Knowledge Sharing and Competition in Bio-tech Consortium

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Abstract

This article deals with a current research project, namely the Amaizing project. The goal of this research project is to create an innovation process in biotechnology. In this aim, firms must share their knowledge. However, private firms are not only in a cooperative framework but also in competition on the market and knowledge sharing increases spillover effects. Indeed, if a firm shares its knowledge it takes the risk that their competitors will develop a new variety that could compete strongly its own product. Therefore, there is a tradeoff between the value of the innovation obtained by the research project and the intensity of competition. Furthermore, a wide heterogeneity between firms in this sector exists. Indeed, some of them compete internationally whereas others only locally. As a consequence their expectations and their actions could be different. A theoretical model is developed where firms, in a first stage, choose whether they share their knowledge and, in a second stage, compete in a Cournot duopoly. The model shows that a high competition between firms decreases the knowledge sharing and a firm with a technological advantage keeps knowledge sharing with higher competition than the other firm except if the firm lags behind can rectify her lag.

Key words: Knowledge sharing, R&D cooperation, Horizontal Differentiation,

Spillovers

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1 Introduction

Innovation is crucial for many firms to create new products and preserve their market share, but it is also expensive and uncertain. To minimize risks and costs associated with innovation, firms can cooperate during the R&D stage. The advantages of cooperation are abundant, particularly firms in cooperation are able to share R&D cost, knowledge and cooperation can lead to commercializing inventions. Moreover spillover increases with cooperative behavior (Branstetter and Sakakibara, 2002; Gomes-Casseres et al., 2006). As a consequence, a rise of R&D cooperation appears during 90s and they stay currently widespread (Hagedoorn, 2002; Letterie et al., 2008; Tomasello et al., 2016). Furthermore, public authorities promote also public and private cooperation in R&D. Indeed, private firms are able to turn public laboratory and university discoveries into commercializing applications.

In the biotechnology sector, plant variety creation plays a key role. Indeed, it provides a considerable contribution to the agricultural sector by improving yields, reducing losses coming from pests and diseases, reducing consumption of water and chemical products and also meeting the food industrial expectations in terms of characteristics provided by crops (e.g. the quality of the flour from different variety of wheat). In this paper, a particular and current research project is analyzed, namely the Amaizing Project. This project incorporates public laboratories and eight private companies in biotechnology, essentially firms specialized in plant variety innovation. The research project has a main goal: provide a process innovation useful for private companies. This innovation will allow private firms to create new varieties of maize that will resist to cold temperature and consume less water and nitrogen. In this aim, private firms should not just share their R&D cost but also their genetic resources and the result of different experimentations. Indeed, the more private firms share their knowledge, the more the innovation provided by the research project will be significant. In fact, this project need the largest number of genetic resources to be efficient. In France, similar consortium exist such as Breedwheat, Rapsodyn and Peamust where knowledge plays also a key role. Internationally, the non-profit organization International Service for the Acquisition of Agri-Biotech Application (ISAAA) creates public/private partnership with knowledge sharing to develop agriculture in developing countries. More generally, knowledge sharing is crucial in many collaboration in high-tech industries (Samaddar and Kadiyala, 2004).

However, private firms are rarely only in a cooperative framework but also in competition on the market and knowledge sharing increases spillover effects. Indeed, if a firm shares its knowledge it takes the risk that their competitors will develop a new variety that could compete strongly its own product. Therefore, there is a tradeoff between the value of the innovation obtained by the consortium and the intensity of competition. Furthermore, a wide heterogeneity between firms in this sector exists. Indeed, some of them compete internationally whereas others only locally. As a consequence their expectations and their actions could be different. Transnational companies require less this research project than local firms. Actually, local firms have more difficulties to invest in biotechnology than transnational companies due to the high cost of R&D. In fact, firms in bio-tech spend more than ten percent of their revenues in R&D. Therefore, a firm with a higher market share has a great advantage to conduct R&D compared to her rivals. In parallel, if an international company obtains technologies of a small company, there is a risk that she could monopolize the whole market share of the small company. Consequently, small companies could also hesitate to share their knowledge. Thus, two problematics emerge of this consortium:

- Does the risk of a higher competition deteriorate the knowledge sharing that is crucial for R&D collaboration?
- What is the consequences when a firm is more advanced in terms of technologies?

Our research questions are linked to the literature on R&D cooperation and knowledge sharing. Many authors, study these problematics starting with Katz (1986) and d'Aspremont and Jacquemin (1988). They modeled R&D cooperation in a Cournot framework where in a first stage, firms cooperate by maximizing the joint profit for the choice of R&D and, in a second stage, firms compete by maximizing their own profit for the commercialization of their products. The spillover plays a role by lowering the production cost. D'Aspremont and Jacquemin (1988) find that, with few firms and high spillovers, R&D cooperation is interesting for firms. Katz (1986) shows that high competition decreases effective R&D whereas strong spillover increases effective R&D. Moreover, when the R&D plays a role on the output, cooperation could reduces incentives to conduct R&D because it helps its rivals. Different authors improved this fields by introducing asymmetry, dynamics, absorptive capacities and knowledge (Katz and Ordover, 1990; Kamien et al., 1992; Motta, 1992; Suzumura, 1992; Ziss, 1994; Salant and Shaffer, 1998; Petit and Tolwinski, 1998, Cellini and Lambertini, 2009; among others). In this literature an interesting paper for the problematics highlight above is Sakakibara (2003). He integrates in the model of d'Asprement and Jacquemin (1988), beside the R&D effort, complementary knowledge as Cohen and Levinthal (1989). Before the maximization of the profit by the R&D effort and quantities, competitors has to choose the level of the knowledge sharing ratio. The model suggests that cooperation when rivals control complementary knowledge increases the endogenous spillover ratio and R&D efforts. In all this fields, spillover does not play a direct role on the degree of competition what is crucial for questions presented above.

An original model, initially developed by Samaddar and Kadiyala (2004) and modified by Ding and Huang (2010), takes into account the R&D effort and the knowledge sharing. They also made a difference between current research creation and prior knowledge. Unlike the previous articles, competitors are in a Stackelberg framework, with a leader and a follower, for the decisions of the R&D effort and the knowledge sharing. However, in this theoretical model, the stage of competition does not exist. The main result of these two articles is that leader would participate only if it marginal gain is high enough comparing to the elasticity of current research creation.

Network theory is an other field working on cooperative R&D. Goyal and Gonzales (2001) and Goyal and Joshi (2003) build a theoretical model with a Cournot competition where R&D and spillovers reduce the unit cost as D'Aspremont and Jacquemin (1988). They highlight that, in a Cournot competition with homogeneous goods, R&D effort is negatively influenced by cooperation. In this literature, empirical works try, in particular, to test whether similarity between firms increases cooperative behavior. Cantner and Graf (2006), Hanaki et al. (2010) and Tomasello et al. (2016) use patent statistics to test the effect of similarity. Cantner and Graff (2006) employ citation data and found a positive effect. Tomasello et al. (2016) obtain also a positive impact but they use categories of patent (IPC classification) instead of citations. Conversely, Hanaki et al. (2010) find a negative effect with subcategories of patents. This different result can be from an issue with the variable of similarity in these articles. Indeed, two firms can have many similarities and then obtain patents in the same category. Simultaneously, these two firms may produce complementary outputs and would not compete in the same market. For example, in bio-tech sector, a firm can innovate on the genome and sell directly this innovation to the other firm that will create a new variety of crop. Therefore, the incentives to cooperate would be important. Nevertheless, these two firms could produce substitutes products and then compete in the same market. In this situation, firms have probably less incentives to cooperate because higher spillovers involve by cooperation could increase competition.

To address the highlighted problematics, a theoretical model is developed where two firms work together in a R&D cooperation. In the first stage they have to choose whether they share their knowledge. In the second stage, they compete in a Cournot duopoly. To represent the intensity of competition between these two firms, plant varieties are not perfectly homogeneous. The innovation provided by the research project allows bio-tech firms to create a new variety that requests less water and less chemical products to grow up. Therefore, the cost for farmers is decreasing and, as a consequence, their willingness to pay for these varieties rise. Thus we assume that the innovation increases the reservation price. The level of the innovation depends on

the knowledge sharing. The highest level of innovation occurs when both firms share their knowledge. If only one firm shares her knowledge, the level of the innovation is lower and the lowest level arises when both firms do not share their knowledge. At the same time, we assume that spillovers, implied by sharing knowledge, involve two effects. In fact, sharing knowledge brings the risks that competitor can include one technology of the other firm in its products. As a consequence, there is more similarity between both products and then more competition. Simultaneously, this technology can be interesting for farmers, then it increases the value of plant variety. Therefore, spillovers involve a positive impact by increasing the reservation price and a negative impact by raising the competition. If only one firm shares her knowledge, this firm will face more competition without an higher reservation price because she will not obtain the competitor technology whereas its competitor will access to a greater reservation price¹. Finally, we introduce heterogeneity between firms in the reservation price, indeed a high technology can give more value to the product, and in the positive impact provided by spillovers. In fact, a firm lags behind in the development of new technologies can acquire more technologies from knowledge sharing than a firm more advanced.

The theoretical model shows interesting results. The more the competition is, the less competitors share their knowledge. In fact, if two firms are in high competition there will not be enough incentives for them to share their knowledge than two firms nearly monopolist. An example of this result on bio-tech firms specialized in plant variety creation is Limagrain and KWS. These two firms cooperate more intensively

¹To give an other explanation of our assumption an example can be made where two varieties have different characteristics. Plant variety 1 has characteristics a, b and c whereas plant variety two has c, d and e. If both firms share their knowledge, they can grab a characteristic of the plant variety of the competitor to insert it into their own plant variety. Therefore, after the research project, the plant variety 1 resist to pest a, b, c and d and the plant variety 2 resist to pest b, c, d and e. The two plant varieties becomes more homogeneous than before, as a consequence, the competition becomes stronger because more farmers are indifferent between plant varieties one and two. Simultaneously, plant variety one and two has more characteristics, then more farmers can be interested by these varieties at a higher value. If only one firm shares its knowledge, the plant variety of the competitor will capture one more characteristic (eg. a, b, c and d vs. c, d and e). Therefore, the competition will increase for both varieties but at a lower level and only the firm that do not share its knowledge will obtain an higher reservation price.

in the North American market where they were not in competition with the creation of a joint venture (AgReliant). Furthermore, we found that a advanced firm keeps knowledge sharing with higher competition than a firm lags behind. Moreover, the positive and the negative externality have negative impacts on the knowledge sharing because they incites firms to let only competitors share their knowledge. Lastly, if the positive externality is sufficiently higher, the firm lags behind can have more incentives to share her knowledge than the advanced firms since the firm lags behind can rectify her lags and compete stronger the advanced firm.

