A Three-Sector Urban Equilibrium Model with By-production Externalities*

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Abstract

As pointed out by Boiteux-Orain and Huriot (2000), "modelling suburbanization [may be] too broad and imprecise to carry out an accurate assessment". We take part in the challenge of giving full account of all economic activities that shape the periurban area. Although the literature on periurban agriculture is quite extensive, it is necessary not just to integrate firms in urban planning but also to build up a territory where firms, households and agriculture can coexist. The purpose of this paper is to develop a theoretical framework enabling to investigate the importance of wages, agricultural amenities and commuting costs as determinants of the urban and suburban spatial structures. Formally, we explore this question by building an urban equilibrium model which combines aspects of urban economics and labor economics in outlining the location choices of these agents. The basis of the model arises from the possibility of creating a mixed land-use that is no longer limited to the presence of households and farmers, or conversely of firms and households, but also becomes an attractive economic center where periurban households may find a job and take into account spatial amenities.

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1 Introduction

Over the last forty years the city has increasingly been reshaping itself with the emergence of a new pattern of urban development, the periurban area (PUA) (Cavailhès et al., 2004). Defined as a mixed land-use, both urban and rural due to, respectively, its economical operation and physiognomy (Le Jeannic, 1997; Cavailhès and Schmitt, 2002), periurban spaces are currently experiencing "dynamics at least as important to consider as those which have gone through, at the relevant times, cities in the 1960s-1990s" (Hellier and Dumont, 2010, p.11). This area is now home to an increasing share of population, looking for natural surroundings and other amenities while bearing higher commuting costs, and increasingly becomes an attractive market place. This phenomenon associated with structural transformation is commonplace in many cities: from Toulouse to Stockholm, to London or Bucharest. Because of the complexity of factors and usages, the periurban belt, as a hybrid area, needs to be analyzed in their proper characters (Torquati and Giacché, 2010). Despite a good understanding of the economic forces favouring the emergence of a multicentric structure, there is no real consensus on the trade-off on environmental amenities, wages, proximity to jobs and housing rents when the working population is allowed to commute outside the central business district (CBD). As pointed out by Boiteux-Orain and Huriot (2000), "modelling suburbanization [may be] too broad and imprecise to carry out an accurate assessment". We take part in the challenge of giving full account of all economic activities that shape the periurban area. Although the literature on periurban agriculture is quite extensive, it is necessary not just to integrate firms in urban planning but also to build up a territory where firms, households and agriculture can coexist.

The purpose of this paper is to develop a theoretical framework enabling to investigate the importance of wages, agricultural amenities and commuting costs as determinants of the urban and suburban spatial structures. Formally, we explore this question by building an urban equilibrium model which combines aspects of urban economics and labor economics in outlining the location choices of these agents. The basis of the model arises from the possibility of creating a mixed land-use that is no longer limited to the presence of households and farmers, or conversely of firms and households, but also becomes an attractive economic center where periurban households may find a job and take into account neighbourhood amenities. The model displays heterogeneity between both firms and households, and relaxes the arbitrary assumption that there is just one CBD. Assuming that workers commute daily to their respective employment locations, they seek out optimal residential and job locations based on commuting costs, wages, and endogenous amenities. Farmers can choose between producing direct-selling (alternative) or homogeneous conventional goods. With urban sprawl, urban costs (housing and commuting) tend to rise. We then study a more general theory of commuting behaviour. Urban economic theories argues that housing price and wage rates compensate workers for their commuting costs (Mills, 1972; Gabriel and Rosenthal, 1996). That is, a worker might choose to live further from employment locations for cheaper housing and more favorable neighbourhood amenities while labor economics suggests that longer commuting journeys are compensated by higher wages. Conversely, the relocation of firms is further facilitated by a fall in wages related to the fact that the central centre retains some high-standard specific services, so that firms established in the periurban area incur lower wage costs. This creates an incentive for firms to move along with households and locate outside the CBD, in the periurban area where urban costs are lower. As a result of an optimization process, agents associate a value to each land unit, corresponding to the different degree of attractiveness that the city displays for each agent. The equilibrium conditions of the land market, which reflects the confrontation between the bid-rent functions of the three agents, gives rise to a PUA. This is achieved through the supply of agricultural, the level of which is determined by the interactions between the three agents whose densities are endogenous.

Moreover, amenities are of two types: that arising in the urban residential area, which are exogenous, such as local public goods (schools, infrastructures), and that arising in PUAs, which depend on the level of agricultural activities (Cavailhès et al., 2004; Coisnon et al., 2014a,b). We define periurban amenities as a net balance of exogenous and persistent externalities (e.g. natural geographical features) and negative by-production externalities through the use of pesticides. The pressing importance of periurban agriculture we study is illustrated by the debate on food security and the limits of sustainability both in its social characters, and its environmental and economical components (Fish et al., 2012). This definition allows households to broadly value its multifunctionality at the local level.

The paper is organized as follows. The next section highlights our contribution to the literature. Section 3 presents the three-sector model with social structure and endogenous by-production amenities. Section 4 derives the equilibrium conditions under three different spatial configurations: monocentric, incompletely mixed and tricentric configurations. Section 5 draws the economic implications of the model.[1]. Section 6 concludes.

2 Related Literature

The paper builds on various strands. The objective is the construction of a model with sufficient richness to capture the basic reality of the periurban spatial structure.

This work contributes to the large pool and long history of literature on urban location theory that governs the phenomenon of residential suburbanization. The classic urban model of Alonso (1964), later followed by a great deal of theoretical and empirical work by Muth (1969) and Mills (1972), has by now become one of cornerstones of urban economics.[2] Much of this literature has analyzed the monocentric city model, in which firms are assumed to locate in a single, prespecified center of production of activities, the CBD, and workers decide how close to live to this area. A household's residential location is determined by the trade-off between accessibility (commuting cost) and space (housing cost). Equilibrium in land market is reached when all the households have no more incentive to move, for example to limit daily trips to and from work by living near the city center (Legras, 2016). The assumption of monocentricity greatly simplifies the study of urban land use and permits straightforward analytical solutions of location where it is necessary only to trace locational choices along any ray from the employment center, but it fails short in two fundamental ways. First, it ignores local amenities that may counterbalance higher commuting costs and then does not address many policy questions. Second, it fails to

[1]This section is not available, it will be done soon.

[2]See Fujita and Thisse (2002) and Duranton and Puga (2004) for theoretical foundations, and Anas, Arnott, and Small (1998) for a review of research on urban spatial structure.

characterize the real pattern of periurban space. Centralized employment is convenient for a modelling perspective but lacks realism in describing current phenomenon of urban sprawl. In these models, the CBD employs the entire labor force and do not allow for other commuting pattern.

The model we propose relies on agglomeration economics, more precisely on models inspired by the seminal models of Fujita and Ogawa (Fujita, 1994; Fujita and Ogawa, 1982; Ogawa and Fujita, 1989; Fujita, Thisse and Zenou, 1997).[3] In these quasi-general equilibrium models, only two agents interact: a set of identical firms and a set of identical households that work in these firms. Firms locate either in the CBD and/or at the periphery with respect to wages and competition for land use, and benefit from positive informational spillovers that fade out gradually with distance. That is, productivity of firms is directly related to its greater proximity to other firms. The spontaneous emergence and development of a new employment center result from two opposing processes of agglomeration force generated by informational externalities and a competitive dispersion force for land use (Boiteux-Orain and Huriot, 2002). In fact, the larger agglomeration of firms, the higher commuting costs for workers, the latter being compensated by higher wages that reduce profits and increase the attractiveness of the periphery. Several equilibrium configurations may emerge outside the center when transportation costs exceed a certain threshold. In these model, space is continuous and the city is assumed to be symmetric, so that distance from the center is a summary statistic for the organization of economic activity within the city. Such framework constitutes a basis for certain analysis in urban economics, but is based upon a number of simplifying assumptions, such as fixed lot size, allowing them to analytically characterize the equilibrium values of the whole system.

From a theoretical viewpoint, this paper belongs to the broader Tiebout-like literature analyzes the geographic sorting of different types of households. The issue of consumer sorting within cities is largely overlooked and the main focus of the literature on community choice is on stratification by income (Epple and Nechyba, 2004). Beckmann (1969) was the first to attempt to cope with a continuum of heterogeneous consumers. Unfortunately, his approach was incorrect (Montesano, 1972). Later on, Hartwick et al. (1976) and Fujita (1989) proposed a rigorous analysis of the residential pattern in the case of a finite number of income classes. Their main finding is as follows. When commuting costs are not distance-dependent but income-independent, the income-classes are ranked by increasing income order as the distance to the CBD rises. Behrens et al. (2013) revisit Beckmann (1969) in a setup where consumers differ along two dimensions, that is, income and per unit distance commuting cost. Among which a small literature specifically deals with the sorting of households by environmental amenities, such as Wu (2006), Banzhaf et al. (2008) and Bayer et al. (2009). My contribution is to embed this mechanism in a general equilibrium setting where the initial sorting of workers around environmental amenities gets magnified by transportation costs. Unlike most of these authors, we assume endogenous lot size.

Finally, a way to model neighbourhood amenities in the PUA is to define it through a by-production process, in line with Cavailhès et al. (2004), Coisnon et al. (2014a,b), or Fournier (2016). In the first two papers periurban periurban amenities are defined through agricul-

[3] Agglomeration economies were first proposed by Marshall (1890) and then further explained by Jacobs (1969).

tural practises. Existing urban economics models assume that amenities are either exogenous (Brueckner et al., 1999; Wu and Plantinga, 2003; Wu, 2006) or proportional to agricultural land share (Cavailhès et al., 2004; Bento et al., 2011). The goal of this paper is to extend the study of environmental issues into urban economics models by studying the interaction between different determinants of location choice, as discussed before. Cavailhès et al. (2004) show that agricultural land offers amenities consumed by households and play an important role. Although the previous model does not ignore the fact that agriculture also causes nuisance, it does not take into account the by-production process of amenities. Recently, Fournier (2016) develops a spatial economic model which takes into account the externalities of urban pollution on agricultural yields while Coisnon et al. (2014b) advocate the introduction of endogenous agricultural amenities in order to investigate the relevance of leapfrog development. We follow their works by dealing with endogenous by-production amenities from agriculture that vary spatially with farmers' behaviour. In this setting, farming is more intensive and more polluting close to the city (that is, in the PUA (if any) than in the rural area). This definition of amenities is consistent with the public economics theory on externalities of production since positive (or negative) externalities can affect the utility function of households and vary differently depending on the type of agricultural production system chosen in the vicinity of housing. Roback (1982) demonstrates that in more amenable places, the wages should lower while the change in rents is uncertain. To the best of our knowledge, interactions between endogenous agricultural amenities, commuting costs and wages have not been explicitly examined yet in urban economics theory. However the study of these different components may lead to conflicting results. Indeed, on the one hand, the higher the density of farmers, the lower the amenity. Negative externalities resulting from farmers' intensive use of pesticides in the same buffer as households affect the locational decision of households, through the decrease of amenity level, by pushing them away from farmers. In other words, households may hardly compensate higher commuting costs (or lower wages) with a high level of amenities, and they may be attracted by the pure residential area, closer to the CBD, which offers an exogenous level of urban amenities and higher wages (whatever its location inside), all other parameters being constant. On the other hand, the lower the density of farmers, the higher the density of both firms and households. In this case, households benefit from a higher level of amenities, which converge to the initial level, but are much more to work in the suburb and then receive lower wages.

