Phosphorus Conservation, Eutrophication Reduction and Social Welfare Improvement: Taxation of Extracted Phosphorus or Subsidy of Recycled Phosphorus?

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Abstract

This paper aims at analyzing the effect of an environmental tax or an environmental subsidy as instruments for preserving phosphate reserves, for improving water quality by reducing eutrophication, and for increasing or decreasing social welfare. Toward that goal, we use a duopoly model "à la Stackelberg", assume the presence of a benevolent government that takes into account the beneficial effect of recycling in the social welfare function and refunds the revenue of the tax to the society. First, we find that taxing extracted phosphorus or subsidizing recycled phospohrus contributes to the postponement of the depletion of the resource and to the reduction of eutrophication. Second, we find that taxing extracted phosphorus reduces consumers'surplus, whereas subsidizing recycled phosphorus increases it. Third, we show that the tax (the subsidy) set by the regulator is higher (lower) than the marginal damage of pollution (marginal benefit of recycling). Fourth, we show that the subsidy increases always social welfare, whereas the effect of the tax on the latter is ambiguous and depends on the size of the market. If the latter exceeds some threshold, the tax reduces always social welfare. Conversely, if the size of the market is below this threshold, the effect of the tax depends on the level of the marginal damage of pollution. If this level is large, the tax reduces social welfare, whereas the latter decreases in the tax rate if the level of the marginal damage is not large. Fifth, by way of comparison, we find that if the regulator aims at saving phosphorus, at reducing eutrophication and at improving social welfare simultaneously, subsidizing recycled phospohrus is the best policy, because the tax reduces social welfare in some specified conditions. Sixth, we find that if he aims only at saving phosphorus and at reducing eutrophication, taxing extracted phosphorus is more optimal than subsidizing recycled phosphorus.

JEL Classification:

Keywords: Environmental Tax, Subsidy, Phosphorus, Eutrophication, Recycling, Competition "à la Stackelberg".

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1 Introduction

A basic economic insight is that a competitive economy, under ideal conditions, will generate a socially efficient or a Pareto optimal allocation of private goods, meaning that it is not possible to reallocate resources in such a way that everyone becomes better off (Sandmo, 2003). One element of the ideal conditions requirement is the absence of external effects. In other words, if one agent generates externalities, the allocation is no longer socially optimal if the market is not regulated. Externalities may be both positive or negative. In this paper, we focus on both types of externalities because extracted phosphorus creates eutrophication phenomenons by polluting water, whereas recycling prevents phosphorus from polluting water. Accordingly, extraction generates a negative externality, while recycling generates a positive externality. Since Pigou (1920), it is well known that negative externalities caused by pollution would be internalized by the market if polluters paid a tax equal to the marginal social cost of polluting emissions (Nimubona and Sinclaire-Desgagné, 2005), while several economists stress that it is desirable to subsidize a polluter in order to induce him to abate pollution or to subsidize green products which generate a positive externality.

In this paper, the polluter is associated with a firm that extracts phosphorus, whereas the environmentally friendly firm refers to the recycler. It is widely recognized that unrecycled phosphorus[1] pollutes waters. In fact, primary[2] phosphorus ends up into the water due to water run-off, soil erosion, drainage from agricultural land, excreta from livestock, municipal and industrial effluents and creates, therefore, eutrophication phenomenons. As stated above, recycling will reduce water pollution by preventing phosphorus from ending up into the water. It is noteworthy to mention that eutrophication is an unwanted explosion of living aquatic-based organisms in lakes and estuaries that results in oxygen depletion, which can destroy an aquatic ecosystem (Liu and al., 2008). Significant eutrophication took place in the 1950s in the Great Lakes of North America, in Cayuga Lake which is in Central New York (Jacobs and Casler, 1979), in the Poyang Lake watershed that is in China (Deng and al, 2011), in the Norfolk broads of United Kingdom (Philipps, 1984) and has been prevalent in many lakes and estuaries around the world (International Lake Environment Committee Foundation, 2003).

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[1] There are other elements like nitrogen, carbon and trace which create eutrophication (Lee, 1973).

[2] We distinguish primary phosphorus to secondary or green phosphorus. The former refers to extracted phosphorus, whereas the latter corresponds to the recycled phosphorus. It is taken as green phosphorus because it reduces water pollution.

One way to reduce[3] eutrophication consists of taxing virgin phosphorus or of subsidizing recycled phosphorus, as mentioned above. This would diminish (increase) extracted phosphorus (recycled phosphorus) and would reduce (boost) extracted phosphorus (recycled phosphorus). Owing to the strategic substitutability of both types of phosphorus, the increase of one triggers the decrease of the other, and vice versa. Taxation of extracted phosphorus has been applied in some countries, including the United States of America (see Jacobs and Casler, 1979; Shakhramanyan and al**.,** 2012) and China. As well as reducing pollution of waters, taxation or subsidy would contribute to prolong the lifetime of phosphorus, whose the exhaustion[4] is predicted in a near future. Although subsidy can be seen as a cost for the government in question, taxation would give him the opportunity to collect some funds in order to finance several goals (see Gersbach and Requate, 2004).

The consideration of positive or negative external effects in the decisions of production of the firms and then in the social welfare function generates a number of interesting questions. Does the tax or the subsidy contribute to prolong the lifetime of phosphorus and to reduce water pollution ? Is the level of the tax (or the subsidy) set above or below the marginal social damage (or the marginal social benefit) ? What is the effect of each of these policies on consumers' surplus and on social welfare ? Is taxation more optimal in saving phosphorus and in reducing eutrophication than subsidy ? What is the best policy in terms of the improvement of social welfare ? In the present paper, we address these and related questions.

In connection with the questions posed above, the aim of this paper is first to analyze the effect of the tax or the subsidy on the depletion of phosphorus and on water pollution. Second, we will see what is the level of the tax or of the subsidy, respectively with respect to the marginal social damage of pollution and with respect to the marginal social benefit of recycling. Third, we aim at comparing the two policies in terms of optimality. Toward that end, we use a duopoly model in which two firms compete "à la Stackelberg" for two consecutive steps. In the first step, firm A chooses the quantity it extracts, whereas in the second step firm B chooses the quantity it recycles. At the very begining of the game, we assume the presence of a benevolent government which sets the level of the tax or that of the subsidy, and both firms produce accordingly.

Summarizing some of our findings, we can state following effects. First, we find that taxing extracted phosphorus or subsidizing recycled phospohrus contributes to delay the depletion of the resource and to reduce eutrophication. Second, we find that taxing extracted phosphorus reduces consumers'surplus, whereas subsidizing recycled phosphorus increases it. Third, we show that the

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[3] The policy of reduction of eutrophication took place in many countries, including the United States and Canada, via the the 1978 Great Lakes Water Quality Agreement which states that the effluents from 400 municipal treatment plants discharging to the lakes should contain a maximum of 1.0 mg litre^{-1} of phosphorus in the upper lakes and 0.5 mg litre^{-1} of phosphorus in the lower lakes (Harrington-Hughes, 1978), China and United Kingdom (Philipps, 1984).

[4] Cordell and al., 2009 highlight that phosphate reserves may be depleted in $50-100$ years.

tax (the subsidy) set by the regulator is higher (lower) than the marginal damage of pollution (marginal benefit of recycling). Fourth, we find that if the regulator aims at saving phosphorus, at reducing eutrophication and at improving social welfare, subsidizing recycled phospohrus is absolutely the best policy. Fifth, we find that if he aims only at saving phosphorus and at reducing eutrophication, taxing extracted phosphorus is more optimal than subsidizing recycled phospohrus.