The paper is organized as follows. In section 2 the model is presented. Section 3 shows the results of the model when firms are similar. Section 4 explores the impacts of heterogeneous firms. Section 5 concludes.

2 The Model

We consider a two-step model with two types of agent: two mono-product bio-tech firms and risk neutral farmers. In the first step both firms choose whether they share their knowledge. The knowledge sharing provides useful resources that make it possible to create an interesting innovation. This innovation represent the collective benefit provided by the knowledge sharing inside the research collaboration. If both firms share their knowledge, the value of the innovation is given by the parameter ω . The innovation parameter increases the demand through the willingness to pay of farmers. If only one firm shares her knowledge, then the value of the innovation will be $\omega_2 = \frac{\omega}{2}$.² When not any of firms decide to share their knowledge the value of the innovation is normalized to zero such that $\omega_3 = 0$. The innovation, ω , will have similar impacts in the following theoretical model than the model of d'Aspremont and Jacquemin (1988). Indeed, the effect of an innovation reducing the unit cost or increasing the reservation price has the same consequences (Spence, 1984). Furthermore, spillovers provided by knowledge sharing involves two externalities: a negative one increasing the competition (for firms), δ , and a positive

²A later discussion will be held in the paper on the impact of an higher or a lower value of ω_2 .

one increasing the reservation price, λ_i . Compare to d'Aspremont and Jacquemin (1988) spillovers do not affect the unit cost of production. Here, high spillovers are the consequence of sharing a part of the prior knowledge³. Indeed, for competitors, sharing prior knowledge could increase the degree of competition between firms for the reason that a competitor could use this knowledge to compete stronger the sharing firms in her demand segment. In the meantime, the technology brought by the competitor can be used by a firm to improve the quality of her product, this represent an individual benefit from knowledge sharing. Moreover, the individual benefit can be different since firms before the collaboration can have different level of technology. In fact, the knowledge sharing, within the collaboration, could bring more new technologies for firms lags behind in the development of these technologies.

After the choice of sharing knowledge, firms will compete in a Cournot duopoly with differentiated goods. To represent this heterogeneity, we assume that the goods are not perfect substitutes, the level of substitution between both goods is γ . We define a quadratic utility function for farmers as Singh and Vives (1984)

$$U = \alpha_1 q_1 + \alpha_2 q_2 - (\beta_1 q_1^2 + \beta_2 q_2^2 + 2\gamma q_1 q_2)/2, \tag{1}$$

which leads to a linear demand system

$$p_1 = \alpha_1 - \beta_1 q_1 - \gamma q_2,$$

$$p_2 = \alpha_2 - \gamma q_1 - \beta_2 q_2$$
(2)

The farmers surplus will be $CS = U - p_1 * q_1 - p_2 * q_2$, the firm profit $\Pi_i = p_i * q_i - c_i * q_i$ and the welfare is the addition of the consumer surplus and the firm profits $W = CS + \sum_{i=1}^{2} \Pi_i$. To obtain positive demand and to the problem makes sense we assume, as Singh and Vives, that $\alpha_i > 0$, $\beta_i > 0$, $\beta_1\beta_2 - \gamma^2 > 0$ and $\alpha_1\beta_2 - \alpha_2\gamma > 0$. Notice that with $\gamma > 0$, goods are substitutes but if $\gamma = 0$ then

³As explained by Samaddar and Kadiyala (2004) there is a difference between prior knowledge and current knowledge. Current knowledge is provided by the R&D cooperation (the innovation) whereas prior knowledge is the stock of knowledge provided by firms (a knowledge created before the collaboration). Samaddar and Kadiyala (2004) show examples of R&D cooperation becoming useless due to the lack of cooperation in the sharing of prior knowledge.

both firms become monopolist. Moreover, if $\gamma < 0$ goods turn into complementary and if $\gamma = \beta_1 = \beta_2$ they would be perfect substitutes.

In order to obtain the most proper resolution possible, we made some simplification. Elasticity parameters, β_i , are normalized to unity⁴. Consequently, equilibrium quantity and price will be equal and the constraint on γ becomes $\gamma \in]0, 1[$ (non perfect substitutes). Moreover, a second difference between firms is the reservation price parameter α_i . In fact, one firm has an advantage in terms of technologies before the collaboration, as a consequence the reservation price is higher for the product offer by this firm. Unit costs, c_i , are normalized to zero. In fact, in this sector (plant variety creation), each firm has a similar unit cost because they all have to comply with the regulation (commercial rules, norms,...) on the specificity of each new and old products. Lastly, we assume that the fixed cost to develop the innovation in the cooperation is equally shared between partners and is normalized to zero⁵. Thus, the return on investment is always positive whatever the level of the R&D fixed cost. As a consequence, the welfare will be simply equal to the farmers utility (W = U).

The model is resolved by backward induction. Thus, we firstly resolved the Cournot competition. To be done, parameters involved by the sharing knowledge activity have to be added to the farmers utility and inverse demand. Actually, four situations are possibles

- case A: both firms share their knowledge,
- case B: only firm 2 shares her knowledge,
- case C: only firm 1 shares her knowledge,
- case D: no one shares its knowledge.

In case A, the innovation, ω , and the positive externality, λ_i , provided by the sharing knowledge increase the market size by raising the willingness to pay of

⁴This simplification will not have an impact on the further main results of the paper.

⁵In the Amaizing consortium, private companies finance it to similar amounts.

farmers interested by this product. Moreover, the negative externality, δ , increases the degree of competition represented by the degree of substitutability. Therefore, farmers utility and inverse demands are

$$U^{A} = (\alpha_{1} + \omega + \lambda_{1})q_{1} + (\alpha_{2} + \omega + \lambda_{2})q_{2} - (q_{1}^{2} + q_{2}^{2} + 2(\gamma + \delta)q_{1}q_{2})/2,$$

$$p_{1}^{A} = \alpha_{1} + \omega + \lambda_{1} - q_{1} - (\gamma + \delta)q_{2},$$

$$p_{2}^{A} = \alpha_{2} + \omega + \lambda_{2} - (\gamma + \delta)q_{1} - q_{2},$$

(3)

where $(\gamma + \delta) \in]0, 1[$. In case *B* only the firm two shares her knowledge. Thus, the consortium obtains a lower innovation, $\frac{\omega}{2}$, than in case *A*. Moreover, the positive externality plays only a role on the demand of the firm one because firm two are not able to use the knowledge of firm one to increase her reservation price. The negative externality continues to exist for both firms but at a lower level, we assume that is divided by two $(\frac{\delta}{2})$ compared to case *A*. Indeed, even if only firm two gives her knowledge, the larger substitutability between products plays for both firms. Thus, farmers utility and inverse demands become

$$U^{B} = (\alpha_{1} + \frac{\omega}{2} + \lambda_{1})q_{1} + (\alpha_{2} + \frac{\omega}{2})q_{2} - (q_{1}^{2} + q_{2}^{2} + 2(\gamma + \frac{\delta}{2})q_{1}q_{2})/2,$$

$$p_{1}^{B} = \alpha_{1} + \frac{\omega}{2} + \lambda_{1} - q_{1} - (\gamma + \frac{\delta}{2})q_{2},$$

$$p_{2}^{B} = \alpha_{2} + \frac{\omega}{2} - (\gamma + \frac{\delta}{2})q_{1} - q_{2},$$
(4)

where $(\gamma + \frac{\delta}{2}) \in]0, 1[$. The case *C* is symmetric to the case *B*. Farmers utility and inverse demands are given by

$$U^{C} = (\alpha_{1} + \frac{\omega}{2})q_{1} + (\alpha_{2} + \frac{\omega}{2} + \lambda_{2})q_{2} - (q_{1}^{2} + q_{2}^{2} + 2(\gamma + \frac{\delta}{2})q_{1}q_{2})/2,$$

$$p_{1}^{C} = \alpha_{1} + \frac{\omega}{2} - q_{1} - (\gamma + \frac{\delta}{2})q_{2},$$
(5)

$$p_{2}^{C} = \alpha_{2} + \frac{\omega}{2} + \lambda_{2} - (\gamma + \frac{\delta}{2})q_{1} - q_{2}.$$

In Case D, there is no longer knowledge sharing. Therefore, all parameters involved by the knowledge sharing are equal to zero. We can write farmers utility and inverse demands as

$$U^{D} = \alpha_{1}q_{1} + \alpha_{2}q_{2} - (q_{1}^{2} + q_{2}^{2} + 2\gamma q_{1}q_{2})/2,$$

$$p_{1}^{D} = \alpha_{1} - q_{1} - \gamma q_{2},$$

$$p_{2}^{D} = \alpha_{2} - \gamma q_{1} - q_{2}.$$
(6)

In the first step, firms have a perfect information on all equilibrium prices, quantities and profits for all cases. The non cooperative Nash game of knowledge sharing is reported in Table 1. Since firms are rational, perfectly informed and profit maximizing, they choose whether they share their knowledge according to the level of their profits.

Table 1 – Knowledge sharing game

		Firm 1					
		shares	does not share				
		knowledge	knowledge				
	shares	Π_1^A	Π_1^B				
Firm	knowledge	Π_2^A	Π_2^B				
2	does not share	Π_1^C	Π_1^D				
	knowledge	Π_2^C	Π_2^D				

The Table 2 summarizes all the parameters of the model that will be used in the following sections.

Table 2 – Parameters

q_i^j	Quantity of firm i in case j
p_i^j	Price of firm i in case j
α_i	Reservation price
ω	The collective benefit: innovation
γ	Degree of substitutability/competition
δ	Increases competition: negative externality
λ_i	The individual benefit: positive externality

3 Homogeneous Firms

This section explores the impact of the innovation and spillovers involved by the knowledge sharing of two symmetric firms $\alpha_1 = \alpha_2 = \alpha$ and with the same positive externality, $\lambda_1 = \lambda_2 = \lambda$. Results provided by the Cournot competition in step two are displayed in Table 3.

In the Case A, where both firms share their knowledge, the innovation and the positive externality increase equilibrium quantities, prices, profits, farmers surplus and the welfare. On the contrary, the negative externality and the degree of competition have negative impacts.