3 The model

Consider a three-sector model defined over a featureless one-dimensional location space X. Location and distance from the origin are identically denoted by $x \in X$. Jobs are available in the CBD and in the PUA (if any) located at $x_j > 0$. The city is assumed to be closed, with no migration, where the population is a constant parameter and equilibrium utility U is determined by the model.[4] Three types of agents compete for land: (i) a continuum of farms with density $\rho_a(x) \geq 0$; (ii) a continuum of firms with density $\rho_b(x) \geq 0$, and (iii) a continuum of households with density $\rho_h(x) \geq 0$ at location x.

[4] On the contrary, in the case of an open city, the spatial equilibrium condition implies that the prevailing level of utility is exogenous and must be equal in every city through costless migration.

Competition for land arises because each type of agent nurtures a particular interest in settling as close as possible to the CBD. As in the classical tradition, an absentee landlord is assumed (Fujita, 1989). Land is finite and the total area occupied by farms, firms and households at each location x is fixed and normalized to 1, as follows:

$$\rho_h(x)s_h(x) + \rho_b(x)s_b(x) + \rho_a(x)s_a(x) = 1 \tag{1}$$

where $s_h(x)$, $s_b(x)$ and $s_a(x)$ stand for lot size of households, firms and farmers, respectively.[5] The cross-product between each density and plot size denotes the total amount of land being used by each type of agent at location x.

Throughout the paper, we focus on the right-hand side of the city, the left-hand side being perfectly symmetrical.

3.1 Heterogenous Households

Assume households that differ in income and amenity level, and commute to their workplaces located in x_j,[6] facing commuting costs. Households derive utility through choice of housing (s_h), composite non-spatial good (q), chosen as a numéraire, and neighbourhood amenities (a), and in equilibrium have no incentive to move to an alternate location. Let $a(x) > 0$ be the level or, equivalently, the utility of spatial amenities, which depends on the location of households. The (relative) value of amenities consists of the sum of an exogenous part a_u, on one hand, that may be viewed as the initial level of amenities provided by the city or residential area, mostly public goods (e.g. local schools, infrastructures, etc.); and an endogenous spatial attribute a_p, on the other hand, that represents periurban amenities generated by agriculture and is defined by (16). These spatial attributes affect directly households' utility but not their budget constraint. We further assume that there are no spillover effects, meaning that amenities are intrinsic to a location. Preferences for a type i household are represented by a Cobb-Douglas utility function *à la* Cavailhès et al. (2004)[7] given by:

$$U_i\left(q, s_h, a\right) = \frac{1}{\alpha^\alpha \beta^\beta} q^\alpha s_h(x)^\beta a(x)^\gamma \tag{2}$$

where, with any loss of generality, we assume that $\alpha, \beta, \gamma \in [0, 1]$ and $\alpha + \beta = 1$. Given this functional form, α is interpreted as a consumption share parameter, or the share of expenditures on goods except land while γ measures the intensity of preferences for amenities.[8]

At this point we make some basic assumptions about the consumer's preference and indifference relations. Our intention is to model the behaviour of what we would consider a rational consumer. Following Fujita (1989), we first describe the utility function.

Assumption 1. *(Well-behaved utility function) The utility function U_i is differentiable, strictly quasi-concave and strictly increasing. Preferences are convex and indifference curves do not cut*

[5]In this paper, land and housing are not the same here as we do model the optimal choice of land developers (see e.g. Brueckner (1987) for further explanation).

[6]We fix $i \in \{u, p\}$ and $j \in \{u, p\}$ depending on the respective residential and job locations, where subscript u denotes the CBD and/or the pure urban residential area and p the PUA.

[7]The authors divide the utility function by $\alpha^\alpha \beta^\beta$ in order to simplify the indirect utility function.

[8]For clarity of presentation, utility function $U_i\left(q, s_h, a\right)$ will henceforth be referred to as $U_i\left(.\right)$.

the axes. That is, if we have two bundles q and $s_h(x)$,

$$\textit{if } q \sim s_h(x) \textit{ then } \theta q + (1-\theta)s_h(x) \succsim q \textit{ where } 0 < \theta < 1.$$

Hence, a worker-consumer household always chooses the same bundle when offered the same set of alternatives, and he always choose one (and only one) alternative from those available.

We next restrict the effects of each bundle on utility.

Assumption 2. *Differentiating the utility function given by (2) yields:*

$$U_i'(q) = \alpha \frac{U_i(.)}{q} > 0, \; U_i'(s_h) = \beta \frac{U_i(.)}{s_h(x)} > 0 \textit{ and } U_i'(a) = \gamma \frac{U_i(.)}{a(x)} > 0.$$

Equivalently, if lot size (or either composite good consumption or amenities) suddenly changes to a higher level, utility will increase *ceteris paribus.*

Assume that each household has a single worker and all workers have the same skill level. A household's gross income is given by ω, with $\omega \in [\omega^p, \omega^u]$ and $0 < \omega^p \leq \omega^u$, and depends on the location of business firms x_j. Wage variation across space may be explained by several reasons. Part of workers living in PUAs bear lower commuting costs than urban workers. Then, periurban firms are assumed to pay their workers a lower wage than that prevailing at the center (White, 1988). Assume that each worker commutes to an employment center and bears a unit commuting cost given by $t > 0$, so that a worker's commuting cost varies according to his residential and job location.[9] The budget constraint of a household residing at $x \in X$ and working in the corresponding CBD is given by:[10]

$$\omega(x_j) - t\,|x - x_j| = q + R_{h,i}(x)s_h(x), \tag{3}$$

where $R_h(x)$ is the residential rent prevailing at location $x \in X$.

At equilibrium, households of the same type i receive the same reservation utility level $\bar{u}_i$, no matter their residential location as they are identical and mobile without cost. The household's demand function for the composite good q is obtained by solving:

$$U_i(.) = \bar{u}_i \tag{4}$$

Similarly, a household residing at x and working at x_j maximizes the utility level U_i with respect to q and $s_h(x)$ subject to (3), yielding the optimal numéraire demand:

$$\begin{aligned} q^* =& q^*(s_h, a, \bar{u}_i) \\ =& \alpha\left(\omega(x_j) - t\,|x - x_j|\right) \end{aligned} \tag{5}$$

and the housing demand:

$$s_h^*(x) = \frac{\beta\left(\omega(x_j) - t\,|x - x_j|\right)}{R_{h,i}} \tag{6}$$

[9]The current model assumes that the commuting cost is a function of distance, but not congestion.

[10]Leisure is ignored and each worker supplies a single unit of labor independently of the wage rate.

where $\omega(x_j) > t\,|x - x_j|$ is assumed to hold. The optimal residential lot size depends negatively on wage and rent. An increase in transportation costs creates incentives for consumers to increase their labor supply to afford them and reduces their housing demand.

Let $\Psi_i(x)$ be the maximum rent (or highest price) type-i households are willing pay for residing at distance x from the CBD while enjoying a fixed level of utility U_i, i.e. $\bar{u_i}$, such that:

$$R^*_{h,i}(x) \equiv \Psi_i(x) = \max_{x_j} \left\{ \frac{1}{s_h} \left[\omega(x_j) - t\,|x - x_j| - q^*\left(s_h, a\right)\right] \mid U_i(.) = \bar{u_i} \right\} \tag{7}$$

Rather than work with the (direct) utility function U_i, one can use the indirect function V_i, in which a household's utility can be expressed as a function of prices at a particular location, income net of transportation costs from that location, and amenities at that location

$$V_i(x) = \left(\omega(x_j) - t\,|x - x_j|\right) R^{-\beta}_{h,i} a(x)^{\gamma} \bar{u_i} \tag{8}$$

Solving the equilibrium condition (4), we determine two bid-rents depending on the households' location. Let $\Psi_u(x)$ and $\Psi_p(x)$ be the urban and periurban housing price at x respectively, such that:

$$\Psi_u(x) = \left(\omega(x_j) - t\,|x - x_j|\right)^{1/\beta} \bar{u_u}^{1/\beta} \tag{9a}$$

$$\Psi_p(x) = \left(\omega(x_j) - t\,|x - x_j|\right)^{1/\beta} a^*_p(x)^{\gamma/\beta} \bar{u_p}^{1/\beta} \tag{9b}$$

where the urban amenity unit equals one as in Cavailhès et al. (2004). From (9a), the urban bid-rent decreases with distance and equals zero at $x = \omega_0/t$, as commuting costs increases. From a static comparative point of view, it is straightforward to check that worker-consumer households have a higher income and that there exists a trade-off between accessibility and environmental quality (Regnier and Legras (2017)). In Case (9a) with no farming, $\Psi_u(x)$ increases with $\omega(x_j)$ but decreases with t. In Case (9b), $\Psi_p(x)$ increases with $\omega(x_j)$ and with $a_p(x)$ but decreases with t. Note that bid-rents paid by households that live at that x depends on the job locations of the households' workers through wage variation across space (that is, $\omega \in \{u, p\}$).

3.2 Land and development decision

On the supply side, developers produce housing under constant return to scale in a competitive market, by converting agricultural land into residential use. A residential developer is defined as "an entrepreneur or member of a land development firm who issues the final approval to purchase land for a major residential subdivision." (Robinson and Robinson, 1986, p.57). Following Coisnon et al. (2014a), we assume that developers choose the level of development (or development density) $\rho_h(x)$ at each location maximizing profit condition on the cost of supplying land and on individual's optimal land consumption $\Psi_i(x)$. The developer's profit is

$$\max_{x} \quad \pi_d(x) = \Psi_i(x)\rho_h(x) - R_d(x) - c\left(\rho_h(x)\right),\ x \in X \tag{10}$$

where Ψ_i and R_d represent the rents paid by type i households and developers, respectively, at location x, and $c\left(\rho_h(x)\right)$ is a technical and labour development cost. We suppose that the development cost is an increasing function of the chosen housing density such that $c\left(\rho_h(x)\right) =$

$\rho_h(x)^\varepsilon$ where $\varepsilon > 1$.

The first-order conditions for this maximization problem are

$$\frac{\partial \pi_d}{\partial \rho_h(x)} = \Psi_i(x) - \varepsilon \rho_h(x)^{\varepsilon-1} = 0$$

These first-order conditions yield the following optimal housing density

$$\rho_h^*(x) = \left(\frac{\Psi_i(x)}{\varepsilon}\right)^{1/\varepsilon-1} \tag{11}$$

The housing density is a function of the residential rent and, through rent, the level of amenities at each location. Additionally, the housing density increases with the residential rent and decreases as it falls. This result is fully discussed in section 4.2.