This paper is based on several strands of the literature: the first one is related to the analysis of the effect of taxation on environmental pollution and on social welfare. Recall that the pigouvian conclusion that the level of the tax must be equal to the marginal social cost of polluting emissions is made within a context of perfect competition. When the market is imperfectly competitive, the tax should be set lower than the marginal social cost of pollution, because it trades off the desire to provide incentives for abatement and the necessity to prevent a greater contraction of output (Nimubona and Sinclaire-Desgagné, 2005). It is possible that the tax exceeds the marginal social cost. David and Sinclaire-Desgagné (2005) state that, under some conditions, an optimal emission tax should be set higher than the marginal social cost of pollution. The intuition underlying this idea is that imperfect competition between environment firms results in abatement prices larger than the marginal social cost of abatement; emission taxes must then be raised in oder to make polluters reduce their emissions sufficiently. In spite of the lack of consensus on the level of the tax, the overall conclusion of this line of research is that taxation reduces pollution. However, the effect of the environmental tax may induce an undesirable effect. Levin (1985) analyzes this issue within a Cournot oligopoly model. He assumes that a tax is imposed on each seller at a uniform rate per unit of output and shows if firms are sufficiently different, pollution increases with the tax. This behavior is similar to what happens in the green paradox[5] framework, usually addressed in dynamic models. Buchanan (1969) highlights that the corrective tax may well lead to a reducton in welfare rather than an increase. In contrast to him, we specify some conditions under which the tax can increase welfare. Even if our two firms are symmetric in terms of costs of production, which are zero, we show, contrary to Levin (1985), that the tax always reduces pollution.

The second strand of the literature concerns the relationship between a subsidy and pollution control. Baumol and Oates (1995) argue that although a subsidy will tend to reduce the emission of the firm, it is apt to increase the emissions of the industry beyond what they would be in the absence of the fiscal incentives. Mestelman (1982) uses a general equilibrium model and analyzes the effects of taxes and subsidies in a competitive economy which is characterized by a production externality. He shows that the use of a subsidy is inefficient. Mestelman (1984) shows that a pollution tax is consistently preferred to a subsidy by majority of individuals. Diamond and Mirrlees (1971) find that the ef-

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[5] The term "green paradox" states that some designs of climate policy, intended to mitigate carbon emissions, might actually increase carbone emissions, at least in the short run (for more details, see Hoel, 2010).

ficient combination of abatement and outpout requires the pigouvian tax alone, because the pollution abatement subsidy distorts the price the input used for reducing pollution, resulting thus in a non-achievement of efficiency. Fredriksson (1997) highlights that pollution abatement subsidies are inefficient instruments for pollution control. He stresses as follows the reasons for which subsidies reduce social welfare. He considers the benchmark as the social optimum situation where the government sets a tax and not a subsidy and argues that if pollution is decreasing in the subsidy rate, the subsidy benefits the environmentalists. Also, the industrialists always gain from receiving the subsidy. Conversely, the remaining groups in society pay a share of the subsidy, but derive not utility from it. Even if aggregate payoffs of the industrialists and the government rise, total welfare declines when one moves away from the social optimum. In contrast to Diamond and Mirrlees (1971), Mestelman (1982), and Fredriksson (1997), we show that the subsidy is efficient in pollution control in the sense that it leads to the reduction of pollution. Unlike Fredriksson (1997), we show clearly that the subsidy improves total social welfare even if one moves away from a situation where no tax is applied[6]. Contrary to Mestelman (1984), we find that a subsidy is preferred to a tax in terms of social welfare improvement.

The remainder of the paper is as follows. The next section introduces the concept of "eutrophication". Section 3 presents the model and the results. The main conclusions and some further research lines are given in section 4 and all proofs are relegated to the appendix in section 5.

2 Eutrophication

Bodies of water can be categorized as being in one of two states on the basis of their nutrient content. Low nutrient oligotrophic waters are clear and have relatively little animal and plant life, whereas the high nutrient content of eutrophic waters encourages the development of fauna and flora (Salerno, 2009). Eutrophication denotes the enrichment in nutrients of lakes and rivers that leads to this state of abundant life and therefore sounds like a positive development for a natural habitat. This enrichment can disrupt the natural balance of the natural system and lead to a complete transformation of the habitat (Ricklefs, 1979). The new altered state is often characterized by rapid plant and algae growth. When the density of the vegetation becomes such the ecosystem can no longer support it, it dies and begins to decay (Salerno, 2009). Since the rate of decomposition enhances, the process consumes so much oxygen that fish and other aquatic animals suffocate (Ricklefs, 1979). In addition, the growth of non-toxic algae results in shade and an rise of the water pH[7], which then favors the abundance of the cyanobacteria or blue-green algae, a bacterium that

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[6] Our calculations show that total welfare in this situation is higher than that obtained in the case where a tax is applied.

[7] The pH measures the acidity or the basicity of a solution. A solution with a pH of 7 is considered to be neutral. If $pH < 7$, the solution is considered to be acid and basic if $pH > 7$ (https://fr.wikipedia.org/wiki/Potentiel_hydrog%C3%A8ne).

can produce lethal toxins (Scheffer, 1998). Algae can also affect treatment of water for potable supply, by blocking filters or passing through them causing bad odeur and taste (Collingwood, 1977).

Eutrophication manifests in four stages:

(*i*) Increasing pollution: phosphorus ends up into waters, due to water run-off, soil erosion, etc. At the beginning, the oxygen content favors aquatic life. Fish are not affected.

(*ii*) Algae growth: phosphorus leads to the development of algae which consume so much oxygen. the oxygen content increases at the surface of the waters but diminishes significantly in the depths of the waters. Some species die.

(*iii*) Anaerobic decomposition: sediments rich in organic matter accumulate more. Aerobic bacteria multiply in order to degrade organic matter and consume oxygen. The oxygen content is strongly weakened on the whole water column.

(*iv*) Extreme degradation of the environment «dystrophy stage»: The oxygen content has significantly fallen. There is an absence of oxygen in the aquatic environment. The depletion of oxygen favors the formation of sulfuric acid and ammoniac in the water, leading to the death of fish. At this stage, there is a health risk for fauna and for humanity that use this water, because some cyanobacteria produce toxins.

3 The model

The economy we consider consists of a benevolent government, consumers and two firms, named firm A and firm B . Firm A holds phosphate rocks, extracts them and transforms them into phosphorus which is used as a fertilizer. This, phophorus is what is, widely, commercialized. Firm B, after consuming phosphorus deriving from firm A, recycles it and sells it. Therefore, both firms compete. Note that it is technically impossible to recycle the whole phosphorus extracted previously, resulting in $r < q$. Phosphorus which is directly extracted from phosphate rocks is called primary phosphorus and that which is recycled is called secondary or green phosphorus. As mentioned above, the primary phosphorus is considered as a polluting resource (Cordell and al., 2011), because it ends up into the water due to water run-off, soil erosion, etc. and creates, consequently, eutrophication phenomenons. Recycling[8] of phosphorus prevents it from ending up into the waters, therefore, reduces water pollution (Weikard and Seyhan, 2009; Cordell and al., 2011; Cogoye, 2009; Beir and Girmens, 2009; Ridder and al., 2012). Another way or an additional means of reducing water pollution would consist of applying a tax to the primary phosphorus or subsidizing recycled phosphorus. Thus, since taxation of virgin phosphorus or subsidizing recycled phosphorus encourages recycling activity, each of these policies reduces eutrophication phenomenons. We assume that the benevolent

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[8] It is noteworthy to mention that if recycled phosphorus ends up into the water it yields the same effect which is triggered by extracted phosphorus. But to focus on the benefit of recycling in the reduction of eutrophication, we assume, in the world of this model, that recycled phosphorus does not end up into the water after its consumption.

government applies this taxation or subsidizes the recycler. In order to compare these two policies, we assume that the benevolent government applies them separately. This analysis could be extended to an international scale if there was a supranational government that regulates pollution.

Both firms compete with the quantities of phosphorus they put in the market. Let q denote the quantity which is extracted by Firm A and r be the quantity that is recycled by firm B. For the sake of simplicity, we assume that the inverse demand function is linear and is $p(Q) = a - Q$, where Q is the total quantity of phosphorus which is sold and a may be interpreted as the size of the market or, the maximum price at which phosphorus can be sold or also as a choke price (Sweeney, 1992; Baksi and Long, 2009). We, also, consider only one tax applied on the polluting resource instead of having another tax applied on the emissions (for this case, see Cremer and Firouz, 2003). It is also considered that there are no extraction and recycling costs. We, also, assume that the tax-revenue (τq) is refunded to the society.