Since firms one and two are symmetric, results of the Cournot competition are also symmetric for quantities and profit in the case B and C. Parameters keep the same influence except for the positive externality. Indeed, equilibrium quantity and price and the profit of the firm that share her knowledge decline with respect to the positive externality $\left(\frac{\partial q_2^B}{\partial \lambda} = \frac{1}{4+2\gamma+\delta} - \frac{1}{4-2\gamma-\delta}\right)$. However, the positive externality maintains a positive impact on the farmers surplus and the welfare. Moreover, its positive influence on the profits of the firm that does not share her knowledge is greater than in the case A $\left(\frac{\partial q_1^B}{\partial \lambda} > \frac{\partial q_1^A}{\partial \lambda}\right)$. In other words, the rise of the profit in the case B for the firm 1 is sharper than in the case A in terms of λ .

In order to solve the first step of the model, threshold values have to be found. These thresholds will indicate when a firm want to share her knowledge whether the other firm share also her knowledge. Knowing that firms are symmetric, thresholds are similar for firm 1 and 2. Therefore, the first thresholds is for $\Pi_1^A > \Pi_1^B$ and $\Pi_2^A > \Pi_2^C$ such that firms want to share their knowledge when the other shares it

$$\lambda < \frac{(4 - 2\gamma - \delta)((2 + \gamma)\omega - \alpha\delta)}{4\gamma(2 + \gamma) + 4(2 + \gamma)\delta + \delta^2},\tag{7}$$

where the right hand side of this inequality is denoted \overline{L}_1 . The second threshold is for $\Pi_1^C > \Pi_1^D$ and $\Pi_2^B > \Pi_2^D$ such that firms want to share their knowledge when

Table 3 – Payoffs for similar firms

q_1^A	$rac{lpha+\omega+\lambda}{2+\gamma+\delta}$	q_1^B	$rac{2lpha+\lambda+\omega}{4+2\gamma+\delta}+rac{\lambda}{4-2\gamma-\delta}$
q_2^A	$rac{lpha+\dot{\omega}+\lambda}{2+\gamma+\delta}$	q_2^B	$rac{2lpha+\lambda+\omega}{4+2\gamma+\delta}-rac{\lambda}{4-2\gamma-\delta}$
Π_1^A	$\frac{(\alpha + \omega + \lambda)^2}{(2 + \gamma + \delta)^2}$	Π_1^B	$\frac{(2\alpha(4-2\gamma-\delta)+\omega(4-2\gamma-\delta)+8\lambda)^2}{(4-2\gamma-\delta)^2(4+2\gamma+\delta)^2}$
Π_2^A	$\frac{(lpha+eta+\lambda)^2}{(2+\gamma+\delta)^2}$	Π_2^B	$\frac{(2\alpha(4-2\gamma-\delta)-(2\gamma+\delta)(2\lambda+\omega)+4\omega)^2}{(4-2\gamma-\delta)^2(4+2\gamma+\delta)^2}$
CS^A	$rac{(lpha+\omega+\lambda)^2(1+\gamma+\delta)}{(2+\gamma+\delta)^2}$	CS^B	$\frac{1}{2} \left(\frac{(2\alpha + \lambda + \omega)^2}{4 + 2\gamma + \delta} - \frac{2(2\alpha + \lambda + \omega)^2}{(4 + 2\gamma + \delta)^2} + \frac{\lambda^2}{4 - 2\gamma - \delta} + \frac{2\lambda^2}{(4 - 2\gamma - \delta)^2} \right)$
W^A	$\frac{(\alpha + \omega + \lambda)^2 (3 + \gamma + \delta)}{(2 + \gamma + \delta)^2}$	W^B	$\frac{(2\alpha+\lambda+\omega)^2}{2(4+2\gamma+\delta)} + \frac{(2\alpha+\lambda+\omega)^2}{(4+2\gamma+\delta)^2} + \frac{\lambda^2}{8-4\gamma-2\delta} - \frac{\lambda^2}{(4-2\gamma-\delta)^2}$
q_1^C	$rac{2lpha+\lambda+\omega}{4+2\gamma+\delta}-rac{\lambda}{4-2\gamma-\delta}$	q_1^D	$\frac{\alpha}{2+\gamma}$
q_2^C	$\frac{2\alpha+\lambda+\omega}{4+2\gamma+\delta}+\frac{\lambda}{4-2\gamma-\delta}$	q_2^D	$\frac{\alpha}{2+\gamma}$
Π_1^C	$\frac{(2\alpha(4-2\gamma-\delta)-(2\gamma+\delta)(2\lambda+\omega)+4\omega)^2}{(4-2\gamma-\delta)^2(4+2\gamma+\delta)^2}$	Π_1^D	$\frac{\alpha^2}{(2+\gamma)^2}$
Π_2^C	$\frac{(2\alpha(4-2\gamma-\delta)+\omega(4-2\gamma-\delta)+8\lambda)^2}{(4-2\gamma-\delta)^2(4+2\gamma+\delta)^2}$	Π^D_2	$\frac{\alpha^2}{(2+\gamma)^2}$
CS^C	$\frac{1}{2} \left(\frac{(2\alpha + \lambda + \omega)^2}{4 + 2\gamma + \delta} - \frac{2(2\alpha + \lambda + \omega)^2}{(4 + 2\gamma + \delta)^2} + \frac{\lambda^2}{4 - 2\gamma - \delta} + \frac{2\lambda^2}{(4 - 2\gamma - \delta)^2} \right)$	CS^D	$\frac{\alpha^2(1+\gamma)}{(2+\gamma)^2}$
W^C	$\frac{(2\alpha+\lambda+\omega)^2}{2(4+2\gamma+\delta)} + \frac{(2\alpha+\lambda+\omega)^2}{(4+2\gamma+\delta)^2} + \frac{\lambda^2}{8-4\gamma-2\delta} - \frac{\lambda^2}{(4-2\gamma-\delta)^2}$	W^D	$\frac{\alpha^2(3+\gamma)}{(2+\gamma)^2}$

the other do not

$$\lambda < \frac{(4 - 2\gamma - \delta)((2 + \gamma)\omega - \alpha\delta)}{2(2 + \gamma)(2\gamma + \delta)}$$
(8)

where the right hand side of this inequality is denoted \overline{L}_2 . \overline{L}_1 and \overline{L}_2 are positive for $(2+\gamma)\omega - \alpha\delta > 0$. In the following, we assume that the innovation is high enough such that $\omega > \frac{\alpha\delta}{(2+\gamma)}$. For the constraint values on γ and δ fixed at $(\gamma + \delta) \in]0, 1[$, \overline{L}_1 is lower than \overline{L}_2 . This last result leads to the following Proposition

Proposition 1

For λ < L
₁

Both firms share their knowledge
For L
₁ < λ < L
₂

There is a multiple equilibrium: one firm share her knowledge

- For $\lambda > \overline{L}_2$

Both firms do not share their knowledge

Proof 1

- For $\lambda < \overline{L}_1$: $\Pi_1^A > \Pi_1^B$, $\Pi_2^A > \Pi_2^C$, $\Pi_1^C > \Pi_1^D$ and $\Pi_2^B > \Pi_2^D$
- For $\overline{L}_1 < \lambda < \overline{L}_2$: $\Pi_1^A < \Pi_1^B$, $\Pi_2^A < \Pi_2^C$, $\Pi_1^C > \Pi_1^D$ and $\Pi_2^B > \Pi_2^D$
- For $\lambda > \overline{L}_2$: $\Pi_1^A < \Pi_1^B$, $\Pi_2^A < \Pi_2^C$, $\Pi_1^C < \Pi_1^D$ and $\Pi_2^B < \Pi_2^D$

The Proposition 1 shows that the more λ is high, the less likely firms will share their knowledge. The effect of other parameters on the threshold values, \overline{L}_1 and \overline{L}_2 , conducts to the Proposition 2

Proposition 2

- The innovation ω has a positive impact on the sharing knowledge.
- The positive externality λ, the negative externality δ, the degree of competition
 γ and the market size parameter α have a negative impact on the sharing
 knowledge.

Proof 2 Partial derivatives of \overline{L}_1 and \overline{L}_2 with respect to δ , γ and α (ω) are negatives (positives). Thus, a rise of δ , γ and α (ω) decrease (increase) the space of parameter values where firms choose to share their knowledge.

As a consequence, the more the competition is strong between two firms, the less they will share their knowledge. Moreover, if the knowledge is heavily strategic and involves a risk that the degree of competition could be increased (δ), then firms are discouraged to share their knowledge. Surprisingly, the positive externality is also an obstacle for the knowledge sharing. In fact, it incites firms to let only the competitor sharing his knowledge. If the positive externality involves by the sharing knowledge is important, then the competitor prefers also to deny knowledge sharing because only the other firm will benefit from it. The negative impact of the market size parameter is specific. In fact, if the value of an innovation is very low compare to the value of the market, therefore it is not interesting to risk a higher competition to obtain it.

We made the assumption that the innovation in case B and C is divided by two compared to the case A. Proposition 2 will not change with a lower ($\omega_2 < \omega/2$) or a higher ($\omega_2 > \omega/2$) innovation but other equilibrium can appear. In fact, when $\omega_2 < \omega/2$, multiple equilibrium where both share their knowledge or both do not are existing. And, when $\omega_2 > \omega/2$, the space of multiple equilibrium where one of them share his knowledge can be higher.

4 Heterogeneous firms

This section analyzes the fact that firms can have different reservation prices such as $\alpha_1 \neq \alpha_2$ but keep the same positive externality, $\lambda_1 = \lambda_2 = \lambda$. The consequences of innovation and spillovers involved by the knowledge sharing are discussed. Results provided by the Cournot competition in step two are shown in Table 4. Profits are not displayed but they are simply the square of the equilibrium quantities (due to $\beta = 1$).

The effect of parameters on payoffs when firms are heterogeneous is very similar to the previous section. In fact, in the case A, the only difference is the change between α_1 and α_2 . The reservation price of the competitor involves a negative impact depending on the level of competition for the profit of the firm. However, both reservation price parameters play positively on the farmer surplus and the welfare.