3.3 Farmer's behaviour

As in Cavailhès et al. (2003) and Coisnon et al. (2014a,b), farmers produce a level of agricultural output per hectare Y_a and sell their crop in the central market with transport costs τ per unit of distance. We suppose a constant-returns-to-scale (CRS) Cobb-Douglas production function $y_a = f(k,x) = Ak^\delta$, where $A > 0$ and $0 < \delta < 1$. To be more specific, δ represents the output elasticity of non-land inputs used per unit of land. This value is a constant determined by available technology. The level k of non-land inputs per unit of land used by a farmer at location x represents the intensity of farming while the amount of land s_a used for production is fixed. The production function is assumed to be increasing and concave. The prices of crops and inputs, respectively p and p_k, are taken as given and the land rent at x is $R_a(x)$.

The profit-maximizing farmer chooses the level of non-land inputs used so as to maximize the difference between total revenue and total cost. The farmer's optimization problem becomes

$$\max_{k,x} \quad (p - \tau x) f(k) - p_k k(x) - R_a(x), \; x \in X \tag{12}$$

where $R_a(x)$ represents the rent per unit of land prevailing at distance x.

Maximizing (12) with respect to $k(x)$ yields the first-order conditions:

$$\frac{\partial \pi_a}{\partial k(x)} = (p - \tau x)\delta A k(x)^{\delta-1} - p_k = 0 \tag{13}$$

From (13), we deduce that at the optimum:

$$k^*(x) = \left(\frac{\delta A(p - \tau x)}{p_k}\right)^{1/1-\delta} \tag{14}$$

The function $k^*(x)$ is monotonically decreasing with distance to the CBD. Intensive farming, with a high ratio k, occurs closest to the city (Coisnon et al., 2014a).

For CRS in perfect competition any farmer with a positive output earns zero profit at equilibrium. Thus, their bid rent function, i.e. the maximum land rent that a farm is willing to pay to locate at $x \in X$, can be expressed as:

$$\Theta(x) = (p - \tau x)^{\frac{1}{1-\delta}} A \left(\frac{\delta A}{p_k}\right)^{\frac{\delta}{1-\delta}} (1 - \delta) \tag{15}$$

The bid-rent function of farmers is a decreasing function of distance ($\partial\Theta(x)/\partial x < 0$) in accordance with the Thünenian behaviour (Coisnon et al., 2014b). Farmers balance the cost of transportation, land, and profit and produce the most cost-effective product for market. They may produce at the city boundary for storage convenience such that products get to market quickly, or agri-food facilities, or direct-sales strategies (Cavailhès et al., 2004). Another straighforward implication is that the closer to the CBD, the higher the land price. In a theoretical study of urban economics, Capozza and Helsley (1989) demonstrate that land prices contain a growth premium equal to the present value of the expected future rate of growth in prices.

Following Cavailhès et al. (2004) and Coisnon et al. (2014a,b), we consider that from a given distance $x \in X$, periurban (alternative) agriculture stops in favour of exporting farming, preserved from any urban influence and hence not prone to any land-use competition. Rural farming is characterised by an exogenous agricultural rent R_A.

By-production of agricultural externalities For the periurban externalities, we adopt and extend the specification of Coisnon et al. (2014b) where by-production externalities are best thought of a net balance of constant level of amenities (e.g. geographical features) and negative by-production externalities from agricultural practices. In this framework, farms located at $x \in X$ generate pollution through an intensive use of non-land inputs such as pesticides. Note that this is an analogous situation arising in PUAs because of urban sprawl. With the reduction of agricultural land, more and more people live in housing adjacent to fields and are exposed to the massive application of agrochemicals. What is more, there exists no legislative provision that defines a buffer zone (or pesticide-free zones) between living areas and pesticide application areas.[11] Numerous studies reveals the relationship between pesticides, and health hazards and the widespread contamination of the environment (especially water). Then, unlike Coisnon et al. (2014b), we consider a local amenity that does not end up in a specific place (e.g. in a lake) and is not absorbed naturally. More precisely, environmental damage from pesticides typically varies with respect to the location of the field to which pesticides are applied. This is also in line with Cavailhès et al. (2006), where households only value environmental amenities inside a buffer of 400 meters around it. Defining $\bar{a_p}$ the constant, local public amenity provision with no pollution and φ the rate of pollution generated per unit of farm intensity k, the periurban amenities are given by:

$$a_p(x) = \bar{a_p} - \rho_a(x) s_a(x) \varphi k^*(x) \tag{16}$$

where $\bar{a_p} > \rho_a(x) s_a(x) \varphi k^*(x)$ is assumed to hold. The externality depends on the density of farmers at location $x \in X$. Furthermore, the closer the CBD, the higher the use of non-land inputs per hectare, the lower amenity level. Households can choose to benefit from these amenities if they locate in the PUA. However, by choosing a location closer from farms, households

[11] Only buildings providing facilities for the sick, the elderly and children are protected by a 50-metre buffer zone.

may bear a higher transportation cost. From (16), we see that $a_p(x) = \bar{a_p}$ when $\varphi = 0$.

3.4 Firms

Drawing upon the theoretical model by Cavailhès et al. (2007), each firm produces one good using fixed labor supply (L) and a plot of endogenous size (s_b), and deliver it through the CBD at a unitary price p_b and a unit cost $\mu > 0$. They are free to locate in etiher the CBD or the PUA (if any) and pay a wage ω depending on which business district they are settled in, with $0 < \omega^p \leq \omega^u$. Firms bear a fixed communication cost $K > 0$ reflecting the fact that some high standard services or facilities (such as banking, airports, etc.) are only available in the CBD : firms located in the periurban area have to visit those services periodically, at a cost. Communication costs equals zero for the CBD-firms and are positive but distance-independent when firms locate outside the CBD.[12] Assume that in equilibrium, full-employment prevails in the city, that is it must be that $N_b = N_h/L$ where N_b denotes the number of firms located in $x \in X$.

The production function is modelled in an analogous manner, with a CRS Cobb-Douglas function:

$$y_b = L^{\phi} s_b(x)^{1-\phi} \tag{17}$$

where $\phi \in [0, 1]$ measures the output elasticity of labor.

Denote by $\pi_{b,u}$ (resp., $\pi_{b,p}$) the profit of a firm set up in the CBD (resp., the PUA). When the firm is located in the CBD, its profit function is given by:

$$\pi_{b,u}(x) = (p_b - \mu x)y_b - R_{b,u}(x)s_b - \omega^u(x)L \tag{18}$$

where $(p_b - \mu x)y_b$ describes the firm's revenue earned, ω^u is the wage prevailing in the CBD, μx is transportation costs and $R_{b,u}(x)$ is the land rent to be paid in the CBD at a distance x from its center.

When the firm is set up in the PUA, its profit function becomes:

$$\pi_{b,p}(x) = (p_b - \mu x)y_b - R_{b,p}(x)s_b - \omega^p(x)L - K \tag{19}$$

where ω^p and $R_{b,p}$ are respectively the wage and the land rent in the periurban area, whereas the firm's revenue and lot size are the same in the CBD. Revenues are assumed to decrease with μx.

Maximizing each profit function, the FOCs with respect to land are the same and are given by:

$$\frac{\partial \pi_{b,j}}{\partial s_b} = (1 - \phi)(p_b - \mu x)\left[L^{\phi} s_b(x)^{(1-\phi)-1}\right] - R_{b,j} = 0$$

[12] In the spirit of models with externalities, Ota and Fujita (1993) distinguish two types of interaction costs for firms, alongside commuting. In the case of front-office firms, on the one hand, information imposes face-to-face interaction costs to allow for external interactions between head offices or other decision-making centers, whereas for back-office firms this information is spread through information and communication technologies and involves distance interaction costs (Guillain and Huriot, 1999). The model thus introduces a concept of proximity that is no longer linked to a short geographical distance in the classical sense. The development of new communication technologies fosters the relocation of firms across space and then, the periurbanization of jobs (Fujita and Thisse, 1997). In this paper, we only consider communication costs periurban firms for simplicity.

From (17) the above first-order conditions can be written as:

$$s_b^*(x) = L\left[\frac{(1-\phi)\,p_b}{R_{b,j}(x)\tau x}\right]^{1/\phi} \tag{20}$$

Demand for land increases with business firms' marginal revenue.

Markets are perfectly competitive then profit is driven to zero at equilibrium and we obtain two different bid-rent functions depending on the location of firms, such that:

$$\Phi_u(x) = \left(\frac{\phi}{\omega^u(x)}\right)^{\frac{\phi}{1-\phi}} (1-\phi)\,(p_b - \mu x)^{\frac{1}{1-\phi}} \tag{21a}$$

$$\Phi_p(x) = \left(\frac{\phi}{\omega^p(x) + K/L}\right)^{\frac{\phi}{1-\phi}} (1-\phi)\,(p_b - \mu x)^{\frac{1}{1-\phi}} \tag{21b}$$

Bid-rents of firms $\Phi_j(x)$ increase with transportation costs and prices, but decrease with wages. In this model, firms gain from suburbanizing because they pay lower land prices and lower wages in the periurban area, but their transport and communication costs are higher.

4 Equilibrium characterization

In this section, we define general equilibrium conditions under different urban configurations. We will rely on this characterization result to spell out further economic implications of our model.

Each equilibrium spatial structure of the city is described by a system

$$\{\rho_h(x), \rho_b(x), \rho_a(x), R(x), W(x), P(x, x_j), U_i\} \tag{22}$$

where $P(x, x_j)$ is number of households locating at x such that

$$P(x, x_j) = \frac{\text{number of households locating at } x \text{ and commuting to job site } x_j}{\text{total number } \rho_h(x) \text{ of households locating at } \ x}, \ x, x_j \in X \tag{23}$$

Equilibrium land use describes a state of the urban system that shows no propensity to change (Fujita, 1989). In the spirit of the von Thünian tradition, the three agents compete for land through an auction mechanism. The land equilibrium is driven by the value each type of agent pegs to a land plot at each possible distance from the CBD, which is given by their bid-rent functions. At the equilibrium, we observe that $R(x) = \max\{\Phi_j(x), \Psi_i(x), \Theta(x), R_A\}$,[13] i.e., land is assigned to the highest bidder process, and no land is vacant (as long as bid-rent functions are positive). We have the following conditions (Fujita, 1989):

Definition 1. *An equilibrium land use with three agents – households, business firms and farmers – consists of a system (22), and a land rent curve $R(x)$ such that, at each x*

i. Land market equilibrium

[13] Recall that, from the definitions of farming production, $\Theta(x)$ denotes the bid-rent function of alternative farming, land rent of conventional agriculture R_A being uniform.