The timing of the game between the regulator and the firms can be described as follows. In the first step, the regulator sets the level of the tax or the level of the subsidy and refunds all the revenue of the tax to the society (in the case of taxation). In the second step, firm A chooses the quantity it extracts. In the third step, firm B chooses the amount it recycles, after consuming phosphorus deriving from firm A.

3.1 Phosphorus conservation and eutrophication reduction

Since Pigou (1920), it is well known that taxation is a means to reduce pollution. Recent studies have also shown that subsidy can yield a similar result, at least within a static context. In this section, we aim at investigating the effects of each of these policies on the reduction of eutrophication. As the reduction of eutrophication coincides here with the decrease of extracted phosphorus, reaching the former goal enables to delay the depletion of phosphorus.

First, we will analyze the effect of the tax on the lifetime of phosphorus and on the reduction of eutrophication. Second, the same issue will be explored within the context of a subsidy.

3.1.1 Taxation of extracted phosphorus

As stated above the timing of the game can be described as follows. In the first step, the regulator sets the level of the tax and decides how much to refund to the society. In the second step, firm A chooses the quantity it extracts. In the third step, firm B chooses the amount it recycles. Knowing the level of the tax charged by the benevolent government, each firm maximizes its own programme. We solve the game by backward induction. Thus,

Step 2: the profit maximization for firm B is:

$$\max_{r>0} \pi^B = (a - q - r)r \tag{1}$$

$$s.t.\ r < q \tag{2}$$

Condition (2) indicates the fact that recycling is never complete (see Weikard and Seyhan, 2009). The first-order condition for the programme above is given by:

$$r(q) = \frac{a-q}{2} \tag{3}$$

Step 1: Since the game is solved by backward induction, firm A inserts the reaction function of firm B in its decision of production and maximizes the following programme:

$$\max_{q>0} \pi^A = (a - q - \frac{1}{2}(a-q))q - \tau q \tag{4}$$

Solving the two programmes above yields the following proposition:

Proposition 1 *The tax levied by the authority decreases (increases) extracted phosphorus (recycled phosphorus). Formally, we have:*

(i)

$$\frac{\partial \hat{q}(\tau)}{\partial \tau} < 0 \tag{5}$$

(ii)

$$\frac{\partial \hat{r}(\tau)}{\partial \tau} > 0 \tag{6}$$

Proof. See Appendix II ■

The comparative statistic results deriving from proposition (1) are quite intuitive. The point (*i*) states that the environmental tax curbs the quantity which is extracted directly from phosphate rocks. This decreasing effect will delay the exhaustion of phosphorus and enables, therefore, the resource to be saved. In fact, the imposition of a tax will make extracted phosphorus more expensive in that the extractor will set higher price, leading consumers to switch towards recycled phosphorus which remains cheaper. Thus, this environmental tax creates a switching effect which consists of boosting recycling activity, resulting in $\frac{\partial \hat{r}(\tau)}{\partial \tau} > 0$. Since the primary resource is polluting, its decrease reduces environmental pollution. The environmental pollution reduction is also strengthened by recycling. Indeed, as mentioned above, recycling prevents phosphorus from ending up into the waters, avoids, therefore, eutrophication phenomenons.Thus, the tax plays a twofold role.

For $a = 1$, this result is illustrated in the following figure:

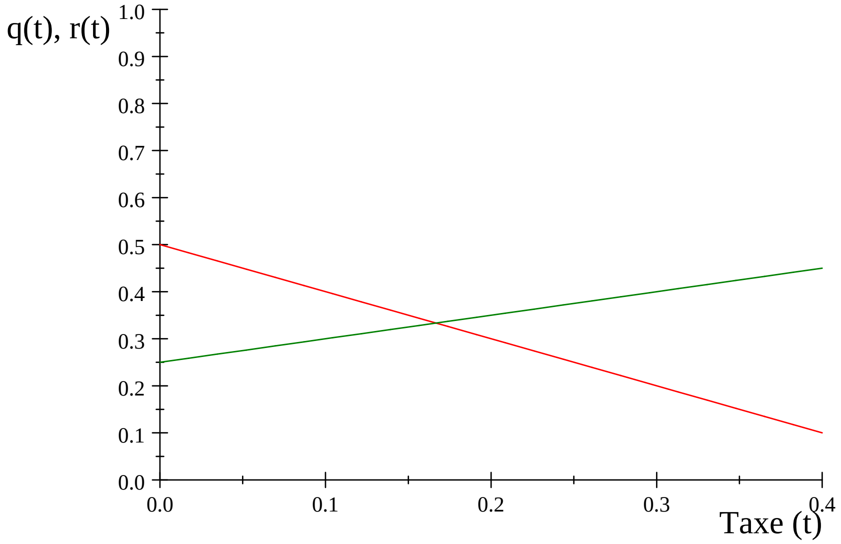

Figure 1: Effect of the tax on the optimal quantities

$$\text{Legend:}\begin{cases} -\text{ Red: Curve of extraction of phosphorus} \\ -\text{ Green: Curve of recycling of phosphorus} \end{cases}$$

This figure shows that the tax reduces extracted phosphorus while it boosts recycled phosphorus. As $r < q$, the graphic is valid only if $\tau \in (0; 0.17)$. Otherwise, the recycling curve is above the extraction curve (one can also verify it through (69) in appendix II). For this level of the tax, pollution diminishes but does not entirely disappear. This figure indicates also that taxation reduces polluting phosphorus from 0.5 to approximatly 0.3. In addition, it increases green phosphorus from 0.25 to 0.32.

Now, let us analyze the impact of the subsidy on the optimal quantities.

3.1.2 Subsidy of recycled phosphorus

We now turn to the equilibrium outcome under the presence of a subsidy. The timing of the game can be described as follows. In the first step, the regulator sets the level of the subsidy that it levies from the society and that it pays to the recycling firm. In the second step, firm A chooses the quantity it extracts. In the third step, firm B chooses the amount it recycles. Knowing the level of the subsidy paid by the benevolent government, each firm maximizes its own programme. As in the case of taxation, we solve the game by backward induction. Thus,

Step 2: the profit maximization for firm B is:

$$\max_{r>0} \pi^B = (a - q - r)r + sr \tag{7}$$

$$s.t.\ r < q \tag{8}$$

The first-order condition deriving from the programme above is then given by:

$$r(q) = \frac{a - q + s}{2} \tag{9}$$

Step 1: Firm A inserts the best-reply function of firm B in its decision of production and maximizes the following programme:

$$\max_{q>0} \pi^A = (a - q - \frac{1}{2}(a - q + s))q \tag{10}$$

Solving the two programmes above leads to the following proposition:

Proposition 2 *The subsidy increases (decreases) recycled phosphorus (extracted phosphorus). Formally, we have:*

(i)

$$\frac{\partial \hat{r}(s)}{\partial s} > 0 \tag{11}$$

(ii)

$$\frac{\partial \hat{q}(s)}{\partial s} < 0 \tag{12}$$

Proof. For the detail of calculations, see appendix V ■

The intuition underlying proposition (2) can be explained as follows. Subsidizing recycled phosphorus increases the profit of the recycling firm, resulting in the rise of the quantity it recycles. Since recycled phosphorus and extracted phosphorus are strategic substitutes, the increase of the former induces the slowdown of the latter.

