally, we explore this question by building a spatial economic model which takes into account the externality of urban pollution on agricultural yields, and where farmers can choose between producing homogeneous conventional goods –under perfect competition– or horizontally and vertically differentiated direct-selling goods –under monopolistic competition. Justifications to relax the classical assumption of perfect competition for local farming can be found in the literature related on purchase motivations and consumers' perception; empirical studies on alternative food networks conducted in the last decade greatly support the idea that, for a majority of consumers, products sold at farmers' markets are perceived of a higher quality than those sold at regular grocery stores [see e.g. Dodds et al. [2014]]. It seems thus more realistic to consider that farmers operating on direct-selling market have a substantial leeway to choose their price.

The framework used in this paper also displays heterogeneity between farmers; the latter face spatial externalities that depend on the city size and induce different productivity levels according to their location within the region. As for the spatial aspect, the model follows the pioneering contribution of Alonso [1964]. We consider a monocentric city in which urban pollution acts as a distance-dependent externality.

As in standard non-spatial model displaying monopolistic competition, we can show that the profit of farmers involved in direct-selling rises as the size of the population increases. However, when accounting for the spatial externalities related to the city size, the relationship and therefore, the incentives for farmers to engage in direct-selling, become much more complex. Notably, we show that the exposure to pollution induces that, in highly urban crowded regions, only the most productive farmers can stay on direct-selling market. Additionally, we highlight that the greater the gap in productivity between the urban fringe and the right-hand side direct-selling boundary, the smaller the opportunities for farmers to engage in direct-selling, and the larger the quality differentiation between varieties.

As regards to the market outcome, we find that urbanization may foster direct-selling farming development provided that the market size effect dominates the net income effect. A corollary of this result is that regions hosting an intermediate-size city are more likely to supply a wider range of varieties. Futhermore, even if accounting for the spatial heterogeneity between farmers does not cancel this result, it nonetheless modifies the value of the variety range achieved at each level of urbanization.

Lastly, we derive that the market equilibrium always leads to a number of varieties which is too low to fully satisfy urban households, but too much high from the farmers' standpoint, this result being all the more compelling for highly urban-crowded regions. These general findings lay some ground for further research on the public policy aspects and seem already to suggest that, as a rule, policies are required to allow direct-selling farming to develop and thrive, especially near highly-crowded cities.

The paper proceeds as follows. Section 2 presents the model. In Section 3 and 4, we determine the land-market equilibrium and deliver some findings on the way spatial externalities affect both quantity and goods quality. Section 5 presents the long-run equilibrium and provides some insights on the relationship between variety, quality, and the city size. We discuss the conditions ensuring that fostering direct-selling development near cities leads to a concomitant welfare improvement in Section 6. Section 7 finally summarizes our main conclusions.

2 The framework

Consider an economy formed by a total population exogenously split into urban and rural households, and two sectors: a perfectly competitive sector providing a homogeneous aggregate good, and an agricultural sector where farmers can choose between direct-selling and conventional market. Agricultural goods are produced using labor, land, and fertilizer. Conventional farmers produce a homogeneous good under perfect competition while farmers engaged in direct-selling operate under monopolistic competition and provide a quality-differentiated good through a short supply chain.

2.1 The spatial structure

The economy is formally described by a one-dimensional space made of an urban area including a CBD and urban households' lots, and a rural area where farmers live and produce agricultural goods. Natural amenities are homogeneously supplied within the region. Distances and locations are denoted by x and measured from the CBD located in the center of the region. Without loss of generality, we focus on the right-hand side of the region, the left-hand side being perfectly symmetrical.

The urban area is entirely used for residential purposes. Urban inhabitants are assumed to be uniformly distributed across the city. They inelastically consume a residential plot of fixed size $\frac{1}{\delta}$, δ capturing the urban density (with $\delta > 1$). Letting λ_u be the size of the urban population, the right endpoint of the city is given by:

$$\bar{x}_u = \frac{\lambda_u}{2\delta}. \tag{1}$$

Farmers live and produce in rural areas located at the periphery of the city. Assuming that each farmer uses one unit of land to produce, the right endpoint of the region is given by:

$$\bar{x} = \bar{x}_u + \frac{\lambda_s + \lambda_c}{2} \tag{2}$$

where λ_s and λ_c stand for the number of direct-selling farmers and conventional farmers, respectively. We finally denote by $\bar{x}_s$ the boundary between direct-selling and conventional farming, and X_s the range of locations hosting direct-selling production. Depending on the regional land allocation, direct-selling farming will take place on plots such that $x \in [\bar{x}_u; \bar{x}_s]$ (near-city farming) or $x \in [\bar{x}_s; \bar{x}]$ (rural farming).

Urban pollution Fields located in the periphery of the city are exposed to urban pollution, causing yield losses that are proportional to the level of pollution encountered in each location. The source of this pollution is located in the CBD. The pollution intensity $h(x, \lambda_u)$ is supposed to be increasing with the level of urban activities ($h_{\lambda_u} > 0$) but decreasing with respect to the distance from the CBD ($h(0, \lambda_u) > 0$ and $h_x < 0$). Moreover, we assume that there is no pollution in the absence of urban households ($h(x, 0) = 0$), and that the urban population size does not interact with the spatial diffusion of the pollution ($h_{x,\lambda_u} = 0$).

2.2 Preferences and demand

In order to capture both the consumer's taste for variety and the consumers' relative valuation of goods' quality, we use the utility specification of Hallak [2006] and Kugler and Verhoogen [2012]. Consumers share the same Cobb-Douglas preferences for two types of goods; a homogeneous aggregate good M *– chosen as the numéraire and including the conventional agricultural good–* and agricultural differentiated products indexed by v:

$$U(Q, M) = Q^{\alpha} M^{1-\alpha} \tag{3}$$

with

$$Q = \left(\int_0^{\lambda_s} \theta(v)^{\beta} q(v)^{\frac{\sigma-1}{\sigma}} dv \right)^{\frac{\sigma}{\sigma-1}} \tag{4}$$

and where $q(v)$ and $\theta(v)$ stand respectively for the quantity and quality of variety v, σ represents the elasticity of substitution between two varieties, and β is the intensity of preference for quality. Utility is increasing with respect to the range of varieties λ_s and the quality. Besides, we assume $0 < \beta < 1$ which implies that the marginal utility of improving the quality of agricultural good is decreasing.

Goods quality Direct-selling goods differ in quality $\theta(v)$. This quality, perceived by the consumers, is assumed to be directly linked to the quantity of synthetic inputs used in the production and can be described as:

$$\theta(v) = \frac{\bar{\theta}}{z(v)^{\kappa}} \tag{5}$$

where $\bar{\theta}$ is the maximum quality level, $z(v)$ the amount of input used to produce the variety v, and κ, the quality sensitiveness to fertilizer use. It is worth noting that quality here rather refers to consumers perception than to real organoleptic properties. In other words, we suppose that consumers are aware of the quantity of synthetic inputs used for each variety and that they are reluctant to purchase goods grown with a large amount of these inputs. Evidences on the link between food quality and safety, and consumers' willingness to pay for synthetic-free products can be found in Roosen et al. [1998], Grunert [2005] or Marette et al. [2012].

Demand Consumers live in the urban area and work in the CBD. They bear urban costs, given by the sum of the commuting costs and the land rent. Letting t_u and $R(x)$ be the per-mile commuting cost and the land rent at x, these costs are such that

$$UC(x) = t_u x + \frac{R(x)}{\delta} \tag{6}$$

Then, denoting by P the price index for the range of direct-selling goods supplied in the region and w_u the urban wage, the budget constraint for any urban household is given by:

$$PQ + M = w_u - UC(x) \tag{7}$$

The individual demand for the composite good and the aggregate demand for direct-selling goods are derived from the maximization of the utility (3) subject to the budget constraint (7):

$$M^d = \frac{1-\alpha}{\alpha}\zeta_u(x) \tag{8}$$

$$Q^d = \frac{\zeta_u(x)}{P} \tag{9}$$

where $\zeta_u(x) \equiv \alpha(w_u - UC(x))$ is the share of the urban net income available for direct-selling goods consumption. Finally, denoting by $p(v)$ the price of the variety v of direct-selling goods and maximizing CES sub-utilities subject to the budget constraint $\zeta_u = \int_0^{\lambda_s} p(v)q(v)dv$ leads to the following demand function for the variety v:

$$q^d(v) = \theta(v)^{\sigma\beta} p(v)^{-\sigma} P^{\sigma-1} \lambda_u \zeta_u \tag{10}$$

with

$$P = \left(\int_0^{\lambda_s} \theta(v)^{\sigma\beta} p(v)^{1-\sigma} dv \right)^{\frac{1}{1-\sigma}} \tag{11}$$

2.3 The direct-selling sector

Transportation and production. Each farmer produces a unique variety v using labor, one unit of land and an amount $z(v)$ of input. To sell their production, they have to carry it to the central market located in the CBD, incurring costs that are increasing with the distance. These costs *–referred to as cost of transportation $t(x)$ in the following–* can be seen as units of working-time required for shipping goods to the market and that cannot be allocated to the production. The net labor supply of any farmer is then obtained by subtracting transportation time from his total time available[1]. Transportation therefore affects the individual production level through a reduction of the time spent in growing agricultural goods: the farthest from the city center, the lower the time available to grow crops, and the fewer the production. It creates an incentive for farmers to locate close to the urban fringe and captures thus, the opportunity cost of remoteness from the city center.

The production function accounts for the effects of both the transportation and the pollution $h(x, \lambda_u)$ on the total output. Denoting by $\bar{q}$ the natural ability of soils to grow crops in the region, we define the individual production for the variety v as:

$$q^s(v, x, \lambda_u) = \bar{q}z(v) \times e(t(x), h(x, \lambda_u)) \tag{12}$$

where $0 < e(t(x), h(x, \lambda_u)) \leq 1$ stands for the agricultural productivity coefficient at x for a city size of λ_u–or similarly, $e(t(x), h(x, \lambda_u))^{-1} \geq 1$ corresponds to the *yield-losses factor*. Its value is influenced by the total space-related effect of location on the production level. Formally, it encompasses the pollution externality cost and the opportunity cost of transportation, that operate in opposite directions as the distance from the city center increases. $e(t(x), h(x, \lambda_u))$ is decreasing with its two arguments $t(x)$ and $h(x, \lambda_u)$. Moreover, we posit $e(0, 0) = 1$ meaning that, without spatial externalities, the agricultural production is given by the combination of soil quality and input use.

In order to keep the discussion as broad as possible, we dot not specify the shape of $e(t(x), h(x, \lambda_u))$. For simplicity however, we assume that the function is additively separable, which implies that there is no correlation between the yield losses stemming from the pollution and transportation time ($e_{t,h} = 0$). Observe finally that when externalities do not vary in space (i.e. $e(x, \lambda_u) = \hat{e}(\lambda_u)\ \forall\ x$), direct-selling farmers are homogeneous producers; they supply a same quantity $\hat{q}^s$ and the quality $\hat{\theta}$ of their variety is identical.