(a) $R^*(x) = \max\{\Phi_j(x), \Psi_i(x), \Theta(x), R_A\}$,

(b) $R(x) = \Phi_j(x)$ *if* $\rho_b(x) \geq 0$,

(c) $R(x) = \Psi_i(x)$ *if* $\rho_h(x) \geq 0$,

(d) $R(x) = \Theta^*(x)$ *if* $\rho_a(x) \geq 0$,

(e) $R(x) = R_A$ *on the rural fringe,*

(f) $\rho_h(x)s_h + \rho_b(x)s_b + \rho_a(x)s_a \leq 1$,

(g) $\rho_h(x)s_h + \rho_b(x)s_b + \rho_a(x)s_a = 1$ *if* $R(x) > R_A$.

ii. Labor market

$\rho_b(x)L = \int_X \rho_h(y)P(y,x)dy$

iii. Total unit number constraints

$\int_X \rho_h(x)dx = N_h, \qquad \int_X \rho_b(x)dx = N_b, \qquad \int_X \rho_a(x)dx = N_a$

iv. Non-negativity constraints

$\rho_h(x) \geqq 0, \qquad \rho_b(x) \geqq 0, \qquad \rho_a(x) \geqq 0, \qquad R(x) \geqq 0$

$\omega(x) \geqq 0, \qquad 0 \leqq P(x, x_j) \leqq 1,$

$\int_X P(x, x_j)dx_j = 1$

Conditions (i.a) in Definition 1 states that the equilibrium land rent curve is the upper envelop of three equilibrium bid-rent curves and the agricultural rent line R_A. Condition (i.f) refers to (1), i.e. that all land is occupied by either a business firm, or a household, or a farm while condition (i.g) precises that if the land rent at x exceeds the rural agricultural land rent R_A, all land must be used for other land uses. Condition (ii.) is a labor market clearing condition, such that the demand for labor must be equal to the supply of labor at all locations in the city. There is then no unemploment in the city. These conditions altogether guarantee that each location x is occupied by the highest-bidding activity.

4.1 Monocentric spatial configuration

Several configurations may be sustained as spatial equilibria. We start with the basic *monocentric spatial configuration*, which corresponds to a city where the majority of households and farmers live separately in the suburbs while firms occupy the center. Formally, we assume that the origin is the center of the city. All firms are located between 0 and x_{0m}, the business district (BD). Households are located between x_{0m} and x_{1m}, the residential area (RA) while farmers are located between x_{1m} and x_{2m}, the alternative farming area (AF). Beyond the urban fringe x_{2m} there are only agricultural lands. Because there are only CBD-firms, households receive a wage ω equivalent to ω^u. Figure 1 represents the monocentric configuration of the city.

As in Ogawa and Fujita (1982), we assume that there is no cross-commuting (such that, for example, between 0 and x_0, individuals work only in firms on their left-hand side). The authors show that each household locating at x optimally chooses its job site x_j, considering

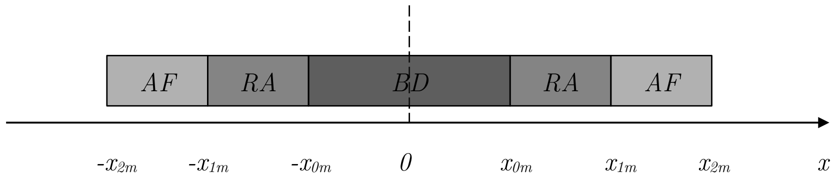

Figure 1: Monocentric urban configuration with three agents.

the trade-off between commuting cost $t\,|x - x_j|$ and wage $\omega(x_j)$. Equilibrium wage profile (or wage gradient) is given by:[14]

$$\begin{aligned}
\omega(x_j) - t\,|x - x_j| &= \omega(x_j) - t(x - x_j) \\
&= \omega_0 - tx_j - t(x - x_j) \\
&= \omega_0 - tx \equiv \omega^*(x)
\end{aligned} \tag{24}$$

The property of no cross-commuting allows us to rewrite the equilibrium conditions (i) in Definition 1 on the land market at each $x \in X$.

Definition 2. *An urban-land-use equilibrium within a monocentric spatial configuration is described as follows.*

$$\begin{aligned}
&R^*(x) = \max\{\Phi_u(x), \Psi_u(x), \Theta(x), R_A\} && \forall x \in [0, x_{2m}] \\
&R(x) = \Phi_u(x) \geq \max\{\Psi_u(x), \Theta(x)\} && \forall x \in [0, x_{0m}) \\
&R(x) = \Phi_u(x) = \Psi_u(x) && at\ x = x_{0m} \\
&R(x) = \Psi_u(x) \geq \max\{\Phi_u(x), \Theta(x)\} && \forall x \in (x_{0m}, x_{1m}) \\
&R(x) = \Psi(x) = \Theta(x) && at\ x = x_{1m} \\
&R(x) = \Theta(x) \geq \max\{\Phi_u(x), \Psi_u(x)\} && \forall x \in (x_{1m}, x_{2m}) \\
&R(x) = \Theta(x) = R_A && at\ x = x_{2m} \\
&\rho_h(x)s_h(x) + \rho_b(x)s_b(x) + \rho_a(x)s_a(x) = 1 && \forall x \in [0; x_{2m}]
\end{aligned}$$

where $\Phi_u(x)$, $\Psi_u(x)$ and $\Theta(x)$ are given by (21a), (9a) and (15), respectively. Appendix C presents the study of behavioural functions for firms, households and farmers.

4.1.1 Existence conditions of boundaries

The Central business district The *Central business district* is the area where business firms are located. Let x_{0m} be the boundary of the city. Land being rented to the highest bidder, the city is represented by the set of locations $\mathcal{C}$:

$$\mathcal{C} = \{x > x_{0m} \mid \Phi_u(x) > \max\{\Psi_u(x), \Theta(x)\}\}$$

The city boundary $x_{0m} \in [0, x_{1m}]$ solves $\Phi_u(x_0) = \Psi_u(x_0)$. In other words, it exists if the firms and households' bid-rent functions intersect at least once within the interval $[0, x_{1m}]$.

[14]See Ogawa and Fujita (1980) for an in-depth discussion of this point.

Because the transport cost function is linear in distance, equilibrium land rents are decreasing and convex.

At equilibrium, denoting $\omega(0) \equiv \omega_0$, we have:[15]

$$\begin{aligned}\Psi_u(0) = \quad & (\omega_0)^{1/\beta} \quad \text{and} \quad \Phi_u(0) = \left(\frac{\phi}{\omega_0}\right)^{\frac{\phi}{1-\phi}} (1-\phi) p_b^{\frac{1}{1-\phi}} \\ \Leftrightarrow \quad & \omega_0 < \phi^{\frac{\phi\beta}{1-\phi(1-\beta)}} (1-\phi)^{\frac{\beta(1-\phi)}{1-\phi(1-\beta)}} p_b^{\frac{\beta}{(1-\phi)[1-\phi(1-\beta)]}}\end{aligned}$$

We also have:

$$\begin{aligned}\Psi_u(x) =& 0 \;\Leftrightarrow\; x = \frac{\omega_0}{t} \\ \Phi_u(x) =& 0 \;\Leftrightarrow\; x = \frac{p_b}{\mu}\end{aligned}$$

The curves will intersect if and only if the following set of conditions is reached:

$$\begin{cases}\Phi_u(0) > \Psi_u(0) \\ x_{\Phi_u(x)=0} < x_{\Psi_u(x)=0}\end{cases}$$

$$\Leftrightarrow \begin{cases}\omega_0 < \phi^{\frac{\phi\beta}{1-\phi(1-\beta)}} (1-\phi)^{\frac{\beta(1-\phi)}{1-\phi(1-\beta)}} p_b^{\frac{\beta}{(1-\phi)[1-\phi(1-\beta)]}} & \text{(25a)} \\ \dfrac{p_b}{\mu} < \dfrac{\omega_0}{t} & \text{(25b)}\end{cases}$$

Observe that, for given ϕ and β, the level of households' income must be relatively low, compared to the price of industrial products. This is consistent with profit optimization problem of firms. Otherwise, if price is less than wage paid to worker-consumer households, then business managers should shut the firm down and produce no output. Intuitively, with $\omega_0 < p_b$, we achieve $\mu > t$ which is necessary for the equilibrium to satisfy the monocentric configuration conditions. That is, the average propensity to commute must be lower that the ability to pay of industries. The trade-off between industrial and residential use can only be made within the interval $[0, x_{0m}]$, as from x_{0m}, all industrial activity stops.

The residential area The CBD is surrounded by a *residential area*, where worker-consumer households live. We denote x_{1m} the boundary of the residential area. Land being rented to the highest bidder, the residential area is defined by the set of locations $\mathcal{R}_u$:

$$\mathcal{R}_u = \{x > x_{0m} \mid \Phi_u^*(x) > \max\{\Psi_u(x), \Theta(x)\}\}$$

Analogously, the location of the boundary x_{1m} is given by: $\Psi_u(x_{1m}) = \Theta(x_{1m})$. x_{1m} exists if the farms and households' bid-rent functions intersect at least once within the interval $[x_{0m}, x_{2m}]$. To reach this possibility, parameters of the models must obey the certain conditions.

Both bid functions are continuous, decreasing and convex within the intervals $\left[0, \frac{\omega_0}{t}\right]$ and

[15] See Appendix D.1 for detailed calculations.

$\left[0, \frac{p}{\tau}\right]$ respectively (see Appendix C). At equilibrium, we have:

$$\Psi_u(0) > \Theta(0) \Leftrightarrow \ \omega_0 > p^{\frac{\beta}{1-\delta}} A^{\beta} \left(\frac{\delta A}{p_k}\right)^{\frac{\beta\delta}{1-\delta}}$$

We also have:

$$\Psi_u(x) = 0 \Leftrightarrow \ x = \frac{\omega_0}{t}$$
$$\Theta(x) = 0 \Leftrightarrow \ x = \frac{p}{\tau}$$

The curves will intersect if and ony if the following set of conditions is reached:

$$\begin{cases} \Psi_u(0) > \Theta(0) \\ x_{\Psi_u(x)=0} < x_{\Theta(x)=0} \end{cases}$$

$$\Leftrightarrow \begin{cases} \omega_0 > p^{\frac{\beta}{1-\delta}} A^{\beta} \left(\frac{\delta A}{p_k}\right)^{\frac{\beta\delta}{1-\delta}} & \text{(26a)} \\ \frac{\omega_0}{t} < \frac{p}{\tau} & \text{(26b)} \end{cases}$$

Any increase in wages must be higher than a rise (decrease) in prices of non-land inputs (agricultural prices) *ceteris paribus*.