In the case B and C, the difference between both firms has an interesting impact. Indeed, the competitor's reservation price parameter continues to play negatively on profit but the firm with the higher reservation price will be less impacted when she shares her knowledge alone whereas its competitor will be more affected if he is the only one to share his knowledge (negative impact on equilibrium quantities for the firm 1 is $-\frac{\alpha_2 - \alpha_1 + \lambda}{4 - 2\gamma - \delta}$ and for the firm 2 $-\frac{\alpha_1 - \alpha_2 + \lambda}{4 - 2\gamma - \delta}$). This result can have a positive (negative) impact for the choice of knowledge sharing for the firm with a superior (inferior) reservation price.

q_1^A	$\frac{2\alpha_1 - \alpha_2(\gamma + \delta) + (2 - \gamma - \delta)(\omega + \lambda)}{(2 - \gamma - \delta)(2 + \gamma + \delta)}$
q_2^A	$rac{2lpha_2-lpha_1(\gamma+\delta)+(2-\gamma-\delta)(\omega+\lambda)}{(2-\gamma-\delta)(2+\gamma+\delta)}$
CS^A	$\frac{\alpha_1^2 \left(4 - 3(\gamma + \delta)^2\right) + 2\alpha_1 \alpha_2 (\gamma + \delta)^3 + \alpha_2^2 \left(4 - 3(\gamma + \delta)^2\right)}{2(2 - \gamma - \delta)^2 (\gamma + \delta + 2)^2} + \frac{2\alpha_1 ((\gamma + \delta + 1)(\omega + \lambda))}{2(\gamma + \delta + 2)^2} + \frac{2\alpha_2 (\gamma + \delta + 1)(\omega + \lambda)}{2(\gamma + \delta + 2)^2} + \frac{2(\gamma + \delta + 1)(\omega + \lambda)}{2(\gamma + \delta + 2)^2}$
W^A	$\frac{1}{4} \left(\frac{(\alpha_1 + \alpha_1 + 2(\omega + \lambda))^2}{2 + \gamma + \delta} + \frac{(\alpha_1 + \alpha_1 + 2(\omega + \lambda))^2}{(2 + \gamma + \delta)^2} + \frac{(\alpha_1 - \alpha_1)^2}{2 - \gamma - \delta} + \frac{(\alpha_1 - \alpha_1)^2}{(2 - \gamma - \delta)^2} \right)$
a_1^B	$\alpha_1 + \alpha_2 + \omega + \lambda + \alpha_1 - \alpha_2 + \lambda$
11 B	$4+2\gamma+\delta$ $4-2\gamma-\delta$
q_2^D	$\frac{\alpha_1 + \alpha_2 + \alpha_1 + \alpha_2}{4 + 2\gamma + \delta} = \frac{\alpha_1 - \alpha_2 + \alpha_1}{4 - 2\gamma - \delta}$
CS^B	$\frac{1}{2} \left(\frac{(\alpha_1 + \alpha_2 + \omega + \lambda)^2}{4 + 2\gamma + \delta} - \frac{2(\alpha_1 + \alpha_2 + \omega + \lambda)^2}{(4 + 2\gamma + \delta)^2} + \frac{(\alpha_1 - \alpha_2 + \lambda)^2}{4 - 2\gamma - \delta} - \frac{2(\alpha_1 - \alpha_2 + \lambda)^2}{(4 - 2\gamma - \delta)^2} \right)$
W^B	$\frac{(\alpha_1 + \alpha_2 + \omega + \lambda)^2}{(\alpha_1 + \alpha_2 + \omega + \lambda)^2} + \frac{(\alpha_1 - \alpha_2 + \lambda)^2}{(\alpha_1 - \alpha_2 + \lambda)^2} + \frac{(\alpha_1 - \alpha_2 + \lambda)^2}{(\alpha_1 - \alpha_2 + \lambda)^2}$
	$(4+2\gamma+\delta)^2 ' 2(4+2\gamma+\delta) ' (4+2\gamma+\delta)^2 ' 2(4+2\gamma+\delta)$
q_1^C	$\frac{\alpha_1 + \alpha_2 + \omega + \lambda}{1 + \omega_2 + \omega_1 + \lambda} = \frac{\alpha_2 - \alpha_1 + \lambda}{2 + \omega_2 + \omega_2 + \omega_2}$
C	$4+2\gamma+o$ $4-2\gamma-o$ $\alpha_1+\alpha_2+\omega+\lambda_{-+}$ $\alpha_2-\alpha_1+\lambda_2$
q_2°	$\frac{-1}{4+2\gamma+\delta} + \frac{-2}{4-2\gamma-\delta}$
CS^C	$\frac{1}{2} \left(\frac{(\alpha_1 + \alpha_2 + \omega + \lambda)^2}{4 + 2\gamma + \delta} - \frac{2(\alpha_1 + \alpha_2 + \omega + \lambda)^2}{(4 + 2\gamma + \delta)^2} + \frac{(\alpha_2 - \alpha_1 + \lambda)^2}{4 - 2\gamma - \delta} - \frac{2(\alpha_2 - \alpha_1 + \lambda)^2}{(4 - 2\gamma - \delta)^2} \right)$
TTC	$(\alpha_1 + \alpha_2 + \omega + \lambda)^2$ $(\alpha_1 + \alpha_2 + \omega + \lambda)^2$ $(\alpha_2 - \alpha_1 + \lambda)^2$ $(\alpha_2 - \alpha_1 + \lambda)^2$
W°	$\frac{\sqrt{4+2\gamma+\delta}^2}{(4+2\gamma+\delta)^2} + \frac{\sqrt{4+2\gamma+\delta}}{2(4+2\gamma+\delta)} + \frac{\sqrt{2}-4\gamma+\delta}{(4+2\gamma+\delta)^2} + \frac{\sqrt{2}-4\gamma+\delta}{2(4+2\gamma+\delta)}$
aD	$2\alpha_1 - \alpha_2\gamma$
q_1	$2q - \gamma^2$
q_2^D	$\frac{2u_2 - u_1\gamma}{4 - \gamma^2}$
aab	$4(\alpha_1^2+\alpha_2^2)+2\alpha_1\alpha_2\gamma^3-3\gamma^2(\alpha_1^2+\alpha_2^2)$
CS^{D}	$\frac{(1-\gamma^2)^2}{2(4-\gamma^2)^2}$
D	$\alpha_1^2(12-\gamma^2) - 2\alpha_1\alpha_2\gamma(8-\gamma^2) + \alpha_2^2(12-\gamma^2)$
W^D	$\frac{1}{2(4-\gamma^2)^2}$

Table 4 – Payoffs for heterogeneous firms with $\alpha_1 \neq \alpha_2$

Four thresholds are obtained to resolve the game of knowledge sharing. The first threshold, \overline{T}_1 , is for $\Pi_1^A > \Pi_1^B$. The second, \overline{T}_2 , is for $\Pi_1^C > \Pi_1^D$. The third, \overline{T}_3 , is for $\Pi_2^A > \Pi_2^C$ and the fourth, \overline{T}_4 , is for $\Pi_2^B > \Pi_2^D$. They are presented and analyzed in Annex A.1. We choose to isolate the parameter α_1 to evaluate the consequences of the heterogeneity of firm 1 and 2. Firm 1 (2) accept to share her knowledge for $\alpha_1 > \overline{T}_1$ ($\alpha_1 < \overline{T}_3$) when the other firm shares it, and, for $\alpha_1 > \overline{T}_2$ ($\alpha_1 < \overline{T}_4$) when the other firm refuses to share it. Moreover, Annex A.2 provides how are ordered each threshold depending on the value of λ . Since there is four thresholds, six equalities can be found between them. For each equality, a different value of λ is obtained and denoted \overline{L}_z , where $z \in [1, 6]$. Each \overline{L}_z is also described and sorted in Annex A.2. Results provided by Annex A.1 and A.2 are presented in Figure 1.

Furthermore, at \overline{L}_1 where $\overline{T}_1 = \overline{T}_3$, replacing \overline{L}_1 in \overline{T}_1 and \overline{T}_3 leads to the result $\overline{T}_1 = \overline{T}_3 = \alpha_2$. The same result is obtained at \overline{L}_2 for \overline{T}_2 and \overline{T}_4 . Therefore, for values of λ lower (higher) than \overline{L}_1 (\overline{L}_2), α_2 is higher (lower) than \overline{T}_1 and \overline{T}_2 and lower (higher) than \overline{T}_3 and \overline{T}_4 . For values of λ between \overline{L}_1 and \overline{L}_2 , α_2 is larger than

 \overline{T}_2 and \overline{T}_3 and inferior to \overline{T}_1 and \overline{T}_4 . These results allow to know whether α_1 is superior to α_2 according to the placement of α_1 between thresholds.

Figure 1 – Comparison of the thresholds

The upper part of the Figure 1 shows what thresholds is higher or lower than other thresholds between each \overline{L}_z . The bottom part shows where α_2 is superior or inferior to thresholds. For example, between zero and \overline{L}_1 we know from Figure 1

- $\overline{T}_4 > \overline{T}_3 > \overline{T}_1 > \overline{T}_2$, - $\alpha_2 > \{\overline{T}_1, \overline{T}_2\}$ and $\alpha_2 < \{\overline{T}_3, \overline{T}_4\}$

Therefore, for α_1 superior to \overline{T}_3 (same result for $\alpha_1 > \overline{T}_4$) we find

- $\alpha_1 > \alpha_2$,

- $\alpha_1 > \{\overline{T}_1, \overline{T}_2\}$, thus Firm 1 wants to share her knowledge,

- $\alpha_1 > \overline{T}_3$, thus Firm 2 declines the knowledge sharing.

For α_1 included between to \overline{T}_3 and \overline{T}_1 we obtain

- $\alpha_1 \ge \alpha_2$ or $\alpha_1 \le \alpha_2$ (low difference),

- $\alpha_1 > \{\overline{T}_1, \overline{T}_2\}$, thus Firm 1 wants to share her knowledge,

- $\alpha_1 < \{\overline{T}_3, \overline{T}_4\}$, thus Firm 2 accepts the knowledge sharing.

For α_1 inferior to \overline{T}_1 (same result for $\alpha_1 < \overline{T}_2$) we find

- $\alpha_1 < \alpha_2$,
- $\alpha_1 < \{\overline{T}_3, \overline{T}_4\}$, thus Firm 2 wants to share her knowledge,
- $\alpha_1 < \overline{T}_1$, thus Firm 1 declines the knowledge sharing.

The analysis of Nash equilibrium for all values of λ continues in Annex A.3 and

leads to Proposition 3.

Proposition 3

- If α_1 and α_2 are close enough
 - $For \lambda < \overline{L}_1$ Both firms share their knowledge
 - $For \overline{L}_1 < \lambda < \overline{L}_2$

There is a multiple equilibrium: one firm share her knowledge

- For $\lambda > \overline{L}_2$

Both firms do not share their knowledge

- If α_1 is high enough compare to α_2
 - Only the firm 1 will share her knowledge
- If α_1 is low enough compare to α_2
 - Only the firm 2 will share her knowledge

Proof 3 See Annex A.3.

Corollary 1 The firm with an higher potential market size, provided by a higher reservation price, has more incentives to share its knowledge than the other firm.