For $a = 1$, the previous result is depicted through the following figure:

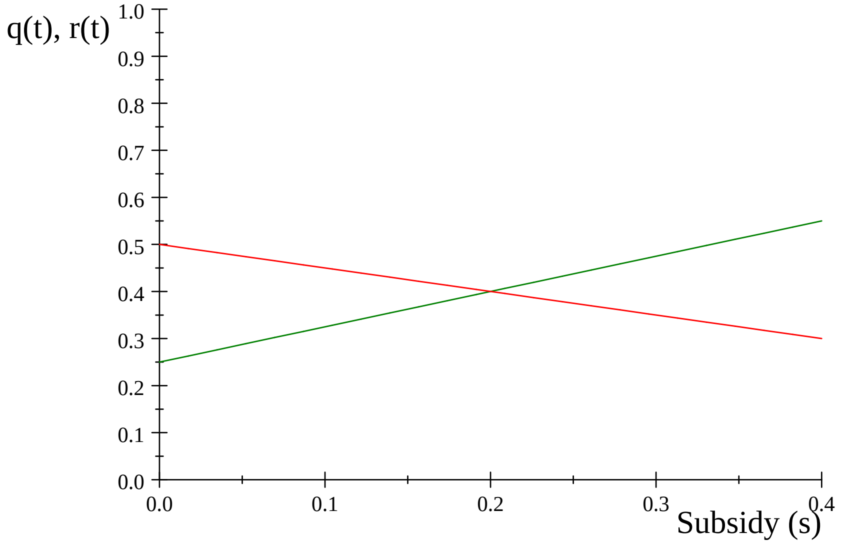

Figure 2: Effect of the subsidy on the optimal quantities

$$\text{Legend:}\begin{cases} -\text{ Red: Curve of extraction of phosphorus} \\ -\text{ Green: Curve of recycling of phosphorus} \end{cases}$$

Figure 2 shows that the subsidy increases recycled phosphorus, whereas it reduces extracted phosphorus. As $r < q$, the graphic is valid only if $s \in (0; 0.2)$. Otherwise, the recycling curve is above the extraction curve. Through the figure, it appears also that the subsidy reduces eutrophication and contributes to delay the depletion of the resource but it does not eliminate the former and will not enable to avoid the exhaustion of phosphorus in the sense that extraction still occurs.

The two previous propositions show that both the tax on extracted phosphorus and the subsidy on recycled phosphorus yield the same results in qualitative terms. The question which obviously arises is whether or not they yield identical results in quantitative terms. In other words, which policy is more optimal in terms of phosphorus saving and in terms of the reduction of eutrophication ? In the next section, we will address this related issue.

3.1.3 Tax on extracted phosphorus or subsidy on recycled phosphorus ?

In order to know which policy is more optimal in terms of phosphorus saving and in terms of the reduction of eutrophication , we will see the extent of the variation of the optimal quantities. In other words, does the tax reduce (increase) the more extracted phosphorus (recycled phosphorus) than the subsidy ? To answer this question, let us consider the two following cases:

case 1: Taxation: the variation of the optimal quantities in moving from the benchmark (zero-tax scenario) to the situation where the market is regulated (presence of a pigovian tax) is:

▶ For extracted phosphorus:

$$\overbrace{\Delta q^{\tau}}^{-} = -\tau \tag{13}$$

▶ For recycled phosphorus:

$$\overbrace{\Delta r^{\tau}}^{+} = \frac{1}{2}\tau \tag{14}$$

For the **proof** of the calculus above, see appendix II. It is straighforward to see through (13) and (14) that the tax decreases extracted phosphorus more than it increases recycled phosphorus. This explains why the total quantity sold by the industry decreases due to the introduction of this taxation scheme, resulting therefore in the decline of consumers' surplus.

case 2: Subsidy: in this case, the variation of the optimal quantities in moving from the benchmark (zero-subsidy scenario) to the situation where the market is regulated (presence of a subsidy) is:

▶ For extracted phosphorus:

$$\overbrace{\Delta q^{s}}^{-} = -\frac{1}{2}s \tag{15}$$

▶ For recycled phosphorus:

$$\overbrace{\Delta r^{s}}^{+} = \frac{3}{4}s \tag{16}$$

For the **proof** of the calculus above, see appendix V. (15) and (16) show clearly that the subsidy increases recycled phosphorus more than it decreases extracted phosphorus. That is why the subsidy increases the total quantity sold by the two firms, resulting then in the rise of consumers' surplus.

Let us turn now to the comparison of the two policies: to get a clearer picture, we sum up (13), (14), (15) and (16) in the following table:

$\Delta\diagdown$policies	tax	subsidy
Δq	$-\tau$	$-\frac{1}{2}s$
Δr	$\frac{1}{2}\tau$	$\frac{3}{4}s$

In order to obtain a standard for a comparison, we will set the tax rate (τ) equal to the subsidy rate (s) as in Ballard and Medema (1993). The analysis of the results of this table yields the proposition below.

Proposition 3 *If the amount of the tax per unit of pollution is equal to the amount of the subsidy per unit of the recycled product:*

(*i*) *The tax is better than the subsidy in terms of the reduction of eutrophication and in terms of the postponement of the depletion of extracted phosphorus. Formally, we have:*

$$\overbrace{\Delta q^{\tau}}^{-} > \overbrace{\Delta q^{s}}^{-} \tag{17}$$

(*ii*) *Conversely, if the government aims at boosting recycling, it is more optimal to subsidize recycled phosphorus than to tax extracted phosphorus. Formally, we have:*

$$\overbrace{\Delta r^{s}}^{+} > \overbrace{\Delta r^{\tau}}^{+} \tag{18}$$

Point (*i*) of proposition (3) states that the reduction of extracted phosphorus is greater than the rise of recycled phosphorus in the situation where taxation is applied, whereas the reduction of extracted phosphorus is lower than the rise of recycled phosphorus in the situation where subisidy is applied, meaning that taxation of virgin phosphorus is more optimal than subsidy of recycled phosphorus in terms of eutrophication reduction and in terms of delaying the depletion of the extracted phosphorus.

The intuition behind point (*ii*) is as follows. Subsidizing recycled phosphorus gives more power to the recycling firm than taxing extracted phosphorus in the sense that it increases its revenue. Even if taxation enhances its revenue by augmenting the quantity it recycles, due to the strategic substitutability, everything happens as if the revenue that it earns directly in the case of the subsidy is higher than that it earns in the case of taxation.

3.2 Effect of each policy on the social welfare

In this section, we will explore whether taxing extracted phosphorus is more optimal than subsidizing recycled phosphorus or not.

3.2.1 Taxation of extracted phosphorus

Consider first the case where the market of virgin phosphorus is not regulated. The social welfare is then given by:

$$w = CS + \pi^A + \pi^B \tag{19}$$

Using a linear inverse demand function, $p(Q) = a - Q$ (where $Q = q + r$), yields:

$$w = aQ - \frac{1}{2}Q^2 = \int_0^Q p(Q)dQ \tag{20}$$

In such a situation, the optimal social welfare is summed up in the following lemma:

Lemma 4

$$w^* = \frac{15}{32}a^2 \tag{21}$$

Proof. See appendix I ■

Now let us consider that the market is regulated. Assume that the revenue of the tax is returned to the society and the positive externality generated by recycling is taken into account by the regulator. To maximize welfare by taxation, the benevolent government must find some tax rate τ per unit of pollution, which will maximize:

$$w(\tau) = CS(\tau) + \pi^A(\tau) + \pi^B(\tau) - D(q(\tau)) + B(r(\tau)) + \tau q(\tau) \tag{22}$$

We use a linear inverse demand function $p(Q) = a - Q$, a linear damage function, i.e. $D(q) = \varepsilon q$ and a linear benefit function, i.e. $B(r) = \delta r$ (where δ is the marginal benefit of recycling). For the sake of simplicity and in order to focus only on the level of the tax $\hat{\tau}$ relatively to the marginal damage of pollution ε, let us assume $\delta = 1$. After making some simplications, the social welfare function writes:

$$w(\tau) = r(\tau) - \varepsilon q(\tau) + \frac{1}{2}aq(\tau) + ar(\tau) - \frac{1}{2}(r(\tau))^2 \tag{23}$$

Inserting the optimal quantities (see appendix II) in (23) and making some simplications yield:

$$w(\tau) = \frac{-4\tau^2 - 4\left(a - 8\varepsilon - 4\right)\tau + a\left(15a + 8\right) - 16a\varepsilon}{32} \tag{24}$$

It has been clearly shown in appendix II that $w(\tau) > 0$. Clearly, it appears that the marginal damage of pollution decreases the social welfare, in the sense that $\frac{\partial w(\tau)}{\partial \varepsilon} = \tau - \frac{1}{2}a < 0$. This is true under equation (69) established in appendix II. This result is quite intuitive.