The alternative farming area Alternative farmers also share the space and settle in the set of locations $\mathcal{F}$ that surrounds the residential area, described as:

$$\mathcal{F} = \{x > x_{1m} \mid \Theta(x) > \max\{\Phi_u(x), \Psi_u(x), R_A\}\}$$

The location of the boundary of periurban farmland x_{2m} is given by $\Theta(x_{2m}) = R_A$, that is,

$$x_{2m} = \frac{1}{\tau}\left[p - R_A^{1-\delta} A(1-\delta)^{\delta-1}\left(\frac{\delta}{p_k}\right)^{-\delta}\right] \tag{27}$$

Setting the rural agriculture rent R_A to zero gives:[16]

$$x_{2m} = \frac{p}{\tau} \tag{28}$$

In this case, Equation (27) indicates that the closer to the CBD, the more intensive farms are, which is characterised by a relatively high level of non-land inputs (Coisnon et al., 2014a).

4.1.2 Definition of the monocentric spatial equilibrium

We sum up the above conditions that together necessary and sufficient for a monocentric urban equilibrium with three sectors to exist.

Proposition 1. *The monocentric configuration is more likely an equilibrium when*

[16] See Appendix D.2 for detailed calculations.

1. *The spatial equilibrium is given by the prevailing land rent at $x \in [0, x_{2m}]$:*

$$R^*(x) = \max\{\Phi_u(x), \Psi_u(x), \Theta(x), R_A\}$$

2. *From Conditions (25a) and (26a), we get:*

$$p^{\frac{\beta}{1-\delta}} A^{\beta} \left(\frac{\delta A}{p_k}\right)^{\frac{\beta\delta}{1-\delta}} < \omega_0 < \phi^{\frac{\phi\beta}{1-\phi(1-\beta)}} (1-\phi)^{\frac{\beta(1-\phi)}{1-\phi(1-\beta)}} p_b^{\frac{\beta}{(1-\phi)[1-\phi(1-\beta)]}}$$

   That is, the level of households' income is low (high) enough compared to the price of industrial (agricultural) products.

3. *From Conditions (25b), (26b) and (27), we have:*

$$\frac{p_b}{\mu} < \frac{\omega_0}{t} < \frac{p}{\tau} < x_{2m}$$

4. *At equilibrium, the delimitations of the different areas are characterised by the city boundary x_{2m} given by (27), the limits of the industrial area x_{0m} and the residential area, x_{1m}.*

4.2 Incompletely mixed spatial configuration

We next illustrate the possibility that the urban configuration is *incompletely mixed.* An incompletely mixed spatial configuration is a generalization of the monocentric configuration. There are four sections in the city. The first two are specialized areas. Firms and urban households still locate, respectively, in the business district (BD) between 0 and x_{0i}, and the residential area (RA) between x_{0i} and x_{1i}. The residential area is surrounded by a periurban belt composed by both periurban households and alternative farming. Finally, alternative farming (AF) is also located in a specialized area, between x_{2i} and x_{3i}. We focus again only on the right-half of the city where $x \geq 0$. Figure 2 represents the incompletely mixed city structure.

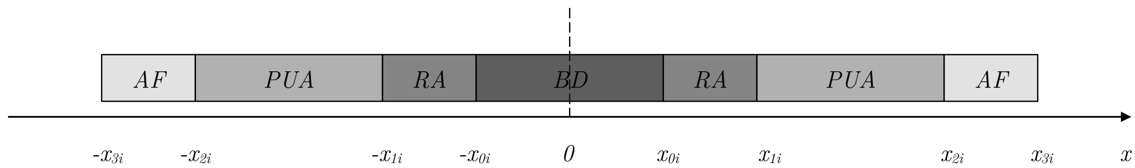

Figure 2: Incompletely mixed spatial configuration with multiple agents.

The property of no cross-commuting allows us to rewrite the equilibrium conditions (i) in Definition 1 on the land market at each $x \in X$.

Definition 3. *An urban-land-use equilibrium within an incompletely mixed spatial configuration* *is described as follows.*

$$
\begin{aligned}
&R^*(x) = \max\{\Phi_u(x), \Psi_u(x), \Psi_p(x), \Theta(x), R_A\} && \forall x \in [0, x_{3i}] \\
&R(x) = \Phi_u(x) \geq \max\{\Psi_u(x), \Psi_p(x), \Theta(x)\} && \forall x \in [0, x_{0i}) \\
&R(x) = \Phi_u(x) = \Psi_u(x) && at\ x = x_{0i} \\
&R(x) = \Psi_u(x) \geq \max\{\Phi_u(x), \Theta(x)\} && \forall x \in (x_{0i}, x_{1i}) \\
&R(x) = \Psi_u(x) = \Theta(x) && at\ x = x_{1i} \\
&R(x) = \Psi_p(x) = \Theta(x) && \forall x \in (x_{1i}, x_{2i}) \\
&R(x) = \Theta(x) \geq \max\{\Phi_u(x), \Psi_u(x)\} && \forall x \in (x_{2i}, x_{3i}) \\
&R(x) = \Theta(x) = R_A && at\ x = x_{3i} \\
&\rho_h(x)s_b(x) + \rho_b(x)s_b(x) + \rho_a(x)s_a(x) = 1 && \forall x \in [0, x_{3i}]
\end{aligned}
$$

4.2.1 Equilibrium conditions

We reach similar results for the city, residential and alternative farming areas as in the monocentric configuration.

The periurban area Within the PUA, land use is shared between farmers and households. A necessary and sufficient condition for the mixed land-use to exist is that houholds' land bid equals the farmers' land bid rents (Cavailhès et al. 2004). The periurban area is defined by the set of locations $\mathcal{P}_i = \{x > x_{1i} \mid \Psi_p(x) = \Theta(x)\}$. We consider that the periurban area develops directly on x_{1i}, allowing households to locate in two areas.

Plugging (11) into (1) when land is shared by both households and farming gives the agricultural land density as

$$\rho_a(x) = 1 - \rho_h^*(x)s_h^*(x) = 1 - \beta\left(\frac{\omega_0 - tx}{2}\right) \tag{29}$$

assuming that $\varepsilon = 2$ and $s_a(x)$ is fixed and equals one.

We have $\Psi_p(x) = \Theta(x)$. We obtain the same type of equilibrium condition for the periurban ring to exist as in Cavailhès et al. (2004), given by

$$a_p^*(x) = \left(\frac{\Theta(x)}{\Psi_u(x)}\right)^{\beta/\gamma} = \left(\frac{\Theta(x)^\beta}{(\omega_0 - tx)\,\bar{u}_p}\right)^{1/\gamma} \quad \text{at any } x \in [x_{1i}, x_{2i}] \tag{30}$$

where the level of amenities available at x, $a_p^*(x)$, is given by (16). The level of amenities, and also farmland size, increases with the distance to the CBD because households need to compensate for a longer commuting by a higher level of periurban externalities. This expression also implies that $a_p^*(x)$ increases (decreases) with the elasticity of utility with respect to amenities (γ) (housing (β)). In other words, residential plot size decreases with distance to the CBD.

From the definition of $a_p(x)$ and (29) we derive the following level of amenities:

$$a_p^*(x) = \bar{a_p} - \varphi\left[1 - \beta\left(\frac{\omega_0 - tx}{2}\right)\right]\left(\frac{\delta A(p - \tau x)}{p_k}\right)^{1/1-\delta} \tag{31}$$

Proposition 2. *The periurban area $\mathcal{P}_i$ exists if farming generates a minimum threshold $\bar{a}_{p_{\min}}$, all other parameters being equal, allowing households to compensate higher commuting costs. Such a threshold is given by:*

$$\bar{a}_{p_{\min}} = \arg\min_x \left(\frac{\Theta(x)}{\Psi_u(x)}\right)^{\gamma/\beta} + \varphi k^*(x) \tag{32}$$

Proof. See Appendix E. ■

4.2.2 Existence of an incompletely spatial equilibrium

As expected the spatial general equilibrium under an incompletely mixed configuration has the same conditions as in the monocentric case. We complete it with the following proposition.

Proposition 3. *The incompletely mixed configuration is more likely an equilibrium when*

1. *The spatial equilibrium is given by the prevailing land rent at $x \in [0, x_{2i}]$: $R^*(x) = \max\{\Phi_u(x), \Psi_u(x), \Psi_p(x), \Theta(x), R_A\}$.*
2. *Proposition 1 is satisfied when bid-rent of firms is given by (22), urban and periurban households' rents by (9a) and (9b) respectively, and farmland rent by (15).*
3. *The level of amenities in the periurban area $a_p^*(x)$ is given by (30) and (31) and depends on farming intensity and development of housing (or equivalently, the proportion that is left to agricultural activity).*

4.3 Triocentric spatial configuration

We finally present the *tricentric urban configuration* in which business firms concentrate and form three distinct employment centers. We widen the definition of the PUA to include activities and services, through the presence of firms, that have gradually created an economic and functional space in order to meet the needs of families at a point in their residential trajectory. This definition is consistent with literature of urban sprawl, as stated in the introduction. Residential mobility has long been correlated with the creation of firms, but firms also follows households. Within this configuration, the PUA is located between x_{1t} and x_{2t}. Then, after studying homogeneous households in both previous urban configurations, we consider that households are heterogeneous in terms of wages and amenity provision but have the same skill level. For simplicity, workers are assumed to commute daily to their respective employment locations. In the absence of cross-commuting, the labor demanded for production in center j is provided by those households located in each subcenter's labor market area ($\rho_{h,i}(x) = \rho_{b,j}(x)$).[17] Urban households (that is, type u) commute to the CBD-firms located at any $x \in [0, x_{0t}]$, and all workers living at $x \in [x_{1t}, x_{2t}]$ commutes to firms located inside the PUA.

Writing place-specific quality-adjusted housing costs (or rents) as $\Psi_i(x)$, area specific ω-adjusted household earnings and the amenity value or quality of life in place i ($a(x)$), the

[17] We will analyze this configuration assuming cross-commuting (that is, when households commute inwardly) later on.

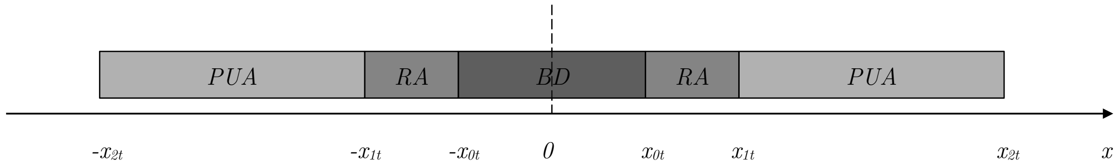

Figure 3: Tricentric urban configuration with multiple agents.

relationship between housing costs, wages, commuting costs and quality of life is:

$$\Delta\Psi_i(x) = \Delta\omega + \Delta t + \Delta a(x)$$

where the Δ means a difference from the baseline reference urban place.