Proposition 3 shows that when firms are sufficiently similar, Nash equilibrium become the same that the situation where firms are exactly the same. In fact, \overline{L}_1 and \overline{L}_2 are the same except that the parameter α is replaced by the parameter α_2 . Therefore, with $\alpha_2 = \alpha$, then \overline{L}_1 and \overline{L}_2 are exactly equivalent to the previous section. The difference between the situation where firms were similar is that when a firm has a sufficiently large reservation price compared to her competitor, then she will share her knowledge even in the situation where, previously, no one firm shares his knowledge ($\lambda > \overline{\lambda}_2$). However, in the situation where both firms shared their knowledge, henceforth only one firm shares her knowledge ($\lambda < \overline{L}_1$). Indeed, the firm with a lower reservation price has less incentives to share her knowledge alone as long as the other firm can obtain a higher part of her demand segment. As the previous section, parameters of the model have similar impacts on the knowledge sharing. Indeed, when $\lambda < \overline{L}_1$ there is situation where both firms share their knowledge whereas with $\lambda > \overline{L}_2$ both firms do not share it. Thus, a rise of λ can involves a lower knowledge sharing. Moreover, Proposition 2 stays true. Indeed, \overline{L}_1 and \overline{L}_2 are similar and the effects of parameters on the other \overline{L}_z are identical.

An other result of the model appears in the case B and C. For a sufficient difference between α_1 and α_2 to be in the case B or C, when only one firm share her knowledge, but if this difference is not too far, then the firm with the higher reservation price can lost her leadership in this market. This result is summarized in Proposition 4.

Proposition 4 For $\alpha_2 < \alpha_1 < \alpha_2 + \lambda$, the firm 1 with an higher reservation price before the knowledge sharing lost her leadership if she shares her knowledge alone.

The profit maximizing behavior leads to this result due to a higher profit in the case where the previous leader share his knowledge alone than the profit in the case where no one share his knowledge. However, this result seems not realistic for the reason that obtaining a higher market share is heavily strategic for multiple reasons such as, for example, the bargaining power.

5 Heterogeneous firms with a different positive externality

This section analyzes the fact that firms have two heterogeneities: firm 1 has a higher reservation price, $\alpha_1 > \alpha_2$, whereas firm 2 has a positive externality superior to firm 1, $\lambda_1 < \lambda_2$. If a firm get both advantage, for example $\alpha_1 > \alpha_2$ and $\lambda_1 > \lambda_2$, it will just reinforce results of the previous section. Results provided by the Cournot competition in step two are shown in Table 5 in the Annex B.1.

Before analyzing the game, it may be interesting to examine the profit, the square of equilibrium quantities in Table 5 (prices are still equal to quantity as in previous section), of both firms obtained in stage 2. As in previous section, parameters of competition, δ and γ , have only a negative impact whereas the collective benefit of the knowledge sharing, ω , increases firms' profit. Contrary to the previous section, individual benefit has a negative impact. However, the competitor's individual benefit still impacts negatively the profit. In fact, parameters representing the heterogeneity of firms will play a crucial role to find Nash equilibrium. Indeed, firm's parameters play positively whereas competitor's parameters have negative effects.

As the previous section, four thresholds are necessary to find Nash equilibrium of the game. Here, the parameter ω , the collective benefit of the knowledge sharing, will be isolated to find the four thresholds. \overline{W}_1 represent the threshold where firm 1 want to share her knowledge when firm 2 shares it as well, so that $\Pi_1^A > \Pi_1^B$. Similarly we obtain \overline{W}_2 for $\Pi_1^C > \Pi_1^D$, \overline{W}_3 for $\Pi_2^A > \Pi_2^C$ and \overline{W}_4 for $\Pi_2^B > \Pi_2^D$. In the four situations, when ω is higher than each threshold it means that firms want to share their knowledge. These four thresholds are presented with more details is Annex B.2. Depending on the value of each parameter of the model, thresholds are sorted differently. To find these different ranking of thresholds, we compare the four thresholds and isolate the absolute difference between reservation prices, $\alpha_1 - \alpha_2$ in the six possible equalities. Thus, six new thresholds are obtained and called \overline{A}_z where $z \in [1, 6]$. The ranking between thresholds is presented in Annex B.3 and summarize in Figure 2.

From Figure 2⁶ and Annex B.4, Nash equilibrium of the game can be found. For example, if $\alpha_1 - \alpha_2 < \overline{A}_6$, meaning that the difference between reservation prices is low, we know that $\overline{W}_3 < \overline{W}_4 < \overline{W}_2 < \overline{W}_1$. Moreover, when ω is inferior to \overline{W}_4 , firm 1 never want to share her knowledge because her profit is lower when she shares it $(\Pi_1^A < \Pi_1^B \text{ and } \Pi_1^C < \Pi_1^D)$, and, firm 2 wants to share it only if firm 1 shares it as

⁶Notice that this Figure is completely exact only when $\frac{\lambda_1(8+8\gamma-2\gamma^2+4\delta-3\gamma\delta-\delta^2)}{8\gamma+6\delta} < \lambda_2 < \frac{\lambda_1(8-\gamma^2-3\gamma\delta-\delta^2)}{2\delta}$. In fact, λ_2 has to be inferior to $\frac{\lambda_1(8-\gamma^2-3\gamma\delta-\delta^2)}{2\delta}$ or the ranking of thresholds, provided by the difference between reservation prices, will be slightly different. Moreover, \overline{A}_6 is positive only if $\lambda_2 > \frac{\lambda_1(8+8\gamma-2\gamma^2+4\delta-3\gamma\delta-\delta^2)}{8\gamma+6\delta}$ and \overline{A}_5 is positive only if $\lambda_2 > \frac{\lambda_1(4(\gamma+\delta)-2\gamma^2-3\gamma\delta-\delta^2)}{8\gamma+6\delta}$. Results presented in this section are similar even if these assumptions are not respected but it adds other possible cases.

well $(\Pi_2^A > \Pi_2^C \text{ and } \Pi_2^B < \Pi_2^D)$. Therefore, both firms do not share their knowledge. When ω is included between \overline{W}_4 and \overline{W}_1 , firm 1 want to share her knowledge only if firm 2 does not share it but firm 2 want to share her knowledge regardless the behavior of firm 1. As a consequence, only the firm 2 shares her knowledge. When ω is superior to \overline{W}_1 then both firms want to share their knowledge because $\Pi_1^A > \Pi_1^B$, $\Pi_1^C > \Pi_1^D$, $\Pi_2^A > \Pi_2^C$ and $\Pi_2^B > \Pi_2^D$.

Figure 2 – Comparison of the thresholds for $\alpha_1 > \alpha_2$ and $\lambda_1 < \lambda_2$

	\overline{W}_3 $\stackrel{\wedge}{\overline{W}_4}$ $\stackrel{\wedge}{\overline{W}_2}$ $\stackrel{\wedge}{\overline{W}_1}$		\overline{W}_4 $\stackrel{\wedge}{\overline{W}_3}$ $\stackrel{\wedge}{\overline{W}_2}$ $\stackrel{\wedge}{\overline{W}_1}$		$\overline{W}_4 \ \wedge \ \overline{W}_2 \ \wedge \ \overline{W}_3 \ \wedge \ \overline{W}_1$		$\overline{W}_2 \ \wedge \ \overline{W}_4 \ \wedge \ \overline{W}_3 \ \wedge \ \overline{W}_1$		\overline{W}_2 \wedge \overline{W}_4 \wedge \overline{W}_1 \wedge \overline{W}_3		\overline{W}_2 \wedge \overline{W}_1 \wedge \overline{W}_4 \wedge \overline{W}_3		
0		\overline{A}_6		\overline{A}_3		\overline{A}_2		\overline{A}_1		\overline{A}_4		\overline{A}_5	$(\alpha_1 - \alpha_2)$

For a high value of the difference between reservation prices $(\alpha_1 - \alpha_2)$, the result is different. For example, if $\overline{A}_4 < \alpha_1 - \alpha_2 < \overline{A}_5$ then thresholds are sorted as follows $\overline{W}_2 < \overline{W}_1 < \overline{W}_4 < \overline{W}_3$. Here, when ω is inferior to \overline{W}_2 then both firms do not want to share their knowledge. However, when ω is included between \overline{W}_2 and \overline{W}_3 , the firm 1 want to share her knowledge if the firm 2 does not share it (for $\omega < \overline{W}_1$) or if the firm 2 shares it (for $\omega > \overline{W}_1$) whereas the firm 2 never want to share her knowledge (for $\omega < \overline{W}_4$) or accepts to share her knowledge only if firm 1 does not share it (for $\omega > \overline{W}_4$). As a consequence, only firm 1 shares her knowledge. Finally, when ω is superior to each threshold, $\omega > \overline{W}_3$, therefore both firms share their knowledge. These two situations and all other possible situations in terms of reservation prices are presented in Annex B.4.

The previous results and the results of Annex B.4 show that if the difference between reservation prices is sufficiently low, for example when $\alpha_1 - \alpha_2 < \overline{A}_6$, then firm 2 has more incentives to share her knowledge whereas if the difference between $\alpha_1 - \alpha_2$ is sufficiently high, for example when $\overline{A}_4 < \alpha_1 - \alpha_2 < \overline{A}_4$, then incentives to share knowledge are larger for firm 1. Moreover, for each threshold \overline{A}_z , λ_2 has a positive impact and a rise λ_1 implies a decrease of thresholds \overline{A}_z . Therefore, a higher value of the individual benefit of firm 2 and/or a lower value of the individual benefit of firm 1 increase the situation where the firm 2 has more incentives to share her knowledge than firm 1. These results lead to the following proposition

Proposition 5

- For a high value of ω
 - Both firms share their knowledge
- For a low value of ω
 - Both firms do not share their knowledge
- For intermediate value of ω
 - When the difference between reservation price, $\alpha_1 \alpha_2$ is low (high), then firm 2 (1) has more incentives to share her knowledge than firm 1 (2).
- Moreover, a higher (lower) λ₁ increases the incentives of firm 1 (2) and λ₂ has a negative (positive) impact on the incentives of firm 1 (2).

Proof 4 See Annex B.3 and B.4.

Proposition 5 highlights that the potential of knowledge sharing in a R&D collaboration highly depends on the heterogeneity between firms, specifically when a firms can use the collaboration to rectify her technological disadvantage. Indeed, in this particular situation, there is a tradeoff in terms of knowledge sharing. In fact, if the knowledge sharing has to be very high for the success of the collaboration, there is a risk that the firm with the technological advantage does not want to share her knowledge because its competitor can catch her technologies and compete more the firm. On the other side, if the knowledge sharing is very low, then the individual benefit will be also very modest. In this case, the firm with the technological disadvantage risk that its competitor use the collaboration to catch a part a her demand segment what will be easier for the firm with the technological advantage.