Assumption 1.

$$\frac{a-4}{8} < \varepsilon < \frac{a-2}{4} \tag{25}$$

Assumption 2.

$$a > 4 \tag{26}$$

Assumption 2 garantees to have $\varepsilon > 0$. Solving $\frac{\partial w(\tau)}{\partial \tau} = 0$, yields the results summarized in the following lemma:

Lemma 5 *The government selects an equilibrium pollution abatement tax $\hat{\tau}$ and an optimal social welfare $\hat{w}^{\hat{\tau}}$ which satisfy:*

(i)

$$\hat{\tau} = 4\varepsilon - \frac{(a-4)}{2} \tag{27}$$

(ii)

$$\hat{w}^{\hat{\tau}} = \frac{4\varepsilon^2 - 2\left(a-2\right)\varepsilon + a^2 + 1}{2} \tag{28}$$

Proof. See appendix II ■

Under (25) and (26), point (i) states that the tax is highger than the marginal damage.

In order to know the effect of the tax on the social welfare, let us now turn to the comparison of $\hat{w}^{\hat{\tau}}$ and w^*. This yields the following preliminary result:

$$w^{\hat{\tau}} - w^* = \frac{\overbrace{64\varepsilon^2 - 32\left(a-2\right)\varepsilon + a^2 + 16}^{N}}{32} \tag{29}$$

The difference $(w^{\hat{\tau}} - w^*)$ takes the sign of N. It is noteworthy to mention that N is a second degree equation in ε. Solving $N = 0$ with respect to ε yields the two following roots:

$$\varepsilon_1 = \frac{a-2}{4} - \frac{\sqrt{a\left(3a-16\right)}}{8} \tag{30}$$

$$\varepsilon_2 = \frac{a-2}{4} + \frac{\sqrt{a\left(3a-16\right)}}{8} \tag{31}$$

Note that ε_2 is to preclude, under (25). Discussing the sign of N leads to the following proposition:

Proposition 6 *The effect of the tax on social welfare depends on the size of the market or on the position of the lower limit of (25) with respect to* ε_1*:*

(1) *If* $a > 8$ *(or* $\frac{a-4}{8} > \varepsilon_1$*), the tax decreases social welfare, resulting in:*

$$w^{\hat{\tau}} < w^* \tag{32}$$

(2) *If* $4 < a < 8$ *(or* $\frac{a-4}{8} < \varepsilon_1$ *), the effect of the tax on social welfare is ambiguous. In this situation, it will depend on the level of pollution or the marginal damage of pollution*

(*i*) *If* $\varepsilon_1 < \varepsilon < \frac{a-2}{4}$*, then, the tax decreases social welfare, resulting in*

$$w^{\hat{\tau}} < w^* \tag{33}$$

(*ii*) *If* $\frac{a-4}{8} < \varepsilon < \varepsilon_1$*, then, the tax increases social welfare, resulting in*

$$w^{\hat{\tau}} > w^* \tag{34}$$

Proof. The detail of all the calculations can be found in appendix III ■

The intuitions behind proposition (6) are as follows. Point (1) states that if the size of the market is large, the tax decreases the social welfare. This can result from the fact that such a situation confers to the polluter more possibility to increase its production. Conversely, Point (2) indicates that if the size of the market is not large , the effect of the tax on the social welfare depends on the level of pollution or the marginal damage of pollution. In fact, if the level ofthe level of pollution or the marginal damage of pollution is high, the tax decreases the social welfare (point (*i*)), meaning that the more generated pollution is important, the more the society is negatively affected. But if the level of pollution or the marginal damage of pollution is low, the tax increases the social welfare (point (*ii*)), meaning that the lesser is generated pollution, the more the society is positively affected.

The detailed explanations of point (1) and point (*i*) are as follows. Owing to the tax applied by the benevolent government, two effects working in opposite directions emerge. On the one hand, the tax enhances the cost incurred by the extracting firm. This curbs the equilibrium extracted quantity, which in turn reduces consumers' surplus and the profit of the extracting firm. On the other hand, the tax increases the quantity which is recycled by the recycling firm, resulting in the increase of the profit of the latter. As mentioned above, this result can be explained by the fact that extracted and recycled phosphorus are strategic substitutes. Then, the decline of the extracted quantity leads, mechanically, the recycling firm to increase the quantity it recycles. The rise of recycled phosphorus may improve the consumers' benefits compared to the situation where the whole demand is met by one supplier. As well as improving quantitatively the total welfare, the increase of the recycled quantity improves it qualitatively because it prevents phosphorus from ending up into the waters.

Consequently, it contributes to the improvement of the quality of the waters, thanks to the reduction of the eutrophication phenomenons. The decreasing effect of the tax on the sum of consumers' surplus and the extracting firm's profit outweighs the enhancing effect of the tax on the recycling firm's profit, resulting in a lower social welfare.

It is noteworthy to mention that if we impose $\varepsilon \leq 1$, (26) turns into:

$$4 < a < 6 \tag{35}$$

Il logically follows that the condition $a > 8$ is to preclude. Therefore, under (25) and (35), point (1) disappears. The results of proposition (6) are confirmed by the two examples established in appendix III.

3.2.2 Subsidy of recycled phosphorus

Consider first the case where the market of virgin phosphorus is not regulated. In such a situation, the social welfare is similar to that established in lemma (3).

Now let us turn to the case where the market is regulated. To maximize welfare by the subsidy, the government must find some subsidy rate s per unit of recycled product, which will maximize:

$$w(s) = SC(s) + \pi^A(s) + \pi^B(s) - D(q(s)) + B(r(s)) - sr(s) \tag{36}$$

We use a linear inverse demand function, a linear damage function, i.e. $D(q) = \varepsilon q$ (where ε is the marginal damage of pollution) and a linear benefit function, i.e. $B(r) = \delta r$ (where δ is the marginal benefit of recycling). For the sake of simplicity and in order to focus only on the level of the subsidy $\hat{s}$ relatively to the marginal benefit of recycling δ, let us assume $\varepsilon = 1$. After making some simplications, the social welfare function writes:

$$w(s) = \frac{(a - s - 2)\, q(s) + (2a - r + 2\delta)\, r(s)}{2} \tag{37}$$

Inserting the optimal quantities given in appendix V yields:

$$w(s) = \frac{a\,(15a + 2\,(s + 4\delta - 8)) + s\,(8\,(3\delta + 2) - s)}{32} \tag{38}$$

Clearly, it appears that the marginal benefit of recycling increases the social welfare, in the sense that $\frac{\partial w(s)}{\partial \delta} = \frac{a+3s}{4}$. This result is quite intuitive. Solving $\frac{\partial w(s)}{\partial s} = 0$, yields the results summarized in the following lemma:

Lemma 7 *The government selects an equilibrium pollution abatement subsidy $\hat{s}$ and an optimal social welfare $\hat{w}^{\hat{s}}$ which satisfy:*

(i)

$$\hat{s} = a + 12\delta + 8 \tag{39}$$

(ii)

$$\hat{w}^{\hat{s}} = \frac{\delta\,(2a + 3\,(3\delta + 4)) + a^2 + 4}{2} \tag{40}$$

Proof. See appendix V ■

Under our assumptions, we show that the optimal level of the subsidy is higher than the marginal benefit generated by recycling activity. The comparison of the two previous social welfares yields the following proposition:

Proposition 8 *The subsidy increases the social welfare. Formally, we have:*

$$\hat{w}^{\hat{s}} > w^* \tag{41}$$

Proof. See appendix V ■

Proposition (8) indicates that the subsidy improves the social welfare. Owing to the subsidy paid by the benevolent government, two effects working in opposite directions emerge. In fact, the subsidy enhances the profit of the recycling firm, resulting in the rise of recycled phosphorus. Since recycled phosphorus and extracted phosphorus are strategic substitutes, this leads to the decline of extracted phosphorus. But as the increasing effect is higher than the decreasiong effect (i.e. $\Delta r^s > \Delta q^s$), the total quantity of the industry increases in subsidy, resulting in the rise of consumers' surplus. As well as improving quantitatively the total welfare, the increase of the recycled quantity improves it qualitatively because it prevents phosphorus from ending up into the waters. Consequently, it contributes to the improvement of the quality of the waters, thanks to the reduction of the eutrophication phenomenons. The increasing effect of the subsidy on the sum of consumers' surplus and the recycling firm's profit outweighs the decreasing effect of the subsidy on the extracting firm's profit, resulting in a higher social welfare.