An equilibrium is such that each household maximizes his utility subject to his budget constraint and, each firm and alternative farm maximizes its profits. In particular, no firm has an incentive to change location within the city, and no worker wants to change his working and/or residence place. We focus on the decision of a resident to live and work in the CBD, or in the PUA given the bid rent. Individuals derive utility (U_i) from consumption (q) and amenities (a) in the place of residence (i) according to the utility function given by (2). In other words, each worker chooses his location so as to maximize utility given his wage and the land rent; let $\Psi_{uu}(x)$ and $\Psi_{pp}(x)$ be the bid rent at $x \in X$ of a worker located respectively in the CBD and the PUA. Similarly, $\Phi_u(x)$ and $\Phi_p(x)$ stand for a firm's bid rents. The bid-rent of alternative farming remains the same, denoted by $\Theta(x)$. We rewrite again the equilibrium conditions (i) in Definition 1 on the land market at each $x \in X$ to fit with the decentralized urban configuration.

Definition 4. *An urban-land-use equilibrium within a tricentric spatial configuration is described as follows.*

$$\begin{aligned}
&R^*(x) = \max\{\Phi_u(x), \Phi_p(x), \Psi_{uu}(x), \Psi_{pp}(x), \Theta(x), R_A\} && \forall x \in [0, x_{2t}] \\
&R(x) = \Phi_u(x) \geq \max\{\Phi_p(x), \Psi_{uu}(x), \Psi_{pp}(x), \Theta(x)\} && \forall x \in [0, x_{0d}) \\
&R(x) = \Phi_u(x) = \Psi_{uu}(x) && at\ x = x_{0t} \\
&R(x) = \Psi_{uu}(x) \geq \max\{\Phi_j(x), \Psi_{pp}(x), \Theta(x)\} && \forall x \in (x_{0t}, x_{1t}) \\
&R(x) = \Phi_p(x) = \Psi_{pp}(x) = \Theta(x) && \forall x \in (x_{1t}, x_{2t}) \\
&R(x) = \Theta(x) = R_A && at\ x = x_{2t} \\
&\rho_{h,i}(x)s_b(x) + \rho_{b,j}(x)s_b(x) + \rho_a(x)s_a(x) = 1 && \forall x \in [0, x_{2t}]
\end{aligned}$$

where $\rho_{b,u}(x)$ is the share of firms located in the CBD, and therefore $\frac{1-\rho_{b,u}(x)}{2}$ the share of firms in each PUA. The equilibrium definition invokes some of the previous conditions on the existence of the CBD and the urban residential area.

4.3.1 The mapping wage and commuting conditions

The location of employment has been treated as exogenous in the equilibrium model described above. Eventually, wages should equalize between centers of employment (White, 1982). To ensure that workers have no incentive to commute to a different location, equilibrium requires a restriction on the wage gradient through commuting costs. The necessary condition in equilibrium is that the difference in wages paid between two locations must be equal to the total commuting cost of traveling between the two locations. The market wage gradient must be negatively related to workplace location x, that is, $\partial\omega(x)/\partial x < 0$. The budget constraint of a worker residing at x and working in the CBD implies that $\omega^u - tx = q + R_{h,u}(x)s_h(x)$, while the budget constraint of a worker living in the PUA is $\omega^p - t\,|x - x_p| = q + R_{h,p}(x)s_h(x)$ where $x_p \in [x_{1t}, x_{2t}]$. At the city equilibrium, the worker living in $x_{1t} \in X$ is indifferent between working in the CBD or in the PUA, which implies:

$$\omega^u - tx_{1t} - q - R_{h,u}(x_{1t})s_h(x_{1t}) = \omega^p - t\,(x_p - x_{1t}) - q - R_{h,p}(x_{1t})s_h(x_{1t})$$

that is,

$$\omega^u - \omega^p = t(2x_{1t} - x_p) + s_h(x_{1t})\,(R_{h,u}(x_{1t}) - R_{h,p}(x_{1t})) \tag{33}$$

Thus, the difference between the wages paid in the CBD and in the PUA compensates exactly the worker for the difference in the corresponding commuting costs. At the border x_{1t},$R_{h,p}(x_{1t}) = R_{h,u}(x_{1t})$, then $\omega^u - \omega^p = t(2x_{1t} - x^p) + s_h(x_{1t})$.

Since both types of firms face the same demand function, setting the profits to zero, we obtain the following equilibrium condition: Hence, the equilibrium wage rates in the CBD and in the PUA must satisfy the conditions $\pi_{b,u}(\omega^{u*}) = 0$ and $\pi_{b,p}(\omega^{p*}) = 0$, respectively. Thus, setting (18) (resp., (19)) equal to zero solving for ω^{u*} (resp., ω^{p*}), we get:

$$\omega^{u*} = \frac{1}{L}\left[(p_b - \mu x)y_b - \Phi_u(x)s_b^*(x)\right] \tag{34}$$

and

$$\omega^{p*} = \frac{1}{L}\left[(p_b - \mu x)y_b - \Phi_p(x)s_b^*(x) - K\right] \tag{35}$$

Hence, the wage premium, denoted by $\Delta\omega$, is the difference between the optimal wage offers is ω^{u*} and ω^{p*} and is given by: $\Delta\omega = \omega^{u*} - \omega^{p*} = \frac{K}{L}$. The equilibrium wage wedge increases with the communication costs faced by firms in the PUA.[18]

Substituting (34) and (35) into (33) and solving with respect to t yields:

$$\begin{aligned} \frac{K}{L} &= t(2x_{1t} - x_p) + s_h(x_{1t}) \\ \Leftrightarrow t &\leq \frac{1}{2x_{it} - x_p}\left[\frac{K}{L} - s_h(x_{it})\right] \end{aligned} \tag{36}$$

Hence, a tricentric city is more likely to occur when commuting costs are large, communication costs are low. It is straightforward that workers will only travel toward wages that are higher than the wages paid where they live.

[18] Equivalently, this result can be found by equalizing bid rents $\Phi_u(x)$ and $\Phi_p(x)$.

Finally, the model requires a labor-market-clearing condition, which states that all workers must be housed within the city. An equivalent condition is that commuting is zero at the edge of the city, which can be formally written as $N_b(x_{2t}) = 0$. The underlying assumption here is that commuting costs from other cities are sufficiently high to prevent such activity.

4.3.2 Equilibrium land rents

As stated above, the PUA is now defined as a mixed space shared by firms, households and farmers. Accordingly, this area exists if and only if the farmland rent is the same as households' and firms' bid-rents and is defined by the set of locations $\mathcal{P}_t = \{x > x_{1t} \mid \Phi_p(x) = \Psi_{pp}(x) = \Theta(x)\}$. We assume that the periurban area develops directly on x_{1t}, allowing households to locate in two areas and that half of business firms from the CBD relocate in the periurban area. Workers maximize utility by choosing their area of work and residence subject to their budget constraint. A worker residing in the CBD is face with a choice between working at the CBD (i.e. staying), which gives an indirect utility:

$$V_{uu} = (\omega^u - tx)R_{h,u}(x)^{-\beta}\overline{u}_u \tag{37}$$

and working and residing in the PUA (i.e. migrating) which yields utility:

$$V_{pp} = (\omega^p - t\,|x - x_p|)R_{h,p}(x)^{-\beta}a_p(x)^{\gamma}\overline{u}_p \tag{38}$$

The critical level of amenity at which a worker is indifferent between staying and migrating ($\widetilde{a_p}$) is given by:

$$\widetilde{a_p} = \left(\frac{\Psi_{pp}}{\Psi_{uu}}\right)^{\beta/\gamma}\left[\frac{\omega^u - tx}{\omega^p - tx}\right]^{1/\gamma} \tag{39}$$

with the worker preferring to stay if $\widetilde{a_p} \geq a_p^*$.[19]

Plugging (11) into (1) when land is shared by households, firms and farming, we define the agricultural land density as

$$\begin{aligned}\rho_a(x) &= 1 - \rho_b(x)s_b^*(x) - \rho_h^*(x)s_h^*(x)\\ &= 1 - \frac{1}{2}\left[\Psi_{pp}L\left(\frac{(1-\phi)\,p_b}{\Phi_p(x)\tau x}\right)^{1/\phi} + \beta\left(\frac{\omega^p - t\,|x - x_p|}{2}\right)\right]\end{aligned} \tag{40}$$

assuming that $\varepsilon = 2$ and $s_a(x)$ is fixed and equals one.

We then obtain derive the following level of amenities:

$$a_p^*(x) = \bar{a}_p - \varphi\left[1 - \frac{1}{2}\left[\Psi_{pp}L\left(\frac{(1-\phi)\,p_b}{\Phi_p(x)\tau x}\right)^{1/\phi} + \beta\left(\frac{\omega^p - t\,|x - x_p|}{2}\right)\right]\right]\left(\frac{\delta A(p - \tau x)}{p_k}\right)^{1/1-\delta} \tag{41}$$

The periurban belt vanishes when all the land is used for farming.

[19]We assume throughout that when $V_{uu} = V_{pp}$ individuals stay.

4.3.3 Existence of a tricentric spatial equilibrium

Again, we complete the monocentric conditions for the general equilibrium under a tricentric urban configuration.

Proposition 4. *The tricentric urban configuration is more likely an equilibrium when*

1. *The spatial equilibrium is given by the prevailing land rent at* $x \in [0, x_{2t}]$*:* $R^*(x) = \max\{\Phi_u(x), \Phi_p(x), \Psi_{uu}(x), \Psi_{pp}(x), \Theta(x), R_A\}$*, that is, the market for land must clear at every location.*

2. *Proposition 1 is satisfied when bid-rent of firms is given by (22), urban and periurban households' rents by (9a) and (9b) respectively, and farmland rent by (15).*

3. *The wage rates must satisfy (36) and the labor market clears.*

4. *The level of amenities in the periurban area* $a_p^*(x)$ *is given by (39) and (41) and depends on farming intensity, the development of housing and the proportion of firms.*

5 Simulation results (work in progress)

6 Caveats and extensions

This paper proposes a three-sector urban model which allows us to better represent the urban and suburban spatial patterns. Our model extends standard urban models that have been developed to cover this issue but never considering the three main actors altogether in the PUA: firms, households and farmers. Our baseline model represents a monocentric pattern, where the three agents compete for land. We introduce a mixed area of housing and alternative farming activities, following Cavailhès et al. (2004) and Coisnon et al. (2014a,b). On the one hand, we have heterogeneous households with a Cobb-Douglas utility whose variables are housing, amenities and a composite good. Households work either in a pre-determined CBD and in the periurban belt. On the other hand, farmers generate negative externalities through an intensive use of pesticides close to the CBD. The third configuration extends the definition of the PUA by taking into account firms. Equilibrium is reached through a competitive land market. However, the complexity of our model does not allow for the determination of analytical solutions, a simulation being carried out later on in order to conclude on the determinants of the internal structure of such a city. Indeed, the first three configurations, plus another tricentric spatial configuration with cross-commuting, will yield a variety of new insights according to the trade-off of households in terms of wage rates, amenities, rents and commuting costs.