6 Conclusion

In this paper we analyze firms already in R&D cooperation and that compete in the same market. In the aim of creating an innovation, firms have to share their knowledge but this imply spillovers that involve a higher competition. In this framework we try to understand whether firms are incited to share their knowledge through a theoretical model. In the theoretical model, both firms can choose whether their share their knowledge in a first step, and compete in Cournot framework in a second step. We introduce two effects provided by spillovers: a negative externality increasing the competition and a positive externality raising the reservation price.

When there is no heterogeneity between firms, we found that spillovers involve by knowledge sharing have negative impacts on the choice of sharing knowledge when firms compete with substitute products. Moreover, the more the competition is between firms before the cooperation, the less competitors share their knowledge. Furthermore, firms with a larger reservation price keeps knowledge sharing with higher competition than firms with a lower reservation price. Indeed, it is riskier to share the knowledge for a firm lags behind in the development of technologies as long as it is easier for the advanced firm to catch her market share.

Finally, we introduce a second heterogeneity in the individual benefit provided by knowledge sharing. Indeed, the firm lags behind in the development of technology can use the knowledge sharing of its competitor to rectify her lag. Here, if the knowledge sharing necessary to the success of the collaboration is very important, the firm advanced in technology can refuse to share her knowledge in the reason that the firm lags behind rectify her lag and became a stronger competitor. However, if the firm lags behind cannot assimilate enough new technology, it is easier for the advanced firm to compete her and then the firm lags behind can refuse the knowledge sharing to avoid this risk.

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A Heterogeneous firms

In the Annex A, the proofs of the different propositions found in section 4 when $\alpha_1 \neq \alpha_2$ and $\lambda_1 = \lambda_2 = \lambda$ are provided.

A.1 Thresholds of the game

The first threshold is when firm 1 want to share their knowledge when the firm 2 shares it such that $\Pi_1^A > \Pi_1^B$

$$\alpha_1 > \frac{1}{2\delta(4\gamma + 3\delta)} (-2(-4+\gamma^2)(2\gamma\lambda + (-2+\gamma)\omega) + (2\alpha_2(4+\gamma^2) - 8(-2+\gamma+\gamma^2)\lambda - 3(-4+\gamma^2)\omega)\delta - (-3\alpha_2\gamma + (6+5\gamma)\lambda + (2+\gamma)\omega)\delta^2 + (\alpha_2 - \lambda)\delta^3),$$

where the right hand side of this inequality is denoted \overline{T}_1 . The second threshold is when firm 1 want to share their knowledge when the firm 2 does not such that $\Pi_1^C > \Pi_1^D$

$$\alpha_1 > \frac{1}{2\delta(4\gamma + 3\delta)} (-2(-4+\gamma^2)(2\gamma\lambda + (-2+\gamma)\omega) + (2\alpha_2(4+\gamma^2) - (-4+\gamma^2)(2\lambda+\omega))\delta + \alpha_2\gamma\delta^2),$$

where the right hand side of this inequality is denoted \overline{T}_2 . The third threshold is when firm 2 want to share their knowledge when the firm 1 shares it such that $\Pi_2^A > \Pi_2^C$

$$\alpha_1 < \frac{1}{\delta(8+\gamma(2\gamma+\delta))} (2(-4+\gamma^2)(2\gamma\lambda+(-2+\gamma)\omega) + (8\alpha_2\gamma+8(-2+\gamma+\gamma^2)\lambda+3(-4+\gamma^2)\omega)\delta + (6\alpha_2+(6+5\gamma)\lambda+(2+\gamma)\omega)\delta^2+\lambda\delta^3),$$

where the right hand side of this inequality is denoted \overline{T}_3 . The fourth threshold is when firm 2 want to share their knowledge when the firm 1 does not such that

$$\Pi_2^B > \Pi_2^D$$

$$\alpha_1 < \frac{2(-4+\gamma^2)(2\gamma\lambda + (-2+\gamma)\omega) + (8\alpha_2\gamma + (-4+\gamma^2)(2\lambda+\omega))\delta + 2\alpha_2\delta^2}{\delta(8+\gamma(2\gamma+\delta))}$$

where the right hand side of this inequality is denoted \overline{T}_4 . We can calculate partial derivatives for each threshold with respect to λ :

$$\frac{\partial \overline{T}_1}{\partial \lambda} = \frac{4\gamma(4-\gamma^2) + 8(1-\gamma)(2+\gamma)\delta - \delta^2(6+5\gamma) - \delta^3}{2\delta(4\gamma+3\delta)},$$

is positive for $0 < \gamma < 2$ and $0 < \delta < 2 - \gamma$.

$$\frac{\partial \overline{T}_2}{\partial \lambda} = \frac{4\gamma(4-\gamma^2)+2(4-\gamma^2)\delta}{2\delta(4\gamma+3\delta)},$$

is positive for $0 < \gamma < 2$ and $\delta > 0$.

$$\frac{\partial \overline{T}_3}{\partial \lambda} = \frac{-4\gamma(4-\gamma^2) - 2(8-4\gamma(1+\gamma)) + \delta^2(6+5\gamma) + \delta^3}{\delta(8+(\gamma+\delta)(\gamma+\delta))},$$

is negative for $0 < \gamma < 2$ and $0 < \delta < 2 - \gamma$.

$$\frac{\partial \overline{T}_4}{\partial \lambda} = \frac{-4\gamma(4-\gamma^2) - 2(4-\gamma^2)\delta}{\delta(8+\gamma(2\gamma+\delta))},$$

is negative for $0 < \gamma < 2$ and $\delta > 0$. Thus a rise of λ increases \overline{T}_1 , \overline{T}_2 and decreases \overline{T}_3 and \overline{T}_4 .

A.2 Comparison of thresholds

In order to find Nash equilibrium of the game we need to know how thresholds evolve. We compare thresholds by isolating the parameter λ :

 $\begin{array}{l} - \ \overline{T}_1 < \overline{T}_3 \\ & \mbox{For} \ \lambda < \frac{(4-2\gamma-\delta)((2+\gamma)\omega-\alpha_2\delta)}{4\gamma(2+\gamma)+4(2+\gamma)\delta+\delta^2}, \end{array}$

the right hand side of this inequality is denoted \overline{L}_1 .

-
$$\overline{T}_2 < \overline{T}_4$$

For $\lambda < \frac{(4-2\gamma-\delta)((2+\gamma)\omega-\alpha_2\delta)}{2(2+\gamma)(2\gamma+\delta)}$,

the right hand side of this inequality is denoted \overline{L}_2 .

 $\begin{array}{l} - \ \overline{T}_1 < \overline{T}_4 \\ & \text{For } \lambda < \frac{(4 - \gamma^2 - \delta)((2 + \gamma)\omega - \alpha_2 \delta)}{8\gamma + 8\gamma^2 + 2\gamma^3 + 8\delta + 6\gamma\delta + 3\gamma^2\delta + \gamma\delta^2}, \end{array}$

the right hand side of this inequality is denoted \overline{L}_3 .

- $\overline{T}_2 < \overline{T}_3$ For $\lambda < \frac{(4-\gamma^2-\delta)((2+\gamma)\omega-\alpha_2\delta)}{2(4\gamma+4\gamma^2+\gamma^3+2\delta+5\gamma\delta+\gamma^2\delta+\delta^2)}$

the right hand side of this inequality is denoted \overline{L}_4 .

- $\overline{T}_2 < \overline{T}_1$ For $\lambda < \frac{(4-2\gamma-\delta)((2+\gamma)\omega-\alpha_2\delta)}{8\gamma^2+2\delta+7\gamma\delta+\delta^2}$,

the right hand side of this inequality is denoted \overline{L}_5 .

- $\overline{T}_3 < \overline{T}_4$

For $\lambda > \frac{2(4-2\gamma-\delta)((2+\gamma)\omega-\alpha_2\delta)}{-16-24\gamma+4\gamma^2+2\gamma^3+16\delta+2\gamma\delta+3\gamma^2\delta+\gamma\delta^2}$ and $\gamma < \overline{\gamma}$, or for $\lambda < \frac{2(4-2\gamma-\delta)((2+\gamma)\omega-\alpha_2\delta)}{-16-24\gamma+4\gamma^2+2\gamma^3+16\delta+2\gamma\delta+3\gamma^2\delta+\gamma\delta^2}$ and $\gamma > \overline{\gamma}$, where $\overline{\gamma} = \frac{1}{6}(-3\delta-4) + \frac{\sqrt[3]{-36\delta^2+\sqrt{4(-3\delta^2-12\delta+128)^3+(-36\delta^2-576\delta+3328)^2}-576\delta+3328}}{6\sqrt[3]{2}} - \frac{-3\delta^2-12\delta+128}{6\sqrt[3]{2}}7$,

We find also for $\omega > \frac{\alpha \delta}{(2+\gamma)}$ (the innovation is high enough) that

- $L_1 < L_3$ For $0 < \delta < 1$ and $\frac{-3\delta}{4} < \gamma < 2$ - $L_3 < L_4$ For $\delta > 0$ and $c < 2 - \delta$ - $L_4 < L_2$ For $0 < \delta < 8/3$ and $\frac{-\delta}{4} < \gamma < 2 - \delta$ - $L_2 < L_5$ For $\delta < 8/3$ and $\frac{-\delta}{4} < \gamma < 2 - \delta$ - $L_1 > L_6$ For $-8 + 4\sqrt{2} < \delta < 1$ and $\frac{1}{2}(-2 - \delta) + 1\sqrt{\delta} < \gamma < \overline{\gamma}$ - $L_5 < L_6$

⁷Note that for $\delta = 0$, then $\overline{\gamma} = 0.59$ and for $\delta = 1$, then $\overline{\gamma} = 0$.

the right hand side of the first inequality is denoted \overline{L}_6^8 .

 $^{{}^{8}\}overline{L}_{6}$ is negative, decreasing and concave for $\gamma \in [0, \overline{\gamma}]$ and positive, decreasing and convex for $\gamma \in [\overline{\gamma}, 1]$.