For the sake of comparision, we can highligh that the subsidy is more optimal than the tax from a social welfare standpoint, in the sense that the former increases it, whereas the latter decreases it under some specified conditions.

4 Conclusion

In the search for an efficient control for negative externalities, economists, generally, use taxes (subsidies) as instruments of pollution reduction. First, this paper shows that taxing polluting phosphorus (subsidizing green phosphorus) permits to reach this target, in the sense that each of them enables to reduce eutrophication phenonemons. Second, we find that taxation (subsidy) allows for saving this resource by fostering recycling activity. Indeed, the latter contributes to postpone the extraction of primary phosphorus. Third, we show that the tax (the subsidy) set by the regulator is higher (lower) than the marginal damage of pollution (marginal benefit of recycling). Fourth, we show that the subsidy increases always social welfare, whereas the effect of the tax on the latter is ambiguous and depends on the size of the market. If the latter exceeds some threshold, the tax reduces always social welfare. Conversely, if the size of the market is below this threshold, the effect of the tax depends on the level of the marginal damage of pollution. If this level is large, the tax reduces social welfare, whereas the latter decreases in the tax rate if the level of the marginal damage is not large. Fifth, by way of comparison, we find that if the regulator aims at saving phosphorus, at reducing eutrophication and at improving social welfare simultaneously, subsidizing recycled phospohrus is the best policy, because the tax reduces social welfare in some specified conditions. Sixth, we find that if he aims only at saving phosphorus and at reducing eutrophication, taxing extracted phosphorus is more optimal than subsidizing recycled phosphorus.

The challenge for the future is to set up a dynamic model which allows for taking into account the problem that entry causes on pollution. In fact, several authors have stated that subsidy gives to the other firms the incentive to enter the market. They have argued that global pollution increases in subsidy even if the individual pollution may decrease. It is noteworthy to mention that these studies are based on the assumption that subsidized firms are polluters. Since in this paper we consider that the subsidized firm is green in that its activity reduces pollution, it is obvious that even if we consider a dynamic contex, subsidy will continue to decrease pollution.

Another interesting issue is related to the level of the subsidy that the government can set to maintain the polluter in the market or to drop it out of the market. As subsidy reduces pollution and improves social welfare, higher is the level of the subsidy, lower is the pollution level and higher is social welfare. One can think that there may be a level of subsidy, named $\tilde{s}$ (higher than the optimal level $\hat{s}$ obtained when pollution occurs) which will drop the polluter out of the market. Then if $s < \hat{s}$, the government gives to the polluter more importance[9]. If $\hat{s} < s < \tilde{s}$, the polluter remains on the market but its importance reduces relatively to the previous situation. If $s \geq \hat{s}$, the polluter is brought out of the market and only the green firm produces. Accordingly, there is no pollution and social welfare increases due to the high level of the subsidy.

For the sake of simplicity, we have not addressed the problem as thae issue of an exhaustible resource. Imposing a capacity constraint to the resource would be interesting and is another challenge for the future.

5 Appendix

5.1 Appendix I: the market is unregulated

5.1.1 Detailed calculus for the optimal quantities

The timing of the game between the extractor (firm A) and the recycler (firm B) is as follows. In the first step, firm A extracts a quantity of phosphorus $q > 0$. After observing the level of firm A's extraction, firm B recycles a quantity $r > 0$,

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[9] This can be the case because the polluter is very strong in terms of lobbying. The government subsidizes only to reduce pollution slightly.

in the second step. We solve this game by backward induction. The programmes of the two firms are:

Step 2: firm B maximizes

$$\max_{r>0} \pi^B = (a - q - r)r \tag{42}$$

$$s.t.\ r < q \tag{43}$$

The lagrangian for this programme is defined as follows:

$$\mathcal{L}(r, \lambda) = (a - q - r)r + \lambda(q - r) \tag{44}$$

The first-order conditions are given by:

$$\frac{d\mathcal{L}(r, \lambda)}{dr} = a - q - 2r = \lambda \tag{45}$$

$$\lambda(q - r) = 0;\ \lambda \geq 0 \tag{46}$$

► case 1: from (46), we know if $\lambda = 0$, $r < q$, then (45) writes

$$a - q - 2r = 0 \tag{47}$$

which results in:

$$r(q) = \frac{a - q}{2} \tag{48}$$

► case 2: from (46), we know if $\lambda > 0$, $r = q$. It is impossible to have $r = q$ because recycling is never complete. Then, there is no recycling. The best-reply function of the recycling firm is then given by (48).

Step 1: the programme of firm A is:

$$\max_{q>0} \pi^A = (a - q - \frac{1}{2}(a - q))q \tag{49}$$

The first-order condition is then given by:

$$\frac{1}{2}a - q = 0 \tag{50}$$

Which results in the following optimal quantities:

$$q^* = \frac{1}{2}a \tag{51}$$

$$r^* = \frac{1}{4}a \tag{52}$$

The total quantity put in the market is given by

$$Q^* = q^* + r^* = \frac{3}{4}a \tag{53}$$

5.1.2 Detailed calculus for the optimal welfare

Proof of lemma 4: Social welfare is given by the sum of producer's profits and consumer's surplus (CS):

$$w = CS + \pi^A + \pi^B \tag{54}$$

Using a linear inverse demand function $p(Q) = a - Q$ (where $Q = q + r$), yields:

$$w = aQ - \frac{1}{2}Q^2 = \int_0^Q p(Q)dQ \tag{55}$$

The combination of (53) and (55) gives the following optimal social welfare:

$$w^* = \frac{15}{32}a^2 \tag{56}$$

5.2 Appendix II: Detailed calculus when the market is regulated

In this section, we assume that only virgin phosphorus is taxed. The possibility of subsidizing recycled phosphorus is not taken into account here. It will be considered later.

5.2.1 Detailed calculus for the optimal quantities

Proof of proposition 1: Under the tax τ applied by the regulator[10], each firm maximizes its own payoff. We solve this game by backward induction.

Step 2: the programme of firm B is defined as follows:

$$\max_{r>0} \pi^B = (a - q - r)r \tag{57}$$

$$s.t. \ r < q \tag{58}$$

The lagrangian for this programme can be established as follows:

$$\mathcal{L}(r, \lambda) = (a - q - r)r + \lambda(q - r) \tag{59}$$

The first-order conditions are given by:

$$\frac{d\mathcal{L}(r, \lambda)}{dr} = a - q - 2r = \lambda \tag{60}$$

$$\lambda(q - r) = 0; \lambda \geq 0 \tag{61}$$

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[10] It is noteworthy to mention that the timing of the game between the regulator and the firms is as follows. In the first step, the regulator sets the level of the tax. In the second step, firm A extracts $q > 0$. In the third step, firm B recycles $r > 0$. The game is solved by the backward induction approach.

▶ case 1: from (61), we know if $\lambda = 0$, $r < q$, then (60) writes:

$$a - q - 2r = 0 \tag{62}$$

which results in:

$$r(q) = \frac{a - q}{2} \tag{63}$$

▶ case 2: from (61), we know if $\lambda > 0$, $r = q$. This is impossible because recycling is never complete. Then, there is no recycling. The best-reply function of the recycling firm is then given by (63).

Step 1: the programme of firm A is defined as follows:

$$\max_{q>0} \pi^A = (a - q - \frac{1}{2}(a - q))q - \tau q \tag{64}$$

The first-order condition for this programme yields the optimal extracted quantity:

$$\hat{q}(\tau) = \frac{1}{2}a - \tau \tag{65}$$

With

$$\frac{d\hat{q}(\tau)}{d\tau} = -1 \tag{66}$$

And the optimal recycled qunatity:

$$\hat{r}(\tau) = \frac{a + 2\tau}{4} \tag{67}$$

With

$$\frac{d\hat{r}(\tau)}{d\tau} = \frac{1}{2} \tag{68}$$

Since $\tau > 0$, $\hat{q}(\tau) \geq 0$ and $\hat{q}(\tau) < \hat{r}(\tau)$ if and only if:

$$0 \leq \tau < \frac{1}{6}a \tag{69}$$

Under our assumptions, (69) explains why in figure 1 the graphic is valid only if $\tau < 0.17$.