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A Description of variables

$R(x)$	equilibrium land rent
x	the distance from the CBD
x_j	location of job site
$\|x - x_j\|$	distance between residence and job site x_j
Households	
$U(.)$	utility function
$\rho_h(x)$	household density function
N_h	number of households
$R_h(x)$	price per unit of housing
$\Psi^*(x)$	bid-rent function
s_h	lot size (or land) consumed by the household
q	the amount of composite good, chosen as a numeraire ($p_q = 1$)
$a(x)$	level of amenities such that: $a(x) = \bar{a_u}(x) + a_p(x)$
t	round-trip commuting cost per kilometer
$\omega(x_j)$	household wage paid by the business firms located at x_j
$P(x, x_j)$	number of households locating at x
Farmers	
$\pi_a(k, x)$	profit function
$\rho_a(x)$	alternative farming density function
N_a	number of farms
$R_a(x)$	land price for alternative farming use
$\Theta^*(x)$	bid-rent function
s_a	lot size (or land) consumed by farmers
τ	transport cost per unit of distance
y_a	production function such that $y_a = f(k, x)$
k	intensity of non-land inputs per hectare
p	crop prices
p_k	non-land input prices
$\bar{a_p}$	local public amenity provision (or environmental quality with no pollution)
φ	rate of pollution generated per unit of farm
Firms	
$\pi_b(x)$	profit function
$\rho_b(x)$	firm density function
N_b	number of business firms
L	labor demand
$R_b(x)$	land price for commercial use
$\Phi^*(x)$	bid-rent function
$s_b(x)$	lot size (or land) consumed by firms
μx	transportation cost

B Mathematical resolution

B.1 Households' behaviour

B.1.1 Optimal land size and composite good

Suppose we want to solve[20]

$$\begin{cases} \max\limits_{q, s_h(x), x, x_j} & \frac{1}{\alpha^\alpha \beta^\beta} q^\alpha s_h(x)^\beta a(x)^\gamma \\ \text{s.t.} & \omega(x_j) - t\,|x - x_j| = q + R_h(x) s_h(x) \end{cases}$$

using Lagrange multipliers.

From the previous problem, we can subtract one side off of both sides of the budget constraint to get:

$$\omega(x_j) - t\,|x - x_j| - q - R_h(x) s_h(x) = 0$$

First we form the Lagrangian

$$\mathcal{L}(.) = \frac{1}{\alpha^\alpha \beta^\beta} q^\alpha s_h(x)^\beta a(x)^\gamma + \lambda \left[\omega(x_j) - t\,|x - x_j| - q - R_h(x) s_h(x)\right]$$

where $\alpha + \beta = 1$.

Now we find the first order conditions by taking derivatives with respect to q, $s_h(x)$ and λ and setting them equal to 0:

$$[1]: \ \frac{\partial \mathcal{L}(.)}{\partial q} = 0 \iff \frac{1}{\alpha^\alpha \beta^\beta} \alpha q^{\alpha-1} s_h(x)^\beta a(x)^\gamma - \lambda = 0$$

$$[2]: \ \frac{\partial \mathcal{L}(.)}{\partial s_h(x)} = 0 \iff \frac{1}{\alpha^\alpha \beta^\beta} q^\alpha \beta s_h(x)^{\beta-1} a(x)^\gamma - \lambda R_h(x) = 0$$

$$[3]: \ \frac{\partial \mathcal{L}(.)}{\partial \lambda} = 0 \iff \omega(x_j) - t\,|x - x_j| - q - R_h(x) s_h(x) = 0$$

Then we obtain the optimality condition by eliminating λ^* from [1] and [2]:

$$\frac{1}{\alpha^\alpha \beta^\beta} \alpha q^{\alpha-1} s_h(x)^\beta a(x)^\gamma = \frac{1}{R_h(x)} \frac{1}{\alpha^\alpha \beta^\beta} q^\alpha \beta s_h(x)^{\beta-1} a(x)^\gamma$$

$$\begin{aligned} &\iff \frac{\alpha}{\beta} \frac{q^{\alpha-1}}{q^\alpha} R_h(x) = \frac{s_h(x)^{\beta-1}}{s_h(x)^\beta} \\ &\iff \beta q = \alpha R_h(x) s_h(x) \\ &\iff q = \frac{\alpha}{\beta} R_h(x) s_h(x) \end{aligned}$$

This give us the relationship between q and $s_h(x)$ when they are optimally chosen.

Finally, we use the third condition [3], called the feasibility condition, to solve the choice

[20] In this Appendix we drop subscripts for simplicity.

variables:

$$\omega(x_j) - t\,|x - x_j| - q - R_h(x)s_h(x) = 0$$

$$\iff \omega(x_j) - t\,|x - x_j| - \frac{\alpha}{\beta}R_h(x)s_h(x) - R_h(x)s_h(x) = 0$$

$$\iff s_h^*(x) = \frac{\beta\left(\omega(x_j) - t\,|x - x_j|\right)}{R_h(x)}$$

and, substituting $s_h^*(x)$ into q^*,

$$q^* = \frac{\alpha}{\beta}R_h(x)s_h^*(x)$$

$$\iff q^* = \frac{\alpha}{\beta}R_h(x)\left[\frac{\beta\left(\omega(x_j) - t\,|x - x_j|\right)}{R_h(x)}\right]$$

$$\iff q^* = \alpha\left(\omega(x_j) - t\,|x - x_j|\right)$$

B.1.2 Households' indirect utility function

Households' indirect utility function is $V(x)$, such that:

$$\begin{aligned} V(x) &= \frac{1}{\alpha^{\alpha}\beta^{\beta}}\left(\alpha\left(\omega(x_j) - t\,|x - x_j|\right)\right)^{\alpha}\left[\frac{\beta\left(\omega(x_j) - t\,|x - x_j|\right)}{R_h(x)}\right]^{\beta} a(x)^{\gamma}\bar{u} \\ &= \left(\omega(x_j) - t\,|x - x_j|\right) R_h((x)^{-\beta} a(x)^{\gamma}\bar{u} \end{aligned}$$

where $\alpha + \beta = 1$.

B.2 Farmer behaviour

If the farmer sells his production directly to the CBD with transport costs τ per unit of distance, **the profit function** is given by

$$\begin{aligned} &\pi_a(x) = \{(p - \tau x)f(k) - p_k k(x) - R_a(x)s_a\} \\ \iff &\pi_a(x) = \{(p - \tau x)Ak(x)^{\delta} - p_k k(x) - R_a(x)\} \end{aligned}$$

Through the FOC, we determine $k^*(x)$, given by:

$$\begin{aligned} &\frac{\partial \pi_a(x)}{\partial k(x)} = 0 \\ \iff &(p - \tau x)\delta A k(x)^{\delta - 1} - p_k = 0 \\ \iff &k^*(x) = \left(\frac{p_k}{\delta A(p - \tau x)}\right)^{1/\delta - 1} \\ \iff &k^*(x) = \left(\frac{\delta A(p - \tau x)}{p_k}\right)^{1/1 - \delta} \end{aligned}$$

We now want to compute the **bid-rent function** of farmers.

We know that:

$$\pi_a^*(x) = (p - \tau x)Ak^*(x)^\delta - p_k k^*(x) - R_a(x)$$

Substituting k^* by its expression in the profit function,

$$\pi_a(x) = (p - \tau x)A \left[\left(\frac{\delta A(p - \tau x)}{p_k} \right)^{\frac{1}{1-\delta}} \right]^\delta - p_k \left(\frac{\delta A(p - \tau x)}{p_k} \right)^{\frac{1}{1-\delta}} - R_a(x)$$

At equilibrium, profit is equal to zero,

$$(p - \tau x)A \left[\left(\frac{\delta A(p - \tau x)}{p_k} \right)^{\frac{1}{1-\delta}} \right]^\delta - p_k \left(\frac{\delta A(p - \tau x)}{p_k} \right)^{\frac{1}{1-\delta}} - R_a(x) = 0$$

We then compute the bid-rent function:

$$\begin{aligned} R_a^*(x) \equiv \Theta(x) &= (p - \tau x)A \left(\frac{\delta A(p - \tau x)}{p_k} \right)^{\frac{\delta}{1-\delta}} - p_k \left(\frac{\delta A(p - \tau x)}{p_k} \right)^{\frac{1}{1-\delta}} \\ &= (p - \tau x)^{\frac{1}{1-\delta}} \left(\frac{\delta}{p_k} \right)^{\frac{\delta}{1-\delta}} A^{\frac{\delta}{1-\delta}} (A - \delta A) \end{aligned}$$

In this case, the **bid-rent function** of farms is:

$$\Theta(x) = (p - \tau x)^{\frac{1}{1-\delta}} A \left(\frac{\delta A}{p_k} \right)^{\frac{\delta}{1-\delta}} (1 - \delta)$$

B.3 Firms' behaviour

$$\begin{aligned} &\max_x \pi_b = (p_b - \mu x)y_b - (R_b(x)s_b + \omega(x)L) \\ \iff &\max_x \pi_b = (p_b - \mu x) \left[L^\phi s_b(x)^{1-\phi} \right] - (R_b(x)s_b + \omega(x)L) \end{aligned}$$

FOCs:

$$\begin{aligned} \frac{\partial \pi_b}{\partial s_b} = 0 &\iff (1 - \phi)(p_b - \mu x) \left[L^\phi s_b(x)^{(1-\phi)-1} \right] - R_b = 0 \\ &\iff L \left[\frac{(1 - \phi)(p_b - \mu x)}{R_b(x)} \right]^{1/\phi} = s_b^*(x) \end{aligned}$$

where $y_b = L^\phi s_b(x)^{1-\phi}$

We now define the **bid-rent function** of firms:

At equilibrium,

$$\pi_b = (p_b - \mu x) \left[L^\phi s_b(x)^{1-\phi} \right] - \omega(x)L - R_b(x)s_b = 0$$

$$\iff (p_b - \mu x)L^\phi \left[L \left(\frac{(1 - \phi)(p_b - \mu x)}{R_b(x)} \right)^{1/\phi} \right]^{1-\phi} - \omega(x)L - R_b(x)L \left[\frac{(1 - \phi)(p_b - \mu x)}{R_b(x)} \right]^{1/\phi} = 0$$

$$\iff R_b(x) = \left(\frac{\phi}{\omega(x)} \right)^{\frac{\phi}{1-\phi}} (1 - \phi) (p_b - \mu x)^{\frac{1}{1-\phi}}$$

C Study the sign and the form of each bid-rent functions

We have:

$$
\begin{aligned}
\Phi(x) &= \left(\frac{\phi}{\omega(x)}\right)^{\frac{\phi}{1-\phi}} (1-\phi)\,(p_b - \mu x)^{\frac{1}{1-\phi}} \\
\Psi(x) &= (\omega(x_j) - t\,|x - x_j|)^{1/\beta} \\
\Theta(x) &= (p - \tau x)^{\frac{1}{1-\delta}} A \left(\frac{\delta A}{p_k}\right)^{\frac{\delta}{1-\delta}} (1-\delta)
\end{aligned}
$$

where $\alpha, \beta, \gamma, \delta \in (0,1)$, $\alpha + \beta = 1$, and $A > 0$.