For $0 < \delta < \frac{64}{17}$ and $\overline{\gamma} < \gamma < 2 - \delta$

All the conditions found respect the constraints of parameters. We can conclude that

- For
$$\gamma < \overline{\gamma}$$
 and $((2+\gamma)\omega - \alpha_2\delta) > 0$
Then $L_6 < 0 < L_1 < L_3 < L_4 < L_2 < L_5$
- For $\gamma > \overline{\gamma}$ and $((2+\gamma)\omega - \alpha_2\delta) > 0$
Then $0 < L_1 < L_3 < L_4 < L_2 < L_5 < L_6$

All these results are regrouped in Figure 1.

A.3 Nash equilibrium

From Annex A.1 we know that

- if $\alpha_1 > \overline{T}_1$ firm 1 accepts to share her knowledge when firm 2 shares her knowledge,
- if $\alpha_1 > \overline{T}_2$ firm 1 accepts to share her knowledge when firm 2 does not share her knowledge,
- if $\alpha_1 < \overline{T}_3$ firm 2 accepts to share her knowledge when firm 1 shares her knowledge,
- if $\alpha_1 < \overline{T}_4$ firm 2 accepts to share her knowledge when firm 1 does not share her knowledge.

Moreover, results provided by Annex A.2 lead to the following Nash equilibrium.

- (i) For $0 < \lambda < \overline{L}_1$ we know that
 - $\overline{T}_4 > \overline{T}_3 > \overline{T}_1 > \overline{T}_2$,
 - $\alpha_2 > \{\overline{T}_1, \overline{T}_2\}$ and $\alpha_2 < \{\overline{T}_3, \overline{T}_4\},\$
 - Therefore, for α_1 superior to \overline{T}_3 (same result for $\alpha_1 > \overline{T}_4$) we find
 - $-\alpha_1 > \alpha_2,$
 - $-\alpha_1 > \{\overline{T}_1, \overline{T}_2\}$, thus Firm 1 wants to share her knowledge,
 - $-\alpha_1 > \overline{T}_3$, thus Firm 2 declines the knowledge sharing.
 - For α_1 included between to \overline{T}_3 and \overline{T}_1 we obtain

 $-\alpha_1 \ge \alpha_2 \text{ or } \alpha_1 \le \alpha_2 \text{ (low difference)},$

- $-\alpha_1 > \{\overline{T}_1, \overline{T}_2\},$ thus Firm 1 wants to share her knowledge,
- $-\alpha_1 < \{\overline{T}_3, \overline{T}_4\},$ thus Firm 2 accepts the knowledge sharing.

- For α_1 inferior to \overline{T}_1 (same result for $\alpha_1 < \overline{T}_2$) we find
 - $\alpha_1 < \alpha_2,$ $- \alpha_1 < \{\overline{T}_3, \overline{T}_4\},$ thus Firm 2 wants to share her knowledge, $- \alpha_1 < \overline{T}_1,$ thus Firm 1 declines the knowledge sharing.
- (ii) For $\overline{L}_1 < \lambda < \overline{L}_3$ we know that
 - $\overline{T}_4 > \overline{T}_1 > \overline{T}_3 > \overline{T}_2,$
 - $\alpha_2 > \{\overline{T}_2, \overline{T}_3\}$ and $\alpha_2 < \{\overline{T}_1, \overline{T}_4\}$
 - Therefore, for α_1 superior to \overline{T}_1 (same result for $\alpha_1 > \overline{T}_4$) we find - $\alpha_1 > \alpha_2$,
 - $-\alpha_1 > \{\overline{T}_1, \overline{T}_2\}$, thus Firm 1 wants to share her knowledge,
 - $-\alpha_1 > \overline{T}_3$, thus Firm 2 declines the knowledge sharing.
 - For α_1 included between to \overline{T}_1 and \overline{T}_3 we obtain
 - $-\alpha_1 \ge \alpha_2 \text{ or } \alpha_1 \le \alpha_2 \text{ (low difference)},$
 - $-\alpha_1 > \overline{T}_2$ and $\alpha_1 < \overline{T}_1$,
 - $-\alpha_1 < \overline{T}_4$ and $\alpha_1 > \overline{T}_3$,
 - thus, there is a multiple equilibrium: one of them shares her knowledge.
 - For α_1 inferior to \overline{T}_3 (same result for $\alpha_1 < \overline{T}_2$) we find
 - $-\alpha_1 < \alpha_2,$
 - $-\alpha_1 < \{\overline{T}_3, \overline{T}_4\},$ thus Firm 2 wants to share her knowledge,
 - $-\alpha_1 < \overline{T}_1$, thus Firm 1 declines the knowledge sharing.

(iii) For $\overline{L}_3 < \lambda < \overline{L}_4$ we know that

- $\overline{T}_1 > \overline{T}_4 > \overline{T}_3 > \overline{T}_2$,
- $\alpha_2 > \{\overline{T}_2, \overline{T}_3\}$ and $\alpha_2 < \{\overline{T}_1, \overline{T}_4\}$
- Therefore, for α_1 superior to \overline{T}_4 (same result for $\alpha_1 > \overline{T}_1$) we find
 - $-\alpha_1 > \alpha_2,$
 - $-\alpha_1 > \{\overline{T}_3, \overline{T}_4\}$, thus Firm 2 declines the knowledge sharing.
 - $-\alpha_1 > \overline{T}_2$, thus Firm 1 wants to share her knowledge,
- For α_1 included between to \overline{T}_4 and \overline{T}_3 we obtain
 - $-\alpha_1 \ge \alpha_2 \text{ or } \alpha_1 \le \alpha_2 \text{ (low difference)},$

- $-\alpha_1 > \overline{T}_2$ and $\alpha_1 < \overline{T}_1$,
- $-\alpha_1 < \overline{T}_4$ and $\alpha_1 > \overline{T}_3$,

- thus, there is a multiple equilibrium: one of them shares her knowledge.

- For α_1 inferior to \overline{T}_3 (same result for $\alpha_1 < \overline{T}_2$) we find
 - $-\alpha_1 < \alpha_2,$
 - $-\alpha_1 < \{\overline{T}_3, \overline{T}_4\},$ thus Firm 2 wants to share her knowledge,
 - $-\alpha_1 < \overline{T}_1$, thus Firm 1 declines the knowledge sharing.
- (iv) For $\overline{L}_4 < \lambda < \overline{L}_2$ we know that
 - $\overline{T}_1 > \overline{T}_4 > \overline{T}_2 > \overline{T}_3$,
 - $\alpha_2 > \{\overline{T}_2, \overline{T}_3\}$ and $\alpha_2 < \{\overline{T}_1, \overline{T}_4\}$
 - Therefore, for α_1 superior to \overline{T}_4 (same result for $\alpha_1 > \overline{T}_1$) we find - $\alpha_1 > \alpha_2$,
 - $-\alpha_1 > \{\overline{T}_3, \overline{T}_4\}$, thus Firm 2 declines the knowledge sharing.
 - $-\alpha_1 > \overline{T}_2$, thus Firm 1 wants to share her knowledge,
 - For α_1 included between to \overline{T}_4 and \overline{T}_2 we obtain
 - $-\alpha_1 \ge \alpha_2$ or $\alpha_1 \le \alpha_2$ (low difference),
 - $-\alpha_1 > \overline{T}_2$ and $\alpha_1 < \overline{T}_1$,
 - $-\alpha_1 < \overline{T}_4$ and $\alpha_1 > \overline{T}_3$,
 - thus, there is a multiple equilibrium: one of them shares her knowledge.
 - For α_1 inferior to \overline{T}_2 (same result for $\alpha_1 < \overline{T}_3$) we find
 - $-\alpha_1 < \alpha_2,$
 - $-\alpha_1 < \{\overline{T}_1, \overline{T}_2\},$ thus Firm 1 declines the knowledge sharing.
 - $-\alpha_1 < \overline{T}_4$, thus Firm 2 wants to share her knowledge,
- (v) The last case regroups four different situations:
 - For $\overline{L}_2 < \lambda < \overline{L}_5$ Then $\overline{T}_1 > \overline{T}_2 > \overline{T}_4 > \overline{T}_3$
 - And for $\gamma < \overline{\gamma}$ and $\lambda > \overline{L}_5$ Then $\overline{T}_2 > \overline{T}_1 > \overline{T}_4 > \overline{T}_3$
 - And for $\gamma > \overline{\gamma}$ and $\overline{L}_5 < \lambda < \overline{L}_6$

Then $\overline{T}_2 > \overline{T}_1 > \overline{T}_4 > \overline{T}_3$

- And for $\gamma > \overline{\gamma}$ and $\lambda > \overline{L}_6$ Then $\overline{T}_2 > \overline{T}_1 > \overline{T}_3 > \overline{T}_4$
- In these four situations $\alpha_2 > \{\overline{T}_3, \overline{T}_4\}$ and $\alpha_2 < \{\overline{T}_1, \overline{T}_2\}$
- Therefore, for α_1 superior to \overline{T}_2 we find
 - $-\alpha_1 > \alpha_2,$
 - $-\alpha_1 > \{\overline{T}_3, \overline{T}_4\},$ thus Firm 2 declines the knowledge sharing.
 - $-\alpha_1 > \overline{T}_2$, thus Firm 1 wants to share her knowledge,
- For α_1 included between to \overline{T}_2 and \overline{T}_4 we obtain
 - $-\alpha_1 \ge \alpha_2$ or $\alpha_1 \le \alpha_2$ (low difference),
 - $-\alpha_1 < \overline{T}_2$, thus Firm 1 declines the knowledge sharing.
 - $-\alpha_1 > \overline{T}_4$, thus Firm 2 declines the knowledge sharing.
- For α_1 inferior to \overline{T}_4 we find
 - $-\alpha_1 < \alpha_2,$
 - $-\alpha_1 < \{\overline{T}_1, \overline{T}_2\}$, thus Firm 1 declines the knowledge sharing.
 - $-\alpha_1 < \overline{T}_4$, thus Firm 2 wants to share her knowledge,

Logically, in these five cases, each time that α_1 and α_2 are close or equal, the Nash equilibrium are similar to the context where both firms was symmetric. However, a different result appears when α_1 and α_2 are sufficiently different, then the firm with the higher potential market size will share her knowledge and her competitor will refuse to share it.

B Heterogeneous firms

In the Annex B, the proofs of the different propositions found in section 5 when $\alpha_1 > \alpha_2$ and $\lambda_1 < \lambda_2$ are provided.

B.1 Results of the Cournot competition in step two



Table 5 – Payoffs for heterogeneous firms with $\alpha_1 > \alpha_2$ and $\lambda_2 > \lambda_1$

B.2 Thresholds of the game

- The Firm 1 shares her knowledge when the Firm 2 shares her knowledge if

$$\omega > \frac{\alpha_2 * a - \alpha_1 * b - \lambda_1 * b + \lambda_2 * c}{d},$$

where the right hand side of this inequality is denoted \overline{W}_1 .