Rewriting (12) so as to isolate z and setting $\bar{q} = 1$ without loss of generality, yields the quantity of

[1] Note that this specification where producers allocate their working time between goods production and another related activity is used by Lucas and Moll [2014]. In their model, firms allocate a fraction of time to production while the remaining part is used for innovative activities.

inputs used by the farmer located at x and producing the variety v:

$$z(v, x, \lambda_u) = \frac{q^s(v)}{e(t(x), h(x, \lambda_u))} \qquad \text{with} \quad z > 0 \tag{13}$$

We easily verify from (13) that supplying a large quantity of any variety v always requires more inputs. Likewise, the use of the input is all the more intensive that the agricultural productivity coefficient at x is low. This offsetting effect lies in the specification of the production function which allows farmers to compensate some of the yield losses due to the space-related factors (i.e. pollution and transportation cost) by using more input.

Productivity, distance and the city size. Differentiating $e(t(x), h(x, \lambda_u))$ with respect to the distance from the city center x yields:

$$\begin{aligned} e_x \equiv \frac{\partial e(t(x), h(x, \lambda_u))}{\partial x} &= \frac{\partial e(t(x), h(x, \lambda_u))}{\partial t(x)} \times \frac{\partial t(x)}{\partial x} + \frac{\partial e(t(x), h(x, \lambda_u))}{\partial h(x, \lambda_u)} \times \frac{\partial h(x, \lambda_u)}{\partial x} \\ &= -\left|e_t \times t'(x)\right| + \left|e_h \times h_x\right| \end{aligned} \tag{14}$$

Eq.(14) displays the comparative effect of transportation and pollution. Locating near the city allows to keep a high productivity since the opportunity cost of transportation is low but can, in the same time, diminish it because of the pollution externality. Hence, from any location to the direct neighboring one, productivity will decrease if the transportation cost effect ($|e_t \times t'(x)|$) outweighs the yield losses due to urban pollution ($|e_h \times h_x|$), and increase otherwise.

The relationship between the spatial variation of productivity and the urban population size is given by:

$$e_{x\lambda_u} \equiv \frac{\partial^2 e(t(x), h(x, \lambda_u))}{\partial x \partial \lambda_u} = e_{hh} \times h_x \times h_{\lambda_u} \tag{15}$$

where e_{hh} is the second order impact of pollution on yields losses which can be either positive or negative, depending on both the nature of the pollution and the type of crops considered. The sign of (15) is given by the opposite sign of e_{hh}: as the urban population size grows, the impact of externalities on productivity –and therefore, the spatial heterogeneity in agricultural production– tends to smooth over space if e is convex in h and to intensify for e concave.

For simplicity of notations, we further denote $e(t(x), h(x, \lambda_u))$ by $e(x, \lambda_u)$.

The market structure Direct-selling farmers operates on a local market under monopolistic competition; in contrast with conventional farming, they can set their own price both because they sell differentiated products and they do not interact with any intermediary. They supply close substitutes and are free to enter and exit the market. They neglect their mutual strategic interdependence and act as if they were monopolists. Since each variety is produced by a single farmer, the number of differentiated goods is given by the number of farmers involved in direct-selling. Any variety v can therefore be identified by the location x where it is grown.

Operating profit and price The profit of a farmer producing a direct-selling variety at x is given by the receipts from his sales minus a total cost which consists of a fixed cost associated with the purchase of one unit of land, and a constant marginal cost of inputs. Hence, letting p_z and $R(x)$ be the unit cost of the input and the unit rent of land at x, we have:

$$\pi(x,\lambda_u) = \underbrace{p(x,\lambda_u) \times q(x,\lambda_u)}_{receipts} - \underbrace{[R(x) + p_z z(x,\lambda_u)]}_{\text{total cost}} \tag{16}$$

where $q(x,\lambda_u)$ is the Marshallian demand for the variety produced at x, obtained by plugging (13) into (5) and by substituting the resulting expression of $\theta(x,\lambda_u)$ into (10):

$$q(x,\lambda_u) = \left(\left[\bar{\theta} e(x,\lambda_u)^{\kappa}\right]^{\frac{\sigma\beta}{\sigma-1}} p(x,\lambda_u)^{-\frac{\sigma}{\sigma-1}} \left(\lambda_u \zeta_u\right)^{\frac{1}{\sigma-1}} P \right)^{\eta} \tag{17}$$

and where $\eta \equiv \frac{\sigma-1}{1+\sigma\beta\kappa}$ is the elasticity of demand with respect to the direct-selling price index, that is, the price-sensitiveness of consumers to purchase direct-selling goods.

Each farmer sets his price so as to maximize his profit, considering that his decision has no impact on the other prices[2]. Taking the price index P as a constant and differentiating $\pi(x,\lambda_u)$ with respect to $p(x,\lambda_u)$, leads to the equilibrium price of the variety produced at x:

$$p(x,\lambda_u) = \frac{\sigma}{\sigma - 1 - \sigma\beta\kappa} \left(\frac{p_z}{e(x,\lambda_u)} \right) \tag{18}$$

where $\sigma > \frac{1}{1-\beta\kappa}$ must hold for $p(x,\lambda_u)$ to be positive.

The first element of (18) is the monopolistic mark-up. It is always greater than 1 and increases with parameters β and κ, reflecting the fact that farmers are fully aware that consumers are concerned by the quality of their product. The term in parentheses represents the marginal cost of production $c(x,\lambda_u)$ for the variety grown at x. It increases with the unit cost of the input pz, but also with the urban pollution externatity cost and the opportunity cost of transportation, highlighting the fact that farmers partially pass on the charge of their own location costs to consumers.

Still from (18), we calculate the *margin rate* m and the *Lerner index* ψ, respectively given by:

$$m \equiv \frac{p(x,\lambda_u) - c(x,\lambda_u)}{c(x,\lambda_u)} = \frac{1+\sigma\beta\kappa}{\sigma - 1 - \sigma\beta\kappa} \qquad (m > 0) \tag{19}$$

$$\psi \equiv \frac{p(x,\lambda_u) - c(x,\lambda_u)}{p(x,\lambda_u)} = \frac{1+\sigma\beta\kappa}{\sigma} \qquad (0 < \psi < 1) \tag{20}$$

Note that m and ψ are common to all farmers regardless of their spatial location within the region, meaning that they both reflect *the market power of the direct-selling sector as a whole.*

$p(x,\lambda_u)$ and $e(x,\lambda_u)$ share similar properties regarding their variation in space. Denoting by x_a and x_b two neighboring locations such that $\bar{x}_u < x_a < x_b < \bar{x}$, it is readily shown that $p(x_a,\lambda_u) < p(x_b,\lambda_u)$ if and only if $e(x,\lambda_u)$ is decreasing from x_a to x_b.

[2] The number of competitors is assumed to be large enough so that the effect of $p(x,\lambda_u)$ on P can be disregarded.

Market share and competition Multiplying (17) by (18), we can derive the market share of the direct-selling farmer located at x, defined as:

$$s(x,\lambda_u) \equiv \frac{p(x,\lambda_u)\times q(x,\lambda_u)}{2\times\int_{X_s}[p(x,\lambda_u)\times q(x,\lambda_u)]dx} = \frac{e(x,\lambda_u)^\eta}{S(\lambda_s,\lambda_u)} \qquad (0 \le s(x,\lambda_u) \le 1) \tag{21}$$

where $S(\lambda_s,\lambda_u) = 2\times\left(\int_{X_s} e(x,\lambda_u)^\eta dx\right)$ captures the *supply-side (or technical) market potential* of direct-selling food production, given in our framework by the sum over all the direct-selling farmers of the spatially-dependent share of their receipts. The higher $S(\lambda_s,\lambda_u)$, the greater the possibility for direct-selling farming to produce large quantities or, alternatively, a high number varieties. Moreover, we show in Appendix A that $S(\lambda_s,\lambda_u)$ is decreasing with λ_u. This can be explained by the combined effect of the urban pollution and the transportation cost; for any larger city, pollution and transportation cost will be higher, inducing a lower agricultural productivity –or equivalently, a higher yield-losses factor– at each location x.

Market shares are location-dependent and it is readily shown that, when externalities do not vary in space ($e(x,\lambda_u) = \hat{e}(\lambda_u)\ \forall\ x$), direct-selling farmers have a same market share given by $\hat{s} = \frac{1}{\lambda_s}$. The properties of the market share with respect to distance from the city and competition are provided in Appendix A. We notably show that:

1. The spatial variation of the market share follows that of $e(x,\lambda_u)$; it is therefore decreasing with the distance from the CBD if the effect of the opportunity cost of transportation dominates that of the urban pollution externality, and increasing otherwise.
2. The market share is always decreasing with the number of competitors λ_s and the larger the weight of the farmer located at x, the greater his loss in market share.
3. The market share is increasing with the urban population size λ_u for the most productive farmers, but decreasing for farmers experiencing a low productivity coefficient. Moreover, it is worth noting that when productivity is homogeneous over space ($e(x,\lambda_u) = \hat{e}(\lambda_u)\ \forall\ x$), the urban population size does not affect the market share ($s_{\lambda_u}(x,\lambda_u) \equiv \partial s(x,\lambda_u)/\partial\lambda_u = 0$).

3 The land market equilibrium.

We now determine the spatial allocation of land between urban households and farmers. In the manner of Von Thünen, we suppose that each plot of land is allocated to the highest bidder. The short-run equilibrium land rent is thus given by the upper envelop of bid rents, that is:

$$R^{sr}(x) = \max\{\Phi_u(x), \Phi_r^s(x), \Phi_c^s(x)\} \tag{22}$$

$\Phi_u(x)$, $\Phi_r^s(x)$, and $\Phi_c^s(x)$ being the bid land rent of urban households, direct-selling farmers, and conventional farmers, respectively. For simplicity, we further assume that the conventional bid land rent equals to the opportunity cost of land $\bar{R}$[3].

The urban bid rent Plugging (8) and (9) into (3) and rearranging gives the indirect utility of urban households:

$$V_u(x) = \left(\frac{\alpha}{P}\right)^{\alpha}(1-\alpha)^{1-\alpha}(w_u - UC) \tag{23}$$

At the residential equilibrium, the urban bid rent $\Phi_u(x)$ must solve $V_u'(x) = 0$ or equivalently, $t_u + \frac{\Phi_u'(x)}{\delta} = 0$, which solution is given by:

$$\Phi_u(x) = \bar{r}_u - \delta t_u x \tag{24}$$

$\bar{r}_u$ being a constant. Using (24) in (6), we obtain the urban net income available for direct-selling goods consumption:

$$\zeta_u(x) \equiv \zeta_u = \alpha\left(w_u - \frac{\bar{r}_u}{\delta}\right) \tag{25}$$

Observe that, because of the fixed lot size assumption, the total value of non-spatial goods consumption at the residential equilibrium does not depend on locations; the equilibrium value of urban costs –and therefore, the share of the urban net income available for direct-selling goods consumption ζ_u– is the same across urban households.