The total quantity sold par the two firms is:

$$\hat{Q}(\tau) = \frac{3a - 2\tau}{4} \tag{70}$$

It is straighforward to see that

$$\frac{d\hat{Q}(\tau)}{d\tau} = -\frac{1}{2} \tag{71}$$

(71) clearly signals that the tax decreases consumers's surplus in that it reduces the global quantity.

5.2.2 Variation of the quantities

$$\Delta q^{\tau} = q(\tau) - q^{*} = -\tau \tag{72}$$

$$\Delta r^{\tau} = r(\tau) - r^{*} = \frac{1}{2}\tau \tag{73}$$

5.2.3 Detailed calculus for the optimal welfare

Proof of lemma 5: we assume that the benevolent regulator refunds the revenue of the tax to the society, takes into account the external effects (positive for r and negative for q) in the damage function . The tax τ maximizes the following social welfare function which is the difference between the sum of producer's profits and consumer's surplus and any technological external costs which are not accounted for in producers profits :

$$w(\tau) = CS(\tau) + \pi^{A}(\tau) + \pi^{B}(\tau) - D(q(\tau)) + B(r(\tau)) + \tau q(\tau) \tag{74}$$

We use a linear inverse demand function $p(Q) = a - Q$, a linear damage function, i.e. $D(q) = \varepsilon q$ and a linear benefit function, i.e. $B(r) = \delta r$ (where δ is the marginal benefit of recycling). For the sake of simplicity and in order to focus only on the level of the tax $\hat{\tau}$ relatively to the marginal damage of pollution ε, let us assume $\delta = 1$. After making some simplications, the social welfare function writes:

$$w(\tau) = r(\tau) - \varepsilon q(\tau) + \frac{1}{2}aq(\tau) + ar(\tau) - \frac{1}{2}(r(\tau))^{2} \tag{75}$$

Inserting the optimal quantities in the social welfare and making some simplications yield:

$$w(\tau) = \frac{8a + 16\tau - 4a\tau - 16a\varepsilon - 4\tau^{2} + 32\tau\varepsilon + 15a^{2}}{32} \tag{76}$$

Deriving the social welfare with respect to the tax τ yields the first-order condition below:

$$8\varepsilon + 4 - a - 2\tau = 0 \tag{77}$$

Which, in turn, yields the optimal level of the tax given by:

$$\hat{\tau} = 4\varepsilon - \frac{1}{2}(a - 4) \tag{78}$$

As $\hat{\tau} > 0$, we have:

$$\varepsilon > \frac{a - 4}{8} \tag{79}$$

Let us show that $w(\tau) > 0$. Solving $w(\tau) = 0$ with respect to τ gives the two following roots:

$$\hat{\tau}_{1} = 4\varepsilon - \frac{1}{2}(a - 4) - 2\sqrt{a^{2} - 2a\varepsilon + 4\varepsilon^{2} + 4\varepsilon + 1} < \hat{\tau} \tag{80}$$

$$\hat{\tau}_2 = 4\varepsilon - \frac{1}{2}(a-4) + 2\sqrt{a^2 - 2a\varepsilon + 4\varepsilon^2 + 4\varepsilon + 1} > \hat{\tau} \tag{81}$$

Since $w(\tau)$ is a second degree equation in τ, $w(\tau) > 0$ if $\hat{\tau}_1 < \hat{\tau} < \hat{\tau}_2$, which is true. Consequently, $w(\tau) > 0$. Inserting the optimal level of the tax in (76) yields the following optimal social welfare:

$$w^{\hat{\tau}} = \frac{a^2 - 2a\varepsilon + 4\varepsilon^2 + 4\varepsilon + 1}{2} > 0$$

Let us investigate the effect of the marginal damage on the social welfare. Thus,

$$\frac{dw^{\hat{\tau}}}{d\varepsilon} = 4\varepsilon - a + 2 < 0 \tag{82}$$

$\frac{dw^{\hat{\tau}}}{d\varepsilon} < 0$ because pollution decreases the global welfare. Such a result is expected. Since the social welfare is negatively affected by the marginal damage,

$$\varepsilon < \frac{a-2}{4} \tag{83}$$

(79) and (83) yield:

$$\frac{a-4}{8} < \varepsilon < \frac{a-2}{4} \tag{84}$$

It is straighforward to show that this interval is not empty. In fact, $\frac{a-4}{8} < \frac{a-2}{4}$ if and only if $a > 0$, which is true.

5.3 Appendix III: Comparison of the two social welfares

Proof of proposition 6: the comparision of the social welfare in the situattion where the market is regulated to that where the market is unregulated gives:

$$w^{\hat{\tau}} - w^* = \frac{\overbrace{64\varepsilon^2 - 32(a-2)\varepsilon + a^2 + 16}^{N}}{32} \tag{85}$$

$(w^{\hat{\tau}} - w^*)$ takes the sign of N. It is noteworthy to mention that N is a second degree equation in ε. Solving $N = 0$ with respect to ε yields the two following roots:

$$\varepsilon_1 = \frac{a-2}{4} - \frac{\sqrt{a(3a-16)}}{8} \tag{86}$$

$$\varepsilon_2 = \frac{a-2}{4} + \frac{\sqrt{a(3a-16)}}{8} \tag{87}$$

$N < 0$ if $\varepsilon \in (\varepsilon_1, \varepsilon_2)$. Note that $\varepsilon_2 > \frac{a-2}{4}$. Then, we have $N < 0$ if $\varepsilon \in (\varepsilon_1, \frac{a-2}{4})$. The sign of N depends on the position of $\frac{a-4}{8}$. We distinguish two cases:

▶ **case 1**: $\frac{a-4}{8} > \frac{a-2}{4} - \frac{1}{8}\sqrt{a\,(3a-16)}$, if $a > 8$. In this case, $N < 0$, resulting in $w^{\hat{\tau}} - w^{*} < 0$. Consequently,

$$w^{\hat{\tau}} < w^{*} \tag{88}$$

Thus, the global welfare decreases with taxation.

Example 1*:* $a = 10$: In this case, $N = 0$ reduces in:$64\varepsilon^2 - 256\varepsilon + 116 = 0$. Solving this equation with respect to ε yields the two following roots:

$$\varepsilon_1^{'} = 2 - \frac{1}{4}\sqrt{35} \tag{89}$$

$$\varepsilon_2^{'} = 2 + \frac{1}{4}\sqrt{35} \tag{90}$$

(84) reduces in:

$$0.75 < \varepsilon < 2 \tag{91}$$

$N < 0$ if $\varepsilon_1^{'} < \varepsilon < \varepsilon_2^{'}$. This is true un (91) because $\varepsilon_1^{'} < 0.75$ and $\varepsilon_2^{'} > 0$. Then

$$w^{\hat{\tau}} < w^{*} \tag{92}$$

This example confirms the fact that $w^{\hat{\tau}} < w^{*}$ if $a > 8$.

▶ **case 2**: $\frac{a-4}{8} < \frac{a-2}{4} - \frac{1}{8}\sqrt{a\,(3a-16)}$, if $4 < a < 8$. In such a situation, the effect of the tax depends on the position of ε.

(i) If $\frac{a-4}{8} < \varepsilon < \varepsilon_1$, we have $N > 0$, resulting in $w^{\hat{\tau}} - w^{*} > 0$. Therefore,

$$w^{\hat{\tau}} > w^{*} \tag{93}$$

The tax increases the social welfare. This means that if the level of pollution or the marginal damage of pollution is low (and the size of the market low), the implementation of a tax increases the social welfare.

(ii) If $\varepsilon_1 < \varepsilon < \frac{a-2}{4}$, we have $N < 0$, resulting in $w^{\hat{\tau}} - w^{*} < 0$. Therefore,

$$w^{\hat{\tau}} < w^{*} \tag{94}$$

Thus, the global welfare decreases with taxation. This means that if the level of pollution or the marginal damage of pollution is high (and the size of the market high), the implementation of a tax decreases the social welfare.