We see that bid-rent functions are continuous, monotonic, and downward sloping with respect to the distance from the CBD because agents associate a higher value with the land plots located closer to the CBD (i.e., their reservation rent decreases with distance). Given that bid-rent functions decrease with x, they become negative at a certain distance. Naturally, land becomes vacant when all of them become negative.

C.1 Households' bid-rent function

Let $\Psi_u(x)$ be a function and defined as

$$
\Psi_u(x) = (\omega(x_j) - t\,|x - x_j|)^{1/\beta}
$$

where $\alpha + \beta = 1$ and $0 < \gamma < 1$.

$$
\frac{\partial \Psi_u(x)}{\partial x} = -\frac{t}{\beta}\,(\omega(x_j) - t\,|x - x_j|)^{\frac{1}{\beta}-1} < 0 \text{ at any } x < \frac{\omega}{t}
$$

$$
\begin{aligned}
\frac{\partial^2 \Psi_u(x)}{\partial x^2} &= \frac{(\frac{1}{\beta} - 1) t^2\,(\omega(x_j) - t\,|x - x_j|)^{\frac{1}{\beta}-2}}{\beta} \\
&= \frac{(\beta - 1) t^2\,(\omega(x_j) - t\,|x - x_j|)^{\frac{1}{\beta}-2}}{\beta^2} > 0 \text{ at any } x < \frac{\omega}{t}
\end{aligned}
$$

C.2 Farmers' bid-rent function

Let $\Theta(x)$ be a function and defined as

$$\Theta(x) = (p - \tau x)^{\frac{1}{1-\delta}} A \left(\frac{\delta A}{p_k}\right)^{\frac{\delta}{1-\delta}} (1 - \delta)$$

where $A > 0$ and $0 < \delta < 1$.

$$\frac{\partial \Theta_d(x)}{\partial x} = -A\tau \left(\frac{A\delta}{p_k}\right)^{\frac{\delta}{1-\delta}} (p - \tau x)^{\frac{1}{1-\delta}-1} < 0 \text{ at any } x < \frac{p}{\tau}$$

$$\frac{\partial^2 \Theta_d(x)}{\partial x^2} = A \left(\frac{1}{1-\delta} - 1\right) \tau^2 \left(\frac{A\delta}{p_k}\right)^{\frac{\delta}{1-\delta}} (p - \tau x)^{\frac{1}{1-\delta}-2} > 0 \text{ at any } x < \frac{p}{\tau}$$

C.3 Business firms' bid-rent function

Let $\Phi(x)$ be a function and defined as

$$\Phi(x) = \left(\frac{\phi}{\omega(x)}\right)^{\frac{\phi}{1-\phi}} (1 - \phi) (p_b - \mu x)^{\frac{1}{1-\phi}}$$

where $0 < \phi < 1$.

$$\frac{\partial \Phi(x)}{\partial x} = -\frac{\phi^2 \left(\frac{\phi}{\omega(x)}\right)^{\frac{\phi}{1-\phi}-1} \omega'(x) (p_b - \mu x)^{\frac{1}{1-\phi}}}{\omega(x)^2} - \mu \left(\frac{\phi}{\omega(x)}\right)^{\frac{\phi}{1-\phi}} (p_b - \mu x)^{\frac{1}{1-\phi}-1} < 0$$

$$\begin{aligned}
\frac{\partial^2 \Phi(x)}{\partial x^2} = & -\frac{\phi^2 \left(\frac{\phi}{\omega(x)}\right)^{\frac{\phi}{1-\phi}-1} \omega''(x) (p_b - \mu x)^{\frac{1}{1-\phi}}}{\omega(x)^2} \\
& + \frac{\phi^3 \left(\frac{\phi}{1-\phi} - 1\right) \left(\frac{\phi}{\omega(x)}\right)^{\frac{\phi}{1-\phi}-2} \omega'(x)^2 (p_b - \mu x)^{\frac{1}{1-\phi}}}{\omega(x)^4} \\
& + \frac{2\mu\phi^2 \left(\frac{\phi}{\omega(x)}\right)^{\frac{\phi}{1-\phi}-1} \omega'(x) (p_b - \mu x)^{\frac{1}{1-\phi}-1}}{(1-\phi)\omega(x)^2} \\
& + \frac{2\phi^2 \left(\frac{\phi}{\omega(x)}\right)^{\frac{\phi}{1-\phi}-1} \omega'(x)^2 (p_b - \mu x)^{\frac{1}{1-\phi}}}{\omega(x)^3} + \mu^2 \left(\frac{1}{1-\phi} - 1\right) \left(\frac{\phi}{\omega(x)}\right)^{\frac{\phi}{1-\phi}} (p_b - \mu x)^{\frac{1}{1-\phi}-2}
\end{aligned}$$

$$\Longleftrightarrow$$

$$\begin{aligned}
\frac{\partial^2 \Phi(x)}{\partial x^2} = & \frac{1}{(\phi - 1)\omega(x)^2} \\
& [-\phi \left(\frac{\phi}{\omega(x)}\right)^{\frac{\phi}{1-\phi}} (p_b - \mu x)^{\frac{1}{1-\phi}-2} \left(\omega'(x)^2 (p_b - \mu x)^2\right. \\
& \left. + \omega(x) (\mu x - p_b) \left((\phi - 1)\omega''(x) (\mu x - p_b) - 2\mu\omega'(x)\right) + \mu^2 \omega(x)^2\right] > 0
\end{aligned}$$

D Existence conditions of boundaries

D.1 Existence of x_0

At equilibrium we have:

$$\begin{aligned}
\Psi(0) =& \quad (\omega(0) - t(0))^{1/\beta} \quad \text{and} \quad \Phi(0) = \left(\frac{\phi}{\omega(0)}\right)^{\frac{\phi}{1-\phi}} (1-\phi)\left(p_b - \mu(0)\right)^{\frac{1}{1-\phi}} \\
\Phi(0) >& \quad \Psi(0) \Leftrightarrow \left(\frac{\phi}{\omega}\right)^{\frac{\phi}{1-\phi}} (1-\phi) p_b^{\frac{1}{1-\phi}} > \omega^{1/\beta} \\
&\Leftrightarrow \quad \phi^{\frac{\phi}{1-\phi}} (1-\phi) p_b^{\frac{1}{1-\phi}} > \omega_0^{1/\beta} \omega_0^{\frac{\phi}{1-\phi}} \\
&\Leftrightarrow \phi^{\frac{\phi}{1-\phi}} (1-\phi) p_b^{\frac{1}{1-\phi}} > \omega_0^{1/\beta + \frac{\phi}{1-\phi}} \\
&\Leftrightarrow \quad \phi^{\frac{\phi}{1-\phi}} (1-\phi) p_b^{\frac{1}{1-\phi}} > \omega_0^{\frac{1-\phi(1-\beta)}{\beta(1-\phi)}} \\
&\Leftrightarrow \quad \left[\phi^{\frac{\phi}{1-\phi}} (1-\phi) p_b^{\frac{1}{1-\phi}}\right]^{\frac{\beta(1-\phi)}{1-\phi(1-\beta)}} > \omega_0 \\
&\Leftrightarrow \quad \boxed{\omega_0 < \phi^{\frac{\phi\beta}{1-\phi(1-\beta)}} (1-\phi)^{\frac{\beta(1-\phi)}{1-\phi(1-\beta)}} p_b^{\frac{\beta}{(1-\phi)[1-\phi(1-\beta)]}}}
\end{aligned}$$

We also have:

$$\begin{aligned}
\Psi(x) =& 0 \;\Leftrightarrow\; \left(\omega(x_j) - t\left|x - x_j\right|\right)^{1/\beta} = 0 \\
&\Leftrightarrow\; \omega_0 - tx = 0 \\
&\Leftrightarrow\; \boxed{x = \frac{\omega_0}{t}} \\
\Phi(x) =& 0 \;\Leftrightarrow\; \left(\frac{\phi}{\omega_0}\right)^{\frac{\phi}{1-\phi}} (1-\phi)\left(p_b - \mu x\right)^{\frac{1}{1-\phi}} \\
&\Leftrightarrow\; \boxed{x = \frac{p_b}{\mu}}
\end{aligned}$$

where $\omega(0) \equiv \omega_0$.

D.2 Existence of x_2

The location of rural-periurban fringe is given by:

$$\Theta(x_2) = R_A$$

that is, $(p - \tau x_2)^{\frac{1}{1-\delta}} A \left(\frac{\delta A}{p_k}\right)^{\frac{\delta}{1-\delta}} (1-\delta) = R_A$

$(p - \tau x_2)^{\frac{1}{1-\delta}} = \frac{R_A}{A\left(\frac{\delta A}{p_k}\right)^{\frac{\delta}{1-\delta}}(1-\delta)}$

$$p - \tau x_2 = \left[\frac{R_A}{A\left(\frac{\delta A}{p_k}\right)^{\frac{\delta}{1-\delta}}(1-\delta)} \right]^{1-\delta}$$

$$p - \tau x_2 = R_A^{1-\delta} \frac{1}{A^{1-\delta} A^{\delta} \left(\frac{\delta}{p_k}\right)^{\delta} (1-\delta)^{1-\delta}}$$

$$p - \tau x_2 = R_A^{1-\delta} A (1-\delta)^{-(1-\delta)} \left(\frac{\delta}{p_k}\right)^{-\delta}$$

$$\boxed{x_2 = \frac{1}{\tau}\left[p - R_A^{1-\delta} A (1-\delta)^{\delta-1} \left(\frac{\delta}{p_k}\right)^{-\delta} \right]}$$

where x_2 represents the boundary of periurban farmlands.

E Proof of Proposition 2

Let us identify the conditions on our parameters which allow the emergence of a periurban area. Such a configuration occurs when $\Psi_p \geq \Theta(x)$, with $x \in [x_{1i}, x_{2i}]$.

$$\begin{aligned}
&\Psi_p(x) - \Theta(x) \geq 0 \\
&\Psi_u(x) a(x)^{\beta/\gamma} \geq \Theta(x) \\
&\bar{a_p} \geq \left(\frac{\Theta(x)}{\Psi_u(x)}\right)^{\gamma/\beta} + \varphi k^*(x)
\end{aligned}$$

There is a minimum agricultural amenity level $\bar{a}_{p_{\min}}$ for a periurban area to emerge. It is given by

$$\bar{a}_{p_{\min}} = \arg\min_x \left(\frac{\Theta(x)}{\Psi_u(x)}\right)^{\gamma/\beta} + \varphi k^*(x)$$