The direct-selling bid rent The farmers location choice is driven by two considerations. On the one hand, producing goods near the urban boundary allows reducing the opportunity cost of transportation. On the other hand, as urban activities generate pollution, locating away from the city center allows farmers to be less affected by this externality and, therefore, to reduce yield losses.

Plugging the price index (11) into the agricultural supply for variety v (17) and substituting $q(x, \lambda_u)$ by the resulting expression in (16) yields the agricultural profit for a farmer located at x:

$$\pi(x, \lambda_u) = [\psi\lambda_u\zeta_u \times s(x, \lambda_u)] - R_r(x) \tag{26}$$

The operating income, given by the term in square brackets, depends on two factors that help qualifying the degree of competition on the direct-selling market: the (regional net) income adjusted by the Lerner index which gives an overview of the power of producers relative to consumers, and the market share that accounts for the power of each producer relative to his competitors.

Differentiating $\pi(x, \lambda_u)$ with respect to x and equating to zero, we get that the direct-selling bid rent must satisfy $\Phi_r^{s\prime}(x) = \psi\lambda_u\zeta_u \times s_x(x, \lambda_u)$, which solution is given by:

$$\Phi_r^s(x) = \bar{r}_r + \psi\lambda_u\zeta_u s(x, \lambda_u) \tag{27}$$

[3]Note that to better account for the empirical evidence, we may have suppose that the opportunity cost of land is an increasing function of the urban population size. This would have strengthen the urban pressure on land cost and restrained the land strip hosting d-s farming.

$\bar{r}_r$ being a constant. Note from (27) that, because of the negative relationship between market shares and the number of competitors, bids from direct-selling farmers are also decreasing with λ_s. The entry of new competitors leads to diminish the receipts in direct-selling farming, due to the market share split. In the same time, the land market equilibrium requires that profits in direct-selling farming equalize over space; they must notably be equal to the profit of the last entrant $\pi(\bar{x}_s)$, which sets a benchmark value. With the entry of an additional competitor on direct-selling market, the new value of the land cost at each location is thus such that it accounts for both the decrease in receipts and the decrease of the benchmark profit. Since the larger the market share, the greater the loss due to additional competitors, it is readily verified that receipts at each location $x \in [\bar{x}_u; \bar{x}_s[$ diminish much more than the profit at $\bar{x}_s$. This implies in turn that land costs must also diminish to allow profits equalization.

Land use equilibrium Depending on the bid rent curves' ranking, several land use configurations can occur. For our study, we concentrate on the case where the zone dedicated to direct-selling farming is located at the periphery of the city and right-bordered by the conventional farming area ($X_s = [\bar{x}_u; \bar{x}_s]$). Mathematically, the direct-selling bid land rent must verify $\Phi_r^s(x) > \bar{R}$ $\forall x \in [\bar{x}_u; \bar{x}_s[$ and $\Phi_r^s(x) < \bar{R}$ $\forall x > \bar{x}_s$ which notably implies that:

1. $\Phi_r^s(x)$ must be decreasing at $\bar{x}_s$, meaning that, far from the city center, the opportunity cost of transportation always dominates the pollution cost (i.e. $|e_t \times t'(\bar{x}_s)| > |e_h \times h_x(\bar{x}_s)|$).
2. The direct-selling bid land rent at the urban fringe must be at least equal to the opportunity cost of land ($\Phi_r^s(\bar{x}_u) \geq \bar{R}$), entailing in turn $e(\bar{x}_u, \lambda_u) \geq e(\bar{x}, \lambda_u)$. This condition ensures that configurations where the direct-selling area is enclosed in the conventional farming area could not occur.

These two conditions are supposed to be verified in the following.

For the ease of reading, the details of calculations for the equilibrium land rent are reported in Appendix B. As showed therein, the short-run equilibrium land rent is given by:

$$R^{sr}(x) = \begin{cases} \delta\left(w_u - \bar{w}_u(\lambda_u) \times \dfrac{\frac{\delta}{\alpha\psi\lambda_u}}{\bar{s}_u - \bar{s} + \frac{\delta}{\alpha\psi\lambda_u}} - t_u x\right) & \text{if } 0 < x \leq \bar{x}_u \text{ \textit{(urban area)}} \\ \delta\bar{w}_u(\lambda_u) \times \dfrac{s(x,\lambda_u) - \bar{s}}{\bar{s}_u - \bar{s} + \frac{\delta}{\alpha\psi\lambda_u}} + \bar{R} & \text{if } \bar{x}_u < x \leq \bar{x}_s \text{ \textit{(direct-selling farming area)}} \\ \bar{R} & \text{if } x > \bar{x}_s \text{ \textit{(conventional farming area)}} \end{cases} \tag{28}$$

where $\bar{w}_u(\lambda_u) \equiv w_u - t_u \frac{\lambda_u}{2\delta} - \frac{\bar{R}}{\delta}$ is the urban net income at the land market equilibrium, and $\bar{s}_u$ and $\bar{s}$ stand respectively for the market share of the farmers located at each edge of the direct-selling area $s(\bar{x}_u, \lambda_u)$ and $s(\bar{x}_s, \lambda_u)$. Note that, since $e(\bar{x}_u, \lambda_u) \geq e(\bar{x}, \lambda_u)$, we know that $\bar{s}_u \geq \bar{s}$.

The representative curve of the bid land rent for direct-selling farming depends on the shape of $e(x, \lambda_u)$ (Figure 1).

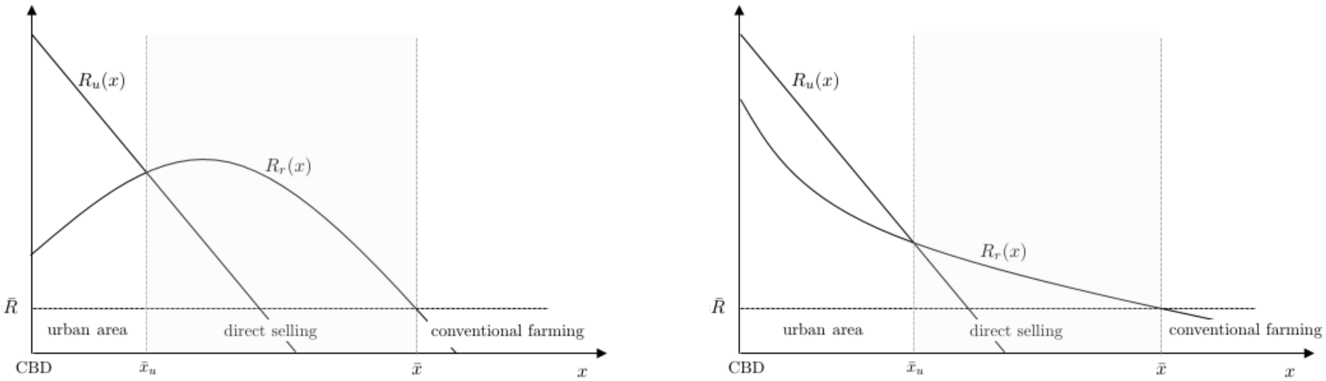

Figure 1: *The short-run equilibrium land rend and regional land allocation*

Observe finally from (28) that the equilibrium land rent is positively linked to the demand-side market effect $\frac{\alpha\psi\lambda_u}{\delta}$, but negatively related to the number of direct-selling competitors λ_s (Figure 2).

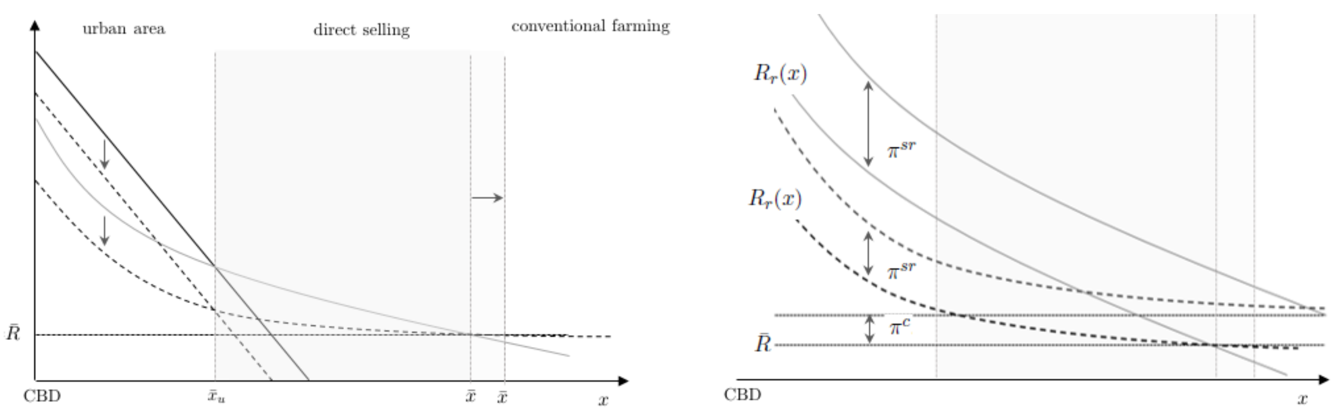

Figure 2: *Direct-selling farmers entrance, land allocation and profits.*

4 Direct-selling market equilibrium and goods quality.

From (11), we calculate the price index of direct-selling goods evaluated at the short-run equilibrium, given by:

$$P^{sr} = \left[\frac{\delta \bar{w}_u(\lambda_u)}{\left(\bar{s}_u - \bar{s} + \frac{\delta}{\alpha\psi\lambda_u} \right) \psi \bar{\theta}^{\frac{1}{\kappa}}} \right]^{\frac{\sigma\beta\kappa}{\sigma-1}} \left(\frac{mp_z}{\psi} \right)^{\frac{1}{m\eta}} S(\lambda_s, \lambda_u)^{-\frac{1}{\eta}} \tag{29}$$

Observe first that, in the case where spatial externalities would not be considered (i.e. $e(x, \lambda_u) = 1$ for all x) and where consumers would not value the quality of the agricultural goods ($\beta = 0$), we recover the standard Dixit–Stiglitz framework where $P = \frac{\sigma}{\sigma-1} p_z \lambda_s^{\frac{1}{1-\sigma}}$. Note also that, though the impact of λ_s on P^{sr} is twofold –*positive through the income effect* ζ_u^{sr} *and negative through the competition effect* $S(\lambda_s, \lambda_u)$–, we can show that the overall effect is non-ambiguous, the price index being always decreasing with the number of direct-selling farmers. Indeed, differentiating (29) with respect to λ_s, we have:

$$\frac{\partial P^{sr}}{\partial \lambda_s} \times \frac{1}{P^{sr}} = -\frac{\bar{s}}{\sigma - 1}\left(1 + \frac{\sigma\beta\kappa}{\frac{\alpha\psi\lambda_u}{\delta} \times (\bar{s}_u - \bar{s}) + 1}\right) < 0 \tag{30}$$