Example 2*:* $a = 6$: In this case, $N = 0$ reduces in: $64\varepsilon^2 - 128\varepsilon + 52 = 0$. Solving this equation with respect to ε yields the two following roots:

$$\varepsilon_1^{''} = 1 - \frac{1}{4}\sqrt{3} \tag{95}$$

$$\varepsilon_2^{''} = 1 + \frac{1}{4}\sqrt{3} \tag{96}$$

And (84) reduces in:

$$0.25 < \varepsilon < 1 \tag{97}$$

(i) If $0.25 < \varepsilon < \varepsilon_1^{''}$, we have $N > 0$, resulting in $w^{\hat{\tau}} - w^* > 0$. Therefore,

$$w^{\hat{\tau}} > w^* \tag{98}$$

The tax increases the social welfare. This means that if the level or the marginal damage of pollution is low (and the size of the market low), the implementation of a tax increases the social welfare.

(ii) If $\varepsilon_1^{''} < \varepsilon < 1$, we have $N < 0$, resulting in $w^{\hat{\tau}} - w^* < 0$. Therefore,

$$w^{\hat{\tau}} < w^* \tag{99}$$

Thus, the global welfare decreases with taxation. This means that if the level or the marginal damage of pollution is high (and the size of the market high), the implementation of a tax decreases the social welfare.

5.4 Appendix IV: Is the level of the tax above or below the marginal damage?

The marginal damage is given by ε. if $\varepsilon > \frac{a}{12}$ (which is true under the assumption above), $\hat{\tau} > \varepsilon$. Then,

$$\hat{\tau} > \varepsilon \tag{100}$$

In contrast to the conventional wisdom that the tax is set above the marginal damage only and only if the two firms have different costs of production, we show that the tax may be greater than the marginal damage when firms are symmetric.

5.5 Appendix V: Subsidizing the recycled phosphorus

In this section, we assume that virgin phosphorus is not taxed. Only recycled phosphorus can be subsidized. We distinguish the following cases:

5.5.1 case 1: benchmark: there is no subsidy

The programmes of the two firms are given by:

Step 2: profit of firm B

$$\max_{r>0} \pi^B = (a - q - r)r \tag{101}$$

$$s.t.\ r < q \tag{102}$$

$$r(q) = \frac{a-q}{2} \tag{103}$$

Step 1: profit of firm A

$$\max_{q>0} \pi^A = (a - q - \frac{1}{2}(a-q))q \tag{104}$$

$$q^* = \frac{1}{2}a \tag{105}$$

And

$$r^* = \frac{1}{4}a \tag{106}$$

5.5.2 case 2: Implementation of a subsidy scheme

Proof of proposition 2

Step 2: the programme of firm B is defined as follows:

$$\max_{r>0} \pi^B = (a - q - r)r + sr \tag{107}$$

The reaction function deriving from this programme is:

$$r(q) = \frac{a - q + s}{2} \tag{108}$$

Step 1: the programme of firm A is defined as follows:

$$\max_{q>0} \pi^A = (a - q - \frac{1}{2}(a - q + s))q \tag{109}$$

The optimal quantity resulting from this programme is:

$$q^s = \frac{a - s}{2} \tag{110}$$

With

$$\frac{dq^s}{ds} = -\frac{1}{2} \tag{111}$$

And the optimal quantity recycled by firm B is given by:

$$r^s = \frac{a + 3s}{4} \tag{112}$$

With

$$\frac{dr^s}{ds} = \frac{3}{4} \tag{113}$$

$q^S > 0$ and $r^S < q^S$ imply:

$$a > 5s \tag{114}$$

The total quantity then is given by:

$$Q^s = \frac{3a + s}{4} \tag{115}$$

With

$$\frac{\partial Q^s}{\partial s} = \frac{1}{4} \tag{116}$$

5.5.3 Variation of quantities

$$\underbrace{\Delta q}_{-} = q^s - q^* = -\frac{1}{2}s \tag{117}$$

$$\underbrace{\Delta r}_{+} = r^s - r^* = \frac{3}{4}s \tag{118}$$

But

$$\Delta r > \Delta q \tag{119}$$

Resulting in:

$$\Delta Q > 0 \tag{120}$$

Or in:

$$\frac{dQ^s}{ds} > 0 \tag{121}$$

The subsidy increases the recycled quantity whereas it decreases the extracted quantity. But, the increasing effect is higher than the decreasing effect, resulting in the rise of the total quantity $(q + r)$ with respect to the increase of the subsidy. Consequently, the subsidy increases consumers' surplus.

5.5.4 Comparison of the social welfares

1. Benchmark: there is no subsidy The social welfare is then given by:

$$w = CS + \pi^A + \pi^B \tag{122}$$

Using a linear demand function, the programme above writes:

$$w = -\frac{1}{2}r^2 + \frac{a\,(q + 2r)}{2} \tag{123}$$

Inserting the optimal quantities in (123) yields:

$$w^* = \frac{15}{32}a^2 \tag{124}$$

2. Implementation of subsidy scheme **Proof of lemma** 7: the social welfare is given by:

$$w(s) = CS + \pi^A + \pi^B - D(q) + B(r) - sr \tag{125}$$

We use a linear inverse demand function, a linear damage function, i.e. $D(q) = \varepsilon q$ (where ε is the marginal damage of pollution) and a linear benefit function, i.e. $B(r) = \delta r$ (where δ is the marginal benefit of recycling). For the sake of simplicity and in order to focus only on the level of the subsidy $\hat{s}$

relatively to the marginal benefit of recycling δ, let us assume $\varepsilon = 1$. After making some simplications, the social welfare function writes:

$$w(s) = \frac{(a-s-2)\,q(s) + (2a-r+2\delta)\,r(s)}{2} \tag{126a}$$

Inserting $q(s)$ and $r(s)$ in (126a) yields:

$$w(s) = \frac{a\,(15a + 2\,(s + 4\delta - 8)) + s\,(8\,(3\delta + 2) - s)}{32} \tag{127}$$

With

$$\frac{\partial w(s)}{\partial s} = \frac{a - s + 12\delta + 8}{16} \tag{128}$$

Deriving the social welfare with respect to the subsidy s yields the first-order condition below:

$$a - s + 12\delta + 8 = 0 \tag{129}$$

Which results in the following optimal level of the subsidy:

$$\hat{s} = a + 12\delta + 8 \tag{130}$$

It is straighforward to show that the marginal benefit increases the subsidy. This results in:

$$\frac{d\hat{s}}{d\delta} = 12 > 0 \tag{131}$$

Let us show that $w(s) > 0$. Solving $w(s) = 0$ with respect to s gives the two following roots:

$$\hat{s}_1 = a + 12\delta + 8 - 4\sqrt{a^2 + 2a\delta + 9\delta^2 + 12\delta + 4} < \hat{s} \tag{132}$$

And

$$\hat{s}_2 = a + 12\delta + 8 + 4\sqrt{a^2 + 2a\delta + 9\delta^2 + 12\delta + 4} > \hat{s} \tag{133}$$

Since $w(s)$ is a second degree equation in s, $w(s) > 0$ if $\hat{s}_1 < \hat{s} < \hat{s}_2$, which is true. Consequently, $w(s) > 0$. Inserting $\hat{s}$ in (127) yields the next optimal social welfare:

$$\hat{w}^s = \frac{\delta\,(2a + 3\,(3\delta + 4)) + a^2 + 4}{2} \tag{134}$$

Let us verify that the subsidy increases the social welfare. To do so, will have comparé $\hat{w}^s$ and w^*, resulting in:

$$\hat{w}^s - w^* = \frac{16\delta\,(2a + 3\,(3\delta + 4)) + a^2 + 64}{32} > 0 \tag{135}$$

The result established in (135) is the **proof of proposition** 8.

5.5.5 Is the level of the subsidy above or below the marginal benefit?

Let us see if $a + 12\delta + 8 > \delta$, resulting in $a + 11\delta + 8 > 0$, which is true because. Then

$$s > \delta \tag{136}$$

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