Competition, location and goods quality Using (29), we obtain the short-run equilibrium quantity and quality of the variety produced at x, respectively given by:

$$q^{sr}(x, \lambda_u) = \frac{\delta\bar{w}_u(\lambda_u)}{mp_z} \times \frac{s(x, \lambda_u)e(x, \lambda_u)}{\bar{s}_u - \bar{s} + \frac{\delta}{\alpha\psi\lambda_u}} \tag{31}$$

and

$$\theta^{sr}(x, \lambda_u) = \frac{\bar{\theta}mp_z}{\delta\bar{w}_u(\lambda_u)} \times \frac{\bar{s}_u - \bar{s} + \frac{\delta}{\alpha\psi\lambda_u}}{s(x, \lambda_u)} \tag{32}$$

$q^{sr}(x, \lambda_u)$ and $\theta^{sr}(x, \lambda_u)$ vary in opposite direction with respect to the distance from the city center; letting x_a and x_b be two neighboring locations such that $\bar{x}_u < x_a < x_b < \bar{x}_s$, we can state that $q^{sr}(x_a, \lambda_u) > q^{sr}(x_b, \lambda_u)$ and $\theta^{sr}(x_a, \lambda_u) < \theta^{sr}(x_b, \lambda_u)$ provided that $s(x_a, \lambda_u) > s(x_b, \lambda_u)$. More generally, we derive the following proposition:

Proposition 4.1 *At the short-run equilibrium, the quantity of any direct-selling variety decreases with the distance from the city center provided that the marginal impact of transportation on productivity is larger than that of the urban pollution externality. In this situation, the farther from the CBD, the lower the quantity of a variety, but the higher its quality.*

The implication of Proposition (4.1) in terms of goods quality may be counter-intuitive; since we have shown from (13) that the use of inputs z is decreasing with respect to $e(x, \lambda_u)$, we may have expected that the quality would be lower for the varieties grown at low-productivity locations ($e(x, \lambda_u)$ low). Instead, we find that the quality of high-productivity varieties is always lower than that produced on locations displaying low-productivity levels. The explanation of this result lies in the relationship between productivity, market share, and goods supply. By definition, the highest market share farmers have to supply a larger quantity of goods, giving them an incentive to use more input so as to meet the demand (see Eq.(13)), and making the quality of their variety lower.

As regard to the features of the competition on direct-selling market, we find that the quality of any variety is improving with the market concentration $\bar{s}_u - \bar{s}$, but decreasing as the Lerner index rises. Additionally, by differentiating (31) and (32) with respect to λ_s, we can show that increasing the number of direct-selling goods always leads to decrease the supply of each variety while improving its quality. Urban households have thus access to a wider range of goods with better quality, but in lower quantity.

Quality, quantity and variety when externalities do not vary in space Suppose now that externalities do not vary in space (i.e. $e(x, \lambda_u) = \hat{e}(\lambda_u) \; \forall \; x$), so that there is no heterogeneity between farmers. In this case, $s(x, \lambda_u) = \frac{1}{\lambda_s}$ for all x and denoting by a hat the non-spatial value of any variable, eqs. (31) and (32) become:

$$\hat{q}^{sr}(\lambda_s, \lambda_u) = \frac{\alpha\psi\lambda_u\bar{w}_u(\lambda_u)}{mp_z\lambda_s} \times \hat{e}(\lambda_u) \tag{33}$$

and

$$\hat{\theta}^{sr}(\lambda_s, \lambda_u) = \frac{\bar{\theta}mp_z\lambda_s}{\alpha\psi\lambda_u\bar{w}_u(\lambda_u)} \tag{34}$$

As previously mentioned, it appears now clearly that increasing the number of direct-selling goods leads to decrease the supply of each variety while improving its quality. Moreover, we can study if there are advantages to increasing the number of varieties in a region instead of another according to their respective urban population size.

The impact of the urban population size on the demand is twofold: a (positive) market size effect, implying that the larger the population, the higher the total demand for each variety, and a (negative) net income effect through the urban land cost, inducing a decrease in individual demand for each variety. These two elements are encapsulated in $\lambda_u\bar{w}_u(\lambda_u)$ which is the *demand-side market potential* or equivalently, the total market size expressed in value[4]. It describes a concave relationship with λ_u, reflecting the fact that the market size effect dominates the income effect in a first step, but becomes lower as the urban population size grows.

Since $\hat{\theta}^{sr}$ is inversely related to the demand-side market potential, we can directly derived from (34) that quality describes a decreasing convex function of λ_u, taking the highest values for the least urban-crowded cities. Regarding quantity, the relation to λ_u is more complex as it is affected by an additional pollution intensity effect $\hat{e}(\lambda_u)$. Precisions can however be obtained by calculating the elasticity of supply and quality to the urban population size, respectively given by:

$$\epsilon_{\hat{q}|\lambda_u} \equiv \frac{\partial\hat{q}}{\partial\lambda_u} \times \frac{\lambda_u}{\hat{q}} = 1 - \frac{1}{\frac{2(\delta w_u - \bar{R})}{t_u\lambda_u} - 1} - \frac{|\hat{e}^{'}(\lambda_u)|}{\hat{e}(\lambda_u)}\lambda_u \quad \text{and} \quad \epsilon_{\hat{\theta}|\lambda_u} \equiv \frac{\partial\hat{\theta}}{\partial\lambda_u} \times \frac{\lambda_u}{\hat{\theta}} = \frac{1}{\frac{2(\delta w_u - \bar{R})}{t_u\lambda_u} - 1} - 1 \tag{35}$$

as well as the second-order elasticities defined as:

$$\epsilon_{\hat{q}|\lambda_s\lambda_u} \equiv \frac{\partial\hat{q}^2}{\partial\lambda_s\lambda_u} \times \frac{\lambda_s\lambda_u}{\hat{q}} = -\epsilon_{\hat{q}|\lambda_u} \quad \text{and} \quad \epsilon_{\hat{\theta}|\lambda_s\lambda_u} \equiv \frac{\partial\hat{\theta}^2}{\partial\lambda_s\lambda_u} \times \frac{\lambda_s\lambda_u}{\hat{\theta}} = \epsilon_{\hat{\theta}|\lambda_u} \tag{36}$$

$\epsilon_{\hat{q}|\lambda_s\lambda_u}$ and $\epsilon_{\hat{\theta}|\lambda_s\lambda_u}$ give the relative sensitivity of the variation of quality and quantity with respect to variety, to the urban population size. Assuming first that pollution is weakly influenced by the urban

[4]Formally, one may observe that when externalities do not vary in space, the share of the urban net income available for direct-selling consumption is $\zeta_u^{sr} = \alpha\bar{w}_u(\lambda_u)$ for each urban household. $\lambda_u\bar{w}_u(\lambda_u)$ is thus the part of the *total (regional) mass of income spent on direct-selling goods* ($\lambda_u\zeta_u^{sr}$) that depends on the urban population size.

population size (i.e. $\hat{e}(\lambda_u) \to \hat{e} \; \forall \; \lambda_u$ so that $\hat{e}^{'}(\lambda_u) \to 0$), it clearly appears that developing direct-selling farming is all the more beneficial in the periphery of small and large cities since it allows to increase the goods quality while decreasing their quantity only in a low proportion (Figure (3) Left).

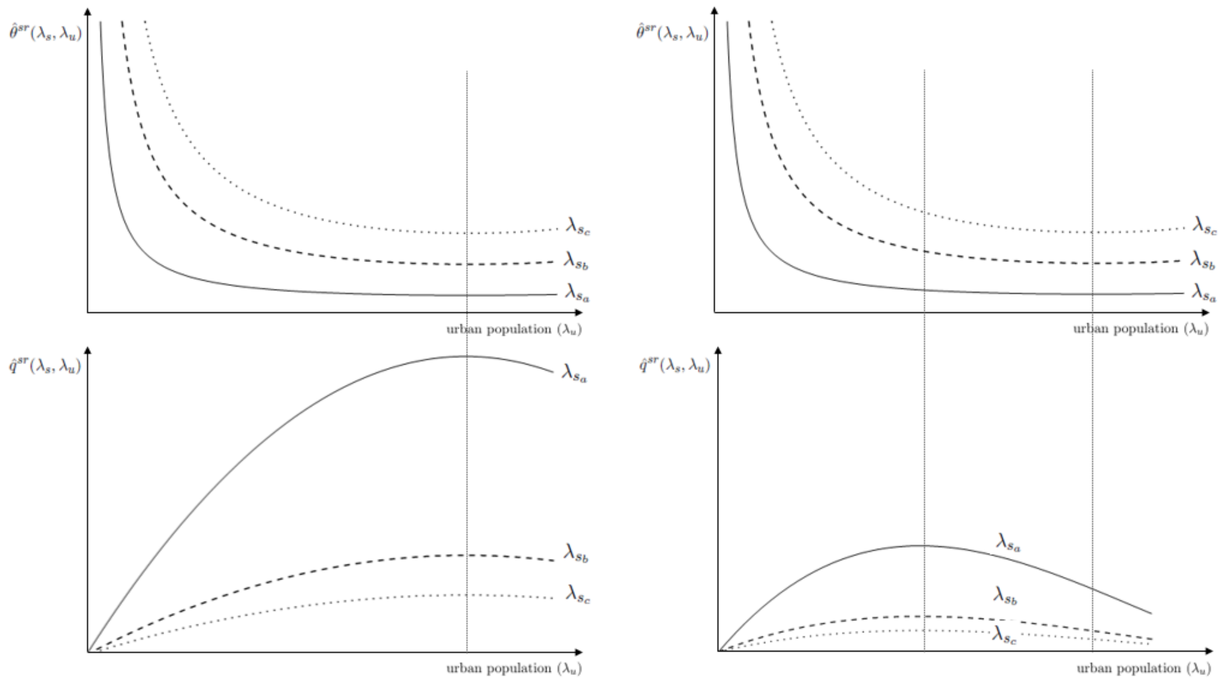

Figure 3: *Quality and quantity for various number of direct-selling varieties ($\lambda_{s_a} < \lambda_{s_b} < \lambda_{s_c}$).*

Relaxing the assumption on the relationship between pollution and urban population size brings some modifications for the quantity supplied in each variety; $\epsilon_{\hat{q}|\lambda_u}$ becomes smaller for all λ_u and the threshold value from which the relationship between quantity and the urban population size reverses is consequently lower than $\frac{\delta w_u - \bar{R}}{t_u}$ (Figure (3) Right).

4.1 Direct-selling profit and spatial externalities.

We finally assess the impact of spatial externalities on the direct-selling market profitability. From (26), we can rewrite the direct-selling profit at the short-run equilibrium as:

$$\pi^{sr}(\lambda_s, \lambda_u) = \frac{\delta \bar{w}_u(\lambda_u) \times \bar{s}}{\bar{s}_u - \bar{s} + \frac{\delta}{\alpha \psi \lambda_u}} - \bar{R} \tag{37}$$

Differentiating $\pi^{sr}(\lambda_s, \lambda_u)$ with respect to λ_s in (37), we can show that the short-run equilibrium profit decreases as the number of farmers involved in direct-selling increases. Given our framework, the latest entrant on the direct-selling market always has a lower market share than his competitors. His operating income is consequently lower than that of the other farmers (see Eq.(26)). However, since profits must equalize over space at the short-run equilibrium, spatial externalities are captured by the equilibrium land rent which, once fed back into the profit, leads to smooth the direct-selling net incomes and results in lower profits for every farmer.

5 The long-run equilibrium.

Farmers enter the direct-selling market as long as the profit they can earn is higher than the (exogenous) equilibrium profit prevailing in conventional farming π^c. In the long run, the number of direct-selling farmers adjusts to ensure that they all earn a profit equal to π^c.

5.1 The equilibrium number of direct-selling varieties.

Since the agricultural profit is decreasing with the number of farmers involved in direct-selling, the long-run equilibrium is ensured to be a unique stable interior solution. Posing $\pi^c \equiv \bar{\pi} - \bar{R}$ and equating it to π^{sr}, we get that the number of direct-selling varieties at the equilibrium λ_s^* must verify:

$$\frac{\alpha\psi\lambda_u}{\delta} = \frac{S(\lambda_s, \lambda_u)}{\phi\bar{e}^\eta - \bar{e}_u^\eta} \tag{38}$$

where $\phi \equiv \frac{\delta\bar{w}_u}{\bar{\pi}} + 1$ can be interpreted as a standard-of-living index and with the simplification of notation $\bar{e}_u = e(\bar{x}_u, \lambda_u)$ and $\bar{e} = e(\bar{x}, \lambda_u)$.

The LHS of (38) stands for the demand-side market effect. It is increasing with the urban population size and the Lerner index, and plays as a Home Market Effect; as the size of the urban population rises, the incentive to enter the direct-selling market increases. The RHS captures the supply-side competition effect and is linearly increasing with the number of direct-selling farmers. Observe that (38) can alternatively be written as $\bar{s} = \bar{\pi} \times \frac{\bar{s}_u + \frac{\delta}{\alpha\psi\lambda_u}}{\delta\bar{w}_u + \bar{\pi}}$, meaning that farmers keep entering the market until the market share of the latest entrant reaches a floor value. Note also that the existence of this equilibrium is ensured only provided that the difference in productivity between the direct-selling farmers settled at the closest and the farthest location from the city center is not too large. Notably, assuming that the productivity at $\bar{x}_s$ can be written as a positive fraction $\epsilon \leq 1$ of the productivity at $\bar{x}_u$ (i.e. $\bar{e} = \epsilon\bar{e}_u$), we draw from (38) that ϵ must be greater than $\phi^{-\frac{1}{\eta}}$ to allow the existence of a positive equilibrium.

Graphically, λ_s^* is given by the abscissa of the intersection point between the demand-side and the supply-side market effect.

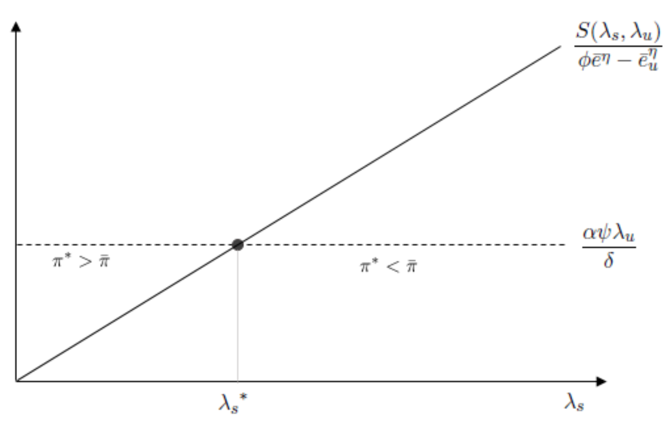

Figure 4: *The long-run equilibrium*

The farther the intersection, the larger the equilibrium number of direct-selling varieties, which is more likely when the representative line of the demand-side market effect is high (i.e. $\frac{\alpha\psi\lambda_u}{\delta}$ high) and/or for a weak slope of the representative line of the supply-side market effect (i.e. $\frac{1}{\phi-\frac{1}{\epsilon\eta}}$ low).

Table (5) summarizes the impact of every parameter of our model on the long-run equilibrium value.

	preferences				economic parameters				spatial parameters		
	α	σ	β	κ	w_u	t_u	$\bar{R}$	$\bar{\pi}$	λ_u	δ	ϵ
Demand-side market effect	high	low	high	high	/	/	/	/	high	low	/
Supply-side market effect	/	high	low	low	high	low	low	low	low	high	high

Figure 5: *Parameters fostering the development of direct-selling farming.*

Interestingly, it highlights the ambivalence of some of the parameters such that the city size (through the density δ and the urban population size λ_u), or the consumers' taste for variety and quality. It also confirms the following proposition:

Proposition 5.1 *At the long-run equilibrium, the number of direct-selling varieties will be all the more important than the spatial externalities affect the agricultural production at the urban fringe and at the right-hand side direct-selling boundary in a similar extent, all things being equal ($\epsilon \to 1$). This condition is notably verified in cases where the opportunity cost of transportation is negligible or sufficiently compensated by the urban pollution damages in the neighboring of the urban fringe.*

5.2 Direct-selling varieties and the city size.

The relation between the urban population size and the number of direct-selling varieties is not trivial as it jointly affects the supply and the demand sides. On the one hand, a highly crowded city creates an incentive for farmers to enter the direct-selling market since they would benefit from a large demand. On the other hand, the city size influences the level of the spatial externalities, playing on both the pollution intensity and the opportunity cost of transportation, and inducing changes in the relative productivity gap between farmers. These externalities, captured by the land rent, modify the level of competition on the land market and imply income changes for both urban and rural households.

In the following, we show that urbanization may favor diversity in direct-selling farming provided that the HME offsets the disincentives occurring on the land market. This result can be analytically derived by studying the variations of the direct-selling profit with respect to the urban population size at the equilibrium. For the sake of clarity, we proceed in two steps.

City size and direct-selling farming with homogeneous productivity over space $\hat{e}(\lambda_u)$. Consider first that externalities do not vary in space. In this case, the relationship between the number of direct-selling varieties and the urban population size is given by the cartesian equation $G(\lambda_s^*, \lambda_u) = 0$ with:

$$G(\hat{\lambda}_s^*, \lambda_u) = \frac{\alpha\psi\lambda_u}{\delta} - \frac{\hat{\lambda}_s^*\bar{\pi}}{\delta\bar{w}_u(\lambda_u)} \tag{39}$$

leading to:

$$\hat{\lambda}_s^*(\lambda_u) = \frac{\alpha\psi\lambda_u}{\delta\bar{\pi}}\left(\delta w_u - \bar{R} - \frac{t_u}{2}\lambda_u\right) \tag{40}$$

Eq.(40) describes a concave relationship, coming from the interplay of two standard competing effects in urban economics: (i) a market size effect that plays positively (and linearly), leading farmers to enter the direct-selling market so as to benefit from the additional outlets, and (ii) a net income effect which restricts the urban households spending at an increasing rate. Direct-selling variety rises as long as the market size effect outweighs the net income effect. It reaches a threshold value $\hat{\lambda}_s^*$ beyond which, any further urban population growth would lead to a decline in goods variety (Figure (6)).

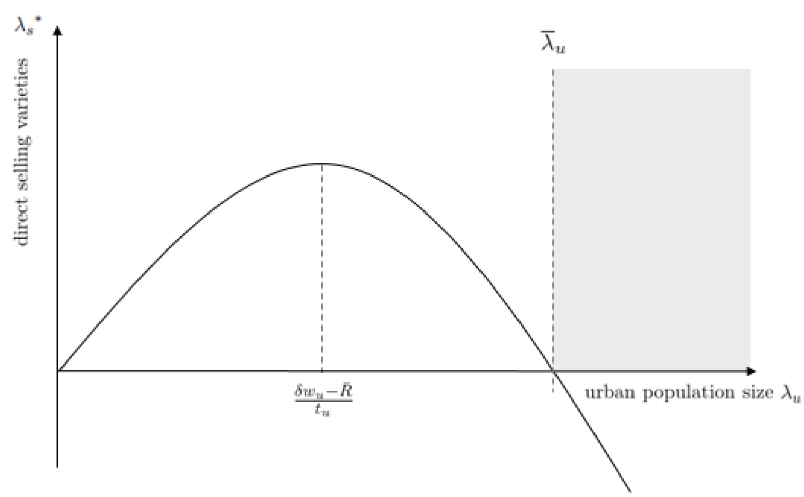

Figure 6: *Direct-selling varieties and urbanization (without spatial externalities)*

Hence, urbanization may foster direct-selling farming development provided that the market size effect dominates the net income effect. A corollary of this result is that direct-selling farming would provide a wider range of varieties in regions hosting an intermediate size city. Observe finally that from a city size $\overline{\lambda}_u$, the urban net income become so low that direct-selling farming can not emerge ($\lambda_s^* = 0\ \forall \lambda_u \geq \overline{\lambda}_u$).

How do spatial externalities change the bell-shaped outcome? Since we do not specify $e(x, \lambda_u)$, solving the implicit cartesian equation when externalities are spatially varying becomes much more complicated. However, recalling that $\pi^{sr}(\lambda_u, \lambda_s^*)$ does not vary in the long-run ($\pi^{sr}(\lambda_u, \lambda_s^*) = \bar{\pi}$) and using the total differential, we can draw the relationship between the urban population size and the number of direct-selling varieties, given by:

$$\frac{\partial \lambda_s^*}{\partial \lambda_u} = \frac{\partial \pi^{sr}(\lambda_u, \lambda_s^*)}{\partial \lambda_u} \times \left|\frac{\partial \pi^{sr}(\lambda_u, \lambda_s^*)}{\partial \lambda_s}\right|^{-1} \tag{41}$$

λ_s^* will be then positively (resp. negatively) correlated to λ_u provided that $\pi^{sr}(\lambda_u, \lambda_s^*)$ is increasing (resp. decreasing) with λ_u. Differentiating (37) with respect to λ_u and rearranging, we get:

$$\frac{\partial \lambda_s^*}{\partial \lambda_u} = \frac{\partial \hat{\lambda}_s^*}{\partial \lambda_u} + \Gamma(\lambda_u) \tag{42}$$

with

$$\Gamma(\lambda_u) \equiv -\frac{\alpha\psi}{\delta \bar{s}} \left(\bar{s}_u - \bar{s} - \lambda_u[\phi s_{\lambda_u}(\bar{x}, \lambda_u) - s_{\lambda_u}(\bar{x}_u, \lambda_u)]\right) \tag{43}$$

Accounting for the spatial variation of externalities induces two major changes:

1. Farmers located at the boundaries do not have the same market share anymore, which has repercussions on the land market. Indeed, using (38), we can calculate the long-run equilibrium land rent given by:

$$R^*(x) = \begin{cases} \delta t_u(\bar{x}_u - x) + \bar{\pi}\left(\dfrac{\bar{s}_u}{\bar{s}} - 1\right) + \bar{R} & \text{if } 0 < x \leq \bar{x}_u \ \textit{(urban area)} \\ \bar{\pi}\left(\dfrac{s(x,\lambda_u)}{\bar{s}} - 1\right) + \bar{R} & \text{if } \bar{x}_u < x \leq \bar{x}_s^* \ \textit{(direct-selling farming area)} \\ \bar{R} & \text{if } x > \bar{x}_s^* \ \textit{(conventional farming area)} \end{cases} \tag{44}$$

   and highlighting the fact that, except when externalities do not vary in space (i.e. $e(x, \lambda_u) = \hat{e}(\lambda_u)$, implying $s(x, \lambda_u) = \hat{s} \ \forall \ x$), the land rent for direct-selling farming is not flat. The cost of land differs from a direct-selling farmer to the other and captures the difference in receipts due to the level of externalities encountered at each location. Hence, for any same city size, the cost of land at the urban fringe and, thereby, urban costs for all the households are inevitably higher. The demand-side market potential is consequently weaker, entailing less incentive to enter direct-selling market in presence of externalities and, as a result, lower varieties –all things being equal. This effect, further referred to as the *land-market spillover effect*, is captured by $-\frac{\alpha\psi}{\delta\bar{s}}(\bar{s}_u - \bar{s})$[5].

2. Spatial externalities also introduce a new effect stemming from the fact that, because of the heterogeneous productivity over space, increasing the urban population size applies with different weight among locations, and captured by $\phi s_{\lambda_u}(\bar{x}, \lambda_u) - s_{\lambda_u}(\bar{x}_u, \lambda_u)$.

Using the expression of $s_{\lambda_u}(x, \lambda_u)$ reported in Appendix A and rearranging, we show in Appendix C that $\Gamma(\lambda_u)$ can be expressed as:

$$\Gamma(\lambda_u) = \frac{\bar{e}_u^\eta - \bar{e}^\eta}{\delta \bar{e}^\eta}(1 - \alpha\psi) + \frac{\eta |e_h h_{\lambda_u}|}{\bar{s}} \times \left(\frac{\int_{\bar{x}_u}^{\bar{x}_s^*} e(x,\lambda_u)^{\eta-1} dx}{\int_{\bar{x}_u}^{\bar{x}_s^*} e(x,\lambda_u)^{\eta} dx} - \frac{\phi \bar{e}^{\eta-1} - \bar{e}_u^{\eta-1}}{\phi \bar{e}^\eta - \bar{e}_u^\eta} \right) \tag{45}$$

The first term of (45) combines the (negative) *land-market spillover effect* and the (positive) *remoteness effect* –due to the benefit for direct-selling farming to move away from the pollution source, that is the

[5]Observe in this respect that, in the very specific case where externalities would be such that $\bar{e}_u = \bar{e}$ and $e(x, \lambda_u) > \bar{e} \ \forall x \in]\bar{x}_u, \bar{x}[$, the land rent would describe a concave parabola that verifies $R^*(\bar{x}_u) = R^*(\bar{x})$ and this effect does not appear.

CBD– on the demand-side market potential. It is always positive, meaning that the second effect always outweighs the first one. The second part of the expression encapsulates the *overall pollution intensity effect* which can be either positive or negative.

To get further insights on the impact of spatial variations and to better understand the trade-off at play, it may be convenient at this stage to structure the discussion according to the relationship between the urban population and the pollution.

(i) The pollution intensity is weakly influenced by the urban population size ($e_h h_{\lambda_u} \to 0$). In this case, $\Gamma(\lambda_u) = \frac{\bar{e}_u^\eta - \bar{e}^\eta}{\delta \bar{e}^\eta}(1-\alpha\psi)$, which is positive and increasing with λ_u. Returning to (42), we have $\frac{\partial \lambda_s^*}{\partial \lambda_u} > \frac{\partial \hat{\lambda}_s^*}{\partial \lambda_u}$ $\forall \lambda_u$, implying in turn that, for any level of urban population, the equilibrium value of direct-selling varieties is higher when agricultural productivity $e(\lambda_u, x)$ is varying over space. Moreover, observe that we still have $\frac{\partial^2 \lambda_s^*}{\partial \lambda_u^2} < 0$, so that the general bell shape of the relationship between urbanization and direct-selling varieties is preserved.

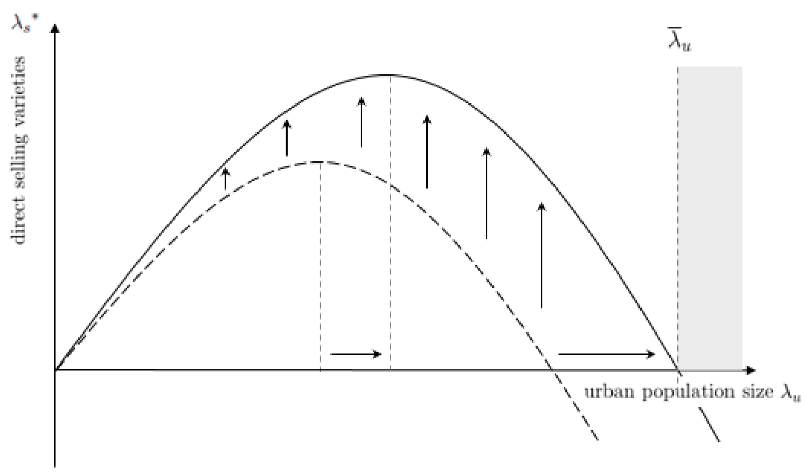

Figure 7: *Direct-selling varieties and urbanization (with low pollution effect)*

Interestingly, we derive that the spatial variation in externalities within the direct-selling area tends to increase diversity in direct-selling farming for any city size. Besides, observe that the higher the urban density δ, the weaker the additive effect from spatial externalities, and the closer from the benchmark equilibrium number of varieties $\hat{\lambda}_r^{s^*}$.

(ii) The pollution intensity is strongly influenced by the urban population size. When accounting for the pollution effect, two elements have to be added in the discussion. The first one, given by the ratio of integrals in (45) is the total losses in yields or the aggregate yield-losses factor; it compares the aggregate theoretical income $\int_{\bar{x}_u}^{\bar{x}_s} \frac{e(x,\lambda_u)^\eta}{e(x,\lambda_u)} dx$, that is the total income in direct-selling farming that would be if there was no yield losses due to externalities, to the effective aggregate income $\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^\eta dx$ (see Appendix D for additional explanations on theoretical income and aggregate yield-losses). The second element is the comparative yield-losses gap at the boundaries $\frac{\phi \bar{e}^{\eta-1} - \bar{e}_u^{\eta-1}}{\phi \bar{e}^{\eta} - \bar{e}_u^{\eta}} > 0$. It compares the gap between theoretical incomes at the boundaries to the gap between effective incomes.

As readily verified from (45), the overall effect of pollution intensity on direct-selling farming will be positive as long as the aggregate yield-losses factor outweighs the comparative yield-losses gap at the boundaries. In this case, even though pollution is damaging yields, direct-selling farming can develop concomitantly with the urban population size. If this condition is not verified, the overall effect is negative, meaning that pollution always tends to restrict direct-selling development.

Using the simplification $\bar{e} = \epsilon \bar{e}_u$ with $0 < \phi^{-\frac{1}{\eta}} < \epsilon \leq 1$, the expression of $\Gamma(\lambda_u)$ becomes:

$$\Gamma(\lambda_u) = \frac{1-\epsilon^{\eta}}{\delta \epsilon^{\eta}}(1-\alpha\psi) + \frac{\eta |e_h h_{\lambda_u}|}{\bar{s}} \times \left(\frac{\int_{\bar{x}_u}^{\bar{x}_s^*} e(x,\lambda_u)^{\eta-1} dx}{\int_{\bar{x}_u}^{\bar{x}_s^*} e(x,\lambda_u)^{\eta} dx} - \frac{\phi \epsilon^{\eta-1} - 1}{(\phi \epsilon^{\eta} - 1)\bar{e}_u} \right) \tag{46}$$

Then, depending on the shape of $e(x, \lambda_u)$, several cases can occur:

If the productivity at the urban fringe and at the right-hand side direct-selling boundary are close and low ($\epsilon \to 1$ and $\bar{e}_u$ low), $\Gamma(\lambda_u) < 0 \ \forall \lambda_u$, so that, compared to the case with homogeneous productivity, the equilibrium number of varieties is lower for any level of urban population. The pollution intensity effect outweighs the benefits due to the remoteness from the pollution source.

If the productivity at the urban fringe and at the right-hand side direct-selling boundary are close and high ($\epsilon \to 1$ and $\bar{e}_u$ high), $\Gamma(\lambda_u) > 0 \ \forall \lambda_u$, so that, compared to the case with homogeneous productivity, the equilibrium number of varieties is higher for any level of urban population.

If the productivity at the urban fringe and at the right-hand side direct-selling boundary differ significantly ($\epsilon \to \phi^{-\frac{1}{\eta}}$), $\Gamma(\lambda_u) \to -\infty$; the equilibrium number of varieties is always decreasing with level of urban population, so that direct-selling farming can not develop ($\lambda_s^* = 0 \ \forall \lambda_u$).

Finally, combining the different steps of the above analysis, we can derive the following proposition:

Proposition 5.2 *Direct-selling farming is likely to provide a wider range of varieties in regions hosting an intermediate-size city, whatever the shape of the spatial externalities. It is however worth noting that, although variations over space tend to favor direct-selling development compared to a situation where every location would experienced a same productivity level, a large gap in productivity between the urban fringe and the right-hand side direct-selling boundary may concurrently counteract this effect and contribute, in the end, to diminish the number of varieties.*

6 Direct-selling farming and regional welfare.

We finally evaluate the welfare implications of direct-selling farming. To do so, we assess the indirect utility of urban households at the short-run equilibrium and we examine whether or not increasing the number of varieties leads to a utility improvement. In a second step, we enlarge the analysis to include the considerations of farmers.

6.1 Urban households utility

Direct-selling farming interacts with urban households utility at two levels: it has a direct impact on consumption through the available range of varieties, the quality and the price level, and a net income spillover effect through the land market.

Equilibrium vs urban households optimum Plugging (29) into (23), the indirect utility at the short-run equilibrium becomes:

$$V_u^{sr}(\lambda_s,\lambda_u) = \Omega(\lambda_u)\times S(\lambda_s,\lambda_u)^{\frac{\alpha}{\eta}}\left[\frac{\frac{\delta}{\alpha\psi\lambda_u}}{(\bar{s}_u-\bar{s})+\frac{\delta}{\alpha\psi\lambda_u}}\right]^{(1-\alpha)+\frac{\alpha}{m\eta}} \tag{47}$$

where $\Omega(\lambda_u)\equiv \bar{w}_u(1-\alpha)^{1-\alpha}\left(\frac{\bar{\theta}^{\frac{1}{\kappa}}}{\psi\lambda_u\bar{w}_u}\right)^{\alpha}\left(\frac{\alpha\psi^2\lambda_u\bar{w}_u}{\bar{\theta}^{\frac{1}{\kappa}}mp_z}\right)^{\frac{\alpha}{m\eta}}$.

Assuming first that externalities do not vary in space, we can easily show that the market outcome always leads to a smaller set of varieties than the optimum; posing $e(x,\lambda_u)=\hat{e}(\lambda_u)$ for all x, we get $V_u^{sr}(\lambda_s,\lambda_u)=\Omega(\lambda_u)\times\hat{\lambda}_s^{\frac{\alpha}{\eta}}\,\hat{e}(\lambda_u)^{\alpha}$, which is concavely increasing with the number of direct-selling varieties. In this case, raising the number of varieties induces a stronger competition between farmers, leading as a result, to lower prices and to a higher satisfaction of urban households. Moreover, as in this case the productivity is the same for all the farmers, the direct-selling bid rent is flat so that new entries in the sector do not affect the urban households net income.

When accounting for the spatial varying externalities, we can show that the result whereby the equilibrium always leads to a smaller range of available varieties than the optimum holds. Indeed, differentiating $V_u^{sr}(\lambda_s,\lambda_u)$ with respect to λ_s gives:

$$\frac{\partial V_u^{sr}}{\partial\lambda_s} = V_u^{sr}(\lambda_s,\lambda_u)\times\bar{s}\left[\frac{\alpha}{\eta}+\left(1-\alpha+\frac{\alpha}{m\eta}\right)\frac{\bar{s}_u-\bar{s}}{\bar{s}_u-\bar{s}+\frac{\delta}{\alpha\psi\lambda_u}}\right]>0 \tag{48}$$

Then, knowing that the indirect utility describes a concave parabola in λ_s, we directly derive from (48) that direct-selling provides less varieties at the equilibrium than optimally wished; given our framework, any rise in goods diversity is always beneficial to consumers, as they get more varieties of higher quality. In this case however, increasing the number of varieties entails changes in the market share distribution; the market share gap increases which implies lower urban net income because of the land-market spillover effect.

Observe anew that the non-ambiguous relationship between the urban households utility and the number of varieties holds because of the monopolistic pricing on direct-selling market which, combined with the bidding process on land market, implies that strengthening the competition on direct-selling market always leads to a lower cost of land at the urban fringe and therefore, to a positive urban net income effect.

6.2 Regional welfare

We finally add the farmers considerations to the analysis. From the previous subsection, we derive that the urban households utility is concavely increasing with the number of varieties. However, since direct-selling profits are decreasing with the number of competitors, there is a conflict between urban and rural wishes, meaning that the welfare-maximizing number of varieties is necessarily lower than the optimal outcome for urban households.

Let the farmers utility be defined as the sum of the rural households profits:

$$V_r^{sr}(\lambda_s, \lambda_u) = \lambda_s \pi^{sr}(\lambda_s, \lambda_u) + (\lambda_r - \lambda_s)\bar{\pi} \tag{49}$$

λ_r being the number of rural households, that is $\lambda_r = \lambda_c + \lambda_s$.

$V_r^{sr}(\lambda_s, \lambda_u)$ describes a concave parabola in λ_s passing through $(0, \bar{\pi})$ and $(\lambda_s^*, \bar{\pi})$. At $\lambda_s = 0$, all the farmers earn a same profit $\bar{\pi}$. The entry on direct-selling market allows some farmers to benefit from the monopolistic competition and, consequently, to get a higher profit $\pi^{sr} > \bar{\pi}$. The utility of farmers is therefore first increasing with the number of competitors, until reaching a threshold from which the gains from imperfect competition vanish. From this value, any new entry would entail a decrease in direct-selling profit.

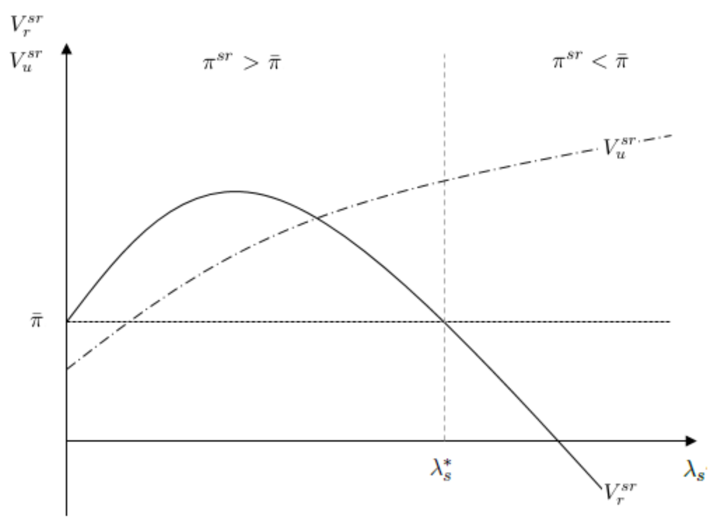

Figure 8: *Direct-selling farming and the welfare components.*

Therefore, as illustrated by the Figure (8), the market equilibrium always leads to a number of direct-selling varieties too much high compared to that which would maximize the farmers utility.

The welfare function can finally be defined as the sum of the urban and the farmers indirect utilities:

$$W^{sr}(\lambda_s, \lambda_u) = \lambda_u V_u^{sr}(\lambda_s, \lambda_u) + \lambda_r V_r^{sr}(\lambda_s, \lambda_u) \tag{50}$$

Because of the non linearity of (50), searching for an analytic solution of the welfare-maximizing problem is intricate. Some general findings can however be drawn; using the two previous subsections, we can easily show that the optimal number of direct-selling farmers is necessarily lower than that allowing to maximize the urban households welfare, but larger than the farmers' optimum. Yet, as indirect utilities are weighed by the population type, this result can be refined if jointly appreciated with the relative size of the urban population. More precisely, it is readily verified from (50) that the optimal outcome would be all the more close to the urban household optimum that the region hosts a highly-crowded city.

7 Conclusion

In this paper, we have investigated the conditions for which direct-selling farming could emerge without public intervention. We have derived that, at the short-run equilibrium, the supply of any direct-selling variety would decrease with the distance from the city center provided that the marginal impact of transportation is larger than that of the urban pollution externality. In this situation, we have shown that the farther from the CBD, the lower the supply of a variety, but the higher its quality since quantity and quality vary in opposite direction with respect to the distance from the city center.

As regards to the relationship between the urban population size and direct-selling farming, we have shown that urbanization may foster direct-selling farming development provided that the market size effect dominates the net income effect. A corollary of this result is that regions hosting an intermediate-size city are more likely to supply a wider range of varieties. Furthermore, although taking into account the spatial heterogeneity between farmers does not cancel this result, it nonetheless modifies the range of variety achieved at each level of urbanization. In this respect, we have found that, even when urban pollution adversely affects agricultural yields, cities may benefit from a large set of varieties provided that the gap in productivity between the urban fringe and the right-hand side direct-selling boundary remains low enough.

Finally, we have found that the market equilibrium always leads to a number of direct-selling farmers which is too low to fully satisfy urban households, but too much high from the farmers standpoint. These general findings on welfare lay some ground for further research on the public policy aspects. Notably, we can logically think that implementing a subsidy to reward farmers who engage in direct-selling may be welfare improving as long as the cost of this measure does not exceed the gains in urban households utility.

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A: Market share

Since the nature of the competition on direct-selling market depends on both the number of farmers involved on the market (supply-side) and the urban population size (demand-side), it is interesting to examine how the market share varies with λ_s and λ_u.

The market share of the direct-selling farmer located at x is given by:

$$s(x,\lambda_u) \equiv \frac{e(x,\lambda_u)^\eta}{2 \times \left(\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^\eta dx\right)} = \frac{e(x,\lambda_u)^\eta}{S(\lambda_s,\lambda_u)} \qquad (0 \leq s(x,\lambda_u) \leq 1) \tag{51}$$

Market share and supply-side competition. Differentiating (51) with respect to λ_s, we obtain the variation of the market shares value in each location with respect to the number of direct-selling farmers:

$$s_{\lambda_s}(x,\lambda_u) = -\frac{e(x,\lambda_u)^\eta \times e(\bar{x},\lambda_u)^\eta}{S(\lambda_s,\lambda_u)^2} = -s(x,\lambda_u)\bar{s} \tag{52}$$

The market share is thus always decreasing with the number of competitors. Additionally, it is readily shown that the larger the weight of the farmer located at x, the greater his loss in market share. This implies notably that the market concentration defined as $\bar{s}_u - \bar{s}$ is always decreasing with λ_s.

Market share and urban population size. Differentiating $e(x,\lambda_u)$ and $S(\lambda_s,\lambda_u)$ with respect to λ_u yields:

$$e_{\lambda_u}(x,\lambda_u) = -|e_h h_{\lambda_u}| \quad \text{and} \quad S_{\lambda_u}(x,\lambda_u) = -2 \times \left[\eta |e_h h_{\lambda_u}| \int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta-1} dx + \frac{\bar{e}_u - \bar{e}}{2\delta}\right] < 0 \tag{53}$$

As shown by (53), increasing the urban population size always leads to diminish the value of the productivity coefficient at x, but also the supply-side market potential $S(x,\lambda_u)$. The latter decreases under the effect of both the pollution intensity $\eta|e_h h_{\lambda_u}| \int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta-1}dx$ and the spatial change of direct-selling boundaries $\frac{\bar{e}_u - \bar{e}}{2\delta}$.

Assessing the overall impact of the urban population size on the market share at x boils down to a comparison between the effect in this location and that on the whole sector. Differentiating $s(x,\lambda_u)$ with respect to λ_u and rearranging, we have:

$$s_{\lambda_u}(x,\lambda_u) = s(x,\lambda_u) \times \left[\eta |e_h h_{\lambda_u}| \left(\frac{\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta-1} dx}{\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta} dx} - \frac{1}{e(x,\lambda_u)}\right) + \frac{\bar{s}_u - \bar{s}}{\delta}\right] \tag{54}$$

The first term in the square brackets captures the overall pollution intensity effect. More precisely, it compares the average yield-losses factor $\left(\frac{\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta-1} dx}{\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta} dx}\right)$ with the yield-losses factor at location x $\left(\frac{1}{e(x,\lambda_u)}\right)$. The second term accounts for the benefit for direct-selling farming of moving away from the pollution source. It is always positive but negatively correlated to the urban density.

We can state from (54) that the market share of a farmer located at x is positively linked to the urban population size provided that the yield-losses factor at location x is sufficiently low:

$$\frac{1}{e(x,\lambda_u)} < \frac{\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta-1} dx + \frac{\bar{e}_u^\eta - \bar{e}^\eta}{2\delta\eta|e_h h_{\lambda_u}|}}{\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta} dx} \tag{55}$$

Note that condition (55) is more likely to be verified in regions hosting a low-density city (δ low) or, as regards to the features of the externality, when pollution causes low damages (E_h low) and/or is weakly correlated to the urban population size (h_{λ_u} low). Moreover, it is readily verified that if the market share of the farmer located at $\bar{x}$ is increasing with the urban population size, then the market share of every farmer involved in direct-selling increases.

Observe finally that when productivity is homogeneous over space, the urban population size does not affect the market share ($s_{\lambda_u} = 0$).

B: Land use equilibrium

Let suppose that conventional farming takes place on plots of land located farthest from the CBD than direct-selling farming. Knowing that the bid rents of conventional and direct-selling farmers must equalize at $\bar{x}_s$, we find $\bar{r}_r = \bar{R} - \psi\lambda_u\zeta_u\bar{s}$, so that we now have:

$$\Phi_r^s(x) = \bar{R} + \psi\lambda_u\zeta_u[s(x, \lambda_u) - \bar{s}] \tag{56}$$

Analogously, we know that urban bid rent and direct-selling bid rent must equal at the urban fringe $\bar{x}_u$. Hence, replacing ζ_u by its value in (56) and equating $\Phi_u(\bar{x}_u)$ to $\Phi_r^s(\bar{x}_u)$ yields:

$$\bar{r}_u = \frac{\bar{R} + \delta t_u\bar{x}_u + \alpha\psi\lambda_u(\bar{s}_u - \bar{s})w_u}{\frac{\alpha\psi\lambda_u}{\delta} \times (\bar{s}_u - \bar{s}) + 1} \tag{57}$$

and the constant of the direct-selling bid rent becomes:

$$\bar{r}_r = \bar{R} - \frac{(\delta w_u - \delta t_u\bar{x}_u - \bar{R}) \times \bar{s}}{\frac{\delta}{\alpha\psi\lambda_u} + (\bar{s}_u - \bar{s})} \tag{58}$$

From (58), we can note that the entry of a new farmer on direct-selling market leads to an increase in the intercept of the bid land rent function $\bar{r}$ but tends, in the same time, to flatten the function since its slope is decreasing with respect to λ_s. As a result, we can show that a rise in direct-selling farmers can either lead to an increase or a decrease of the bid, depending on the location within the region.

The explanation of this result is to be found in the variation of the direct-selling profit with respect to the number of varieties; as previously mentioned, a new entrant always leads to a decrease in the market share of all the competitors already engaged in direct-selling. Their operating profit is consequently lower, as a result of a loss in terms of location rent. However, in the same time, the new competitor enters the market with a smaller share, leading to lower the benchmark value to which the profit of all the farmers should equalize at the land market equilibrium $\pi(\bar{x}_s, \lambda_u)$. In the end, each farmer can either make a larger or a lower bid, depending on his own loss in operating profit relative to the overall decrease in direct-selling profits.

Plugging $\bar{r}_u$ into the urban and the direct-selling bid land rents leads to:

$$\Phi_u(x) = \delta \left(w_u - t_u x - \frac{\delta w_u - \delta t_u x_u - \bar{R}}{\delta + \alpha\psi\lambda_u \left(\bar{s}_u - \bar{s}\right)} \right) \tag{59}$$

for the urban households and to:

$$\Phi_r^s(x) = \left(\delta w_u - t_u \frac{\lambda_u}{2} - \bar{R} \right) \times \frac{s(x, \lambda_u) - \bar{s}}{\bar{s}_u - \bar{s} + \frac{\delta}{\alpha\psi\lambda_u}} + \bar{R} \tag{60}$$

for the direct-selling farmers.

The direct-selling bid rent follows the spatial variations of $e(x, \lambda_u)$; it is thus decreasing with the distance from the CBD if the effect of the opportunity cost of transportation dominates that of the urban pollution externality, and increasing otherwise. Still from (60), we can show that the bid land rent is positively linked to the market size effect $\frac{\alpha\psi\lambda_u}{\delta}$, but negatively related to the market share gap $\bar{s}_u - \bar{s}$. The latter reflects the power of direct-selling farmers relative to urban households and conventional farmers on the land market; the lower $\bar{s}_u - \bar{s}$, the flatter the direct-selling bid land rent, and the smaller the part of the direct-selling profit captured by the land rent.

Combining (59) and (60), the short-run equilibrium land rent is finally given by:

$$R^{sr}(x) = \begin{cases} \delta t_u \left(\bar{x}_u - x\right) + R_r(\bar{x}_u) & \text{if } 0 < x \leq \bar{x}_u \\ \left(\delta w_u - t_u \frac{\lambda_u}{2} - \bar{R} \right) \times \frac{s(x, \lambda_u) - \bar{s}}{\bar{s}_u - \bar{s} + \frac{\delta}{\alpha\psi\lambda_u}} + \bar{R} & \text{if } \bar{x}_u < x \leq \bar{x}_s \\ \bar{R} & \text{if } x > \bar{x}_s \end{cases} \tag{61}$$

Assuming that

C: Long-run equilibrium and the urban population size

When externalities do not vary in space, the number of direct-selling varieties at the long-run equilibrium is given by:

$$\hat{\lambda}_s^*(\lambda_u) = \frac{\alpha\psi\lambda_u}{\delta\bar{\pi}} \left(\delta w_u - \bar{R} - \frac{t_u}{2}\lambda_u \right) \tag{62}$$

and its derivative with respect to λ_u is:

$$\frac{\partial \hat{\lambda}_s^*}{\partial \lambda_u} = \frac{\alpha\psi}{\delta\bar{\pi}} \left(\delta w_u - \bar{R} - t_u\lambda_u \right) \tag{63}$$

Expression (63) has to be compared with that obtained when externalities are varying over space:

$$\frac{\partial \lambda_s^*}{\partial \lambda_u} = \frac{1}{\bar{s}} \times \left[\frac{1}{\lambda_u} - \left(\frac{t_u}{2\bar{\pi}} \times \frac{\bar{s}}{\phi\bar{s} - \bar{s}_u} \right) + \frac{\phi s_{\lambda_u}(\bar{x}, \lambda_u) - s_{\lambda_u}(\bar{x}_u, \lambda_u)}{\phi\bar{s} - \bar{s}_u} \right] \tag{64}$$

Rearranging (64), we get:

$$\begin{aligned}\frac{\partial\lambda_s^*}{\partial\lambda_u} =& \frac{\alpha\psi}{\delta\bar{\pi}}\left[\delta w_u - \bar{R} - t_u\lambda_u - \frac{\bar{\pi}}{\bar{s}}\left(\bar{s}_u - \bar{s} - \lambda_u[\phi s_{\lambda_u}(\bar{x},\lambda_u) - s_{\lambda_u}(\bar{x}_u,\lambda_u)]\right)\right] \\ =& \frac{\partial\hat{\lambda}_s^*}{\partial\lambda_u} - \frac{\alpha\psi}{\delta\bar{s}}\left(\bar{s}_u - \bar{s} - \lambda_u[\phi s_{\lambda_u}(\bar{x},\lambda_u) - s_{\lambda_u}(\bar{x}_u,\lambda_u)]\right) \\ =& \frac{\partial\hat{\lambda}_s^*}{\partial\lambda_u} + \Gamma(\lambda_u)\end{aligned} \tag{65}$$

with

$$\Gamma(\lambda_u) \equiv -\frac{\alpha\psi}{\delta\bar{s}}\left(\bar{s}_u - \bar{s} - \lambda_u[\phi s_{\lambda_u}(\bar{x},\lambda_u) - s_{\lambda_u}(\bar{x}_u,\lambda_u)]\right) \tag{66}$$

Recalling that

$$s_{\lambda_u}(x,\lambda_u) = s(x,\lambda_u) \times \left[\eta|e_h h_{\lambda_u}|\left(\frac{\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta-1}dx}{\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta}dx} - \frac{1}{e(x,\lambda_u)}\right) + \frac{\bar{s}_u - \bar{s}}{\delta}\right] \tag{67}$$

we have:

$$\phi s_{\lambda_u}(\bar{x},\lambda_u) - s_{\lambda_u}(\bar{x}_u,\lambda_u) = (\phi\bar{s} - \bar{s}_u) \times \left[\frac{\bar{s}_u - \bar{s}}{\delta} + \eta|e_h h_{\lambda_u}|\frac{\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta-1}dx}{\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta}dx}\right] - \frac{\phi\bar{e}^{\eta-1} - \bar{e}_u^{\eta-1}}{2\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta}dx} \tag{68}$$

so that (66) becomes:

$$\Gamma(\lambda_u) = \frac{\bar{e}_u^{\eta} - \bar{e}^{\eta}}{\delta\bar{e}^{\eta}}(1-\alpha\psi) + \frac{2\eta|e_h h_{\lambda_u}|}{\bar{e}^{\eta}} \times \left(\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta-1}dx - \frac{\phi\bar{e}^{\eta-1} - \bar{e}_u^{\eta-1}}{\phi\bar{e}^{\eta} - \bar{e}_u^{\eta}}\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta}dx\right) \tag{69}$$

D: Income and aggregate yield-loss factor

As noted in Section 2, $e(x,\lambda_u)^{-1} \geq 1$ corresponds to the yield-losses factor, that is the factor differential between the effective yields and the yields that would be obtained without externalities (theoretical yields). We show in (21) that $e(x,\lambda_u)^{\eta}$ was the location-dependent part of the receipts at x. Multiplying these two elements gives $e(x,\lambda_u)^{\eta-1}$, that can be thus interpreted as the receipts that would be theoretically made without externalities, or equivalently, a theoretical income. When it is summed over the whole direct-selling market, we obtain the aggregate theoretical income for the sector $\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta-1}dx$. Finally, reported on the aggregate effective receipts $\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta}dx$, we get:

$$\frac{\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta-1}dx}{\int_{\bar{x}_u}^{\bar{x}_s} e(x,\lambda_u)^{\eta}dx} \tag{70}$$

which captures the total losses in yields in direct-selling farming, referred to as aggregate yield-loss factor.

Similarly, we can derive that $\phi\bar{e}^{\eta-1} - \bar{e}_u^{\eta-1}$ and $\phi\bar{e}^{\eta} - \bar{e}_u^{\eta}$ correspond respectively to the gap between theoretical and effective incomes at the boundaries (in the long-run equilibrium).