- The Firm 1 shares her knowledge when the Firm 2 does not share her knowl-

edge if

$$\omega > \frac{\alpha_2 * g - \alpha_1 * f + \lambda_2 * e}{h},$$

where the right hand side of this inequality is denoted \overline{W}_2 .

- The Firm 2 shares her knowledge when the Firm 1 shares her knowledge if

$$\omega > \frac{\alpha_1 * a - \alpha_2 * b - \lambda_2 * b + \lambda_1 * c}{d},$$

where the right hand side of this inequality is denoted \overline{W}_3 .

- The Firm 2 shares her knowledge when the Firm 1 does not share her knowledge if

$$\omega > \frac{\alpha_1 * g - \alpha_2 * f + \lambda_1 * e}{h},$$

where the right hand side of this inequality is denoted \overline{W}_4 . In these four thresholds, the value of a, b, c, d, e, f, g, h are

$$- a = \delta(8 + (\gamma + \delta)(2\gamma + \delta))
- b = 2\delta(4\gamma + 3\delta)
- c = (\gamma + \delta)(4 - 2\gamma - \delta)(4 + 2\gamma + \delta)
- d = (2 + \gamma)(2 - \gamma - \delta)(4 - 2\gamma - \delta)
- e = 2(4 - \gamma^2)(2\gamma + \delta)
- f = 2\delta(4\gamma + \delta)
- g = \delta(8 + \gamma(2\gamma + \delta))
- h = (4 - \gamma^2)(4 - 2\gamma - \delta)$$

B.3 Comparison of thresholds

In order to find Nash equilibrium of the game we need to know how thresholds evolve. We compare thresholds by isolating the difference between parameters α_1 and α_2 :

-
$$\overline{W}_1 < \overline{W}_3$$

if $\alpha_1 - \alpha_2 > \frac{(\lambda_2 - \lambda_1)(b+c)}{a+b}$,
where the right hand side of this inequality is denoted \overline{A}_1 .

-
$$\overline{W}_2 < \overline{W}_4$$

if $\alpha_1 - \alpha_2 > \frac{(\lambda_2 - \lambda_1)(e)}{g + f}$,

where the right hand side of this inequality is denoted \overline{A}_2 .

- $\overline{W}_2 < \overline{W}_3$ if $\alpha_1 - \alpha_2 > \frac{\lambda_2(e*x+b) - \lambda_1 * c}{n}$, where the right hand side of t

where the right hand side of this inequality is denoted \overline{A}_3 , $x = \frac{2-\gamma-\delta}{2-\gamma}$ and $n = \frac{\delta(4+2\gamma+\delta)(4-\gamma(\gamma+\delta))}{2-\gamma}$. - $\overline{W}_1 < \overline{W}_4$

if $\alpha_1 - \alpha_2 > \frac{\lambda_2 * c - \lambda_1(e * x + b)}{n}$,

where the right hand side of this inequality is denoted \overline{A}_4 .

- $\overline{W}_1 < \overline{W}_2$ if $\alpha_1 - \alpha_2 > \frac{\lambda_2(c-e*x) - \lambda_1 * b}{m}$,

where the right hand side of this inequality is denoted \overline{A}_5 and $m = \frac{2\delta^2(2\gamma+\delta+4)}{2-\gamma}$.

- $\overline{W}_3 > \overline{W}_4$ if $\alpha_1 - \alpha_2 > \frac{\lambda_2 * b - \lambda_1 * (c - e * x)}{m}$,

where the right hand side of this inequality is denoted \overline{A}_6 .

Knowing that $\gamma > 0$, $\delta > 0$, $\gamma + \delta < 1$, $\lambda_2 > \lambda_1$, $\lambda_1 > 0$ by assumption and using the Reduce function of Mathematica software, we found that $\overline{A}_6 < \overline{A}_3 < \overline{A}_2 < \overline{A}_1 < \overline{A}_4 < \overline{A}_5$ if $\lambda_2 < \frac{\lambda_1(8-\gamma^2-3\gamma\delta-\delta^2)}{2\delta}$,

and

 $\overline{A}_3 < \overline{A}_2 < \overline{A}_6 < \overline{A}_1 < \overline{A}_4 < \overline{A}_5 \text{ if } \lambda_2 > \frac{\lambda_1(8-\gamma^2-3\gamma\delta-\delta^2)}{2\delta}.$

B.4 Nash equilibrium

From Annex B.2 we know that

- if $\omega > \overline{W}_1$ firm 1 accepts to share her knowledge when firm 2 shares her knowledge,
- if $\omega > \overline{W}_2$ firm 1 accepts to share her knowledge when firm 2 does not share her knowledge,
- if $\omega > \overline{W}_3$ firm 2 accepts to share her knowledge when firm 1 shares her knowledge,
- if $\omega > \overline{W}_4$ firm 2 accepts to share her knowledge when firm 1 does not share her knowledge.

Moreover, results provided by Annex B.3 lead to the following Nash equilibria, beginning by the situation where the interval between α_1 and α_2 is very low and finishing with a very high gap.

(i) For $\alpha_1 - \alpha_2 < \overline{A}_6$ we know that

$$- \overline{W}_3 < \overline{W}_4 < \overline{W}_2 < \overline{W}_1,$$

- For ω inferior to \overline{W}_4
 - Both firms do not share their knowledge
- For ω included between \overline{W}_4 and \overline{W}_1
 - Only the firm 2 shares her knowledge,
- For ω superior to \overline{W}_1
 - Both firms share their knowledge.

Note that this case exists only if $\lambda_2 > \frac{\lambda_1(8+8\gamma-2\gamma^2+4\delta-3\gamma\delta-\delta^2)}{8\gamma+6\delta}$, otherwise \overline{A}_6 is lower than 0 and $\alpha_1 - \alpha_2$ can not be lower than 0 knowing that $\alpha_1 > \alpha_2$. Therefore, this case does not exist for $\lambda_2 < \frac{\lambda_1(8+8\gamma-2\gamma^2+4\delta-3\gamma\delta-\delta^2)}{8\gamma+6\delta}$. Note also that $\frac{\lambda_1(8+8\gamma-2\gamma^2+4\delta-3\gamma\delta-\delta^2)}{8\gamma+6\delta} < \frac{\lambda_1(8-\gamma^2-3\gamma\delta-\delta^2)}{2\delta}$ for all values of γ and δ .

- (ii) For $\overline{A}_6 < \alpha_1 \alpha_2 < \overline{A}_3$ we know that
 - $\overline{W}_4 < \overline{W}_3 < \overline{W}_2 < \overline{W}_1$,
 - For ω inferior to \overline{W}_4
 - Both firms do not share their knowledge
 - For ω included between \overline{W}_4 and \overline{W}_1
 - Only the firm 2 shares her knowledge,
 - For ω superior to \overline{W}_1
 - Both firms share their knowledge.

Note that this case exists only if $\lambda_2 > \frac{\lambda_1(4(\gamma+\delta)-2\gamma^2-3\gamma\delta-\delta^2)}{4\gamma+2\delta-2\gamma^2-2\gamma\delta}$ knowing that $\frac{\lambda_1(4(\gamma+\delta)-2\gamma^2-3\gamma\delta-\delta^2)}{4\gamma+2\delta-2\gamma^2-2\gamma\delta} < \frac{\lambda_1(8+8\gamma-2\gamma^2+4\delta-3\gamma\delta-\delta^2)}{8\gamma+6\delta}$ for all values of γ and δ .

(iii) For $\overline{A}_3 < \alpha_1 - \alpha_2 < \overline{A}_2$ we know that - $\overline{W}_4 < \overline{W}_2 < \overline{W}_3 < \overline{W}_1$, - For ω inferior to \overline{W}_4 – Both firms do not share their knowledge

- For ω included between \overline{W}_4 and \overline{W}_2

- Only the firm 2 shares her knowledge,

- For ω included between \overline{W}_2 and \overline{W}_3
 - There is a multiple equilibrium: one of them share her knowledge
- For ω included between \overline{W}_3 and \overline{W}_1
 - Only the firm 2 shares her knowledge,
- For ω superior to \overline{W}_1
 - Both firms share their knowledge.
- (iv) For $\overline{A}_2 < \alpha_1 \alpha_2 < \overline{A}_1$ we know that
 - $\overline{W}_2 < \overline{W}_4 < \overline{W}_3 < \overline{W}_1$,
 - For ω inferior to \overline{W}_2
 - Both firms do not share their knowledge
 - For ω included between \overline{W}_2 and \overline{W}_4
 - Only the firm 1 shares her knowledge,
 - For ω included between \overline{W}_4 and \overline{W}_3
 - There is a multiple equilibrium: one of them share her knowledge
 - For ω included between \overline{W}_3 and \overline{W}_1
 - Only the firm 2 shares her knowledge,
 - For ω superior to \overline{W}_1
 - Both firms share their knowledge.
- (v) For $\overline{A}_1 < \alpha_1 \alpha_2 < \overline{A}_4$ we know that
 - $\overline{W}_2 < \overline{W}_4 < \overline{W}_1 < \overline{W}_3$,
 - For ω inferior to \overline{W}_2
 - Both firms do not share their knowledge
 - For ω included between \overline{W}_2 and \overline{W}_4
 - Only the firm 1 shares her knowledge,
 - For ω included between \overline{W}_4 and \overline{W}_1
 - There is a multiple equilibrium: one of them share her knowledge

- For ω included between \overline{W}_1 and \overline{W}_3
 - Only the firm 1 shares her knowledge,
- For ω superior to \overline{W}_3
 - Both firms share their knowledge.

(vi) For
$$\overline{A}_4 < \alpha_1 - \alpha_2 < \overline{A}_5$$
 we know that
 $\overline{W} = \overline{W} = \overline{W}$

- $-\overline{W}_2 < \overline{W}_1 < \overline{W}_4 < \overline{W}_3,$
- For ω inferior to \overline{W}_2
 - Both firms do not share their knowledge
- For ω included between \overline{W}_2 and \overline{W}_3
 - Only the firm 1 shares her knowledge,
- For ω superior to \overline{W}_3
 - Both firms share their knowledge.
- (vii) For $\alpha_1 \alpha_2 > \overline{A}_5$ we know that
 - $\overline{W}_1 < \overline{W}_2 < \overline{W}_4 < \overline{W}_3,$
 - For ω inferior to \overline{W}_2
 - Both firms do not share their knowledge
 - For ω included between \overline{W}_2 and \overline{W}_3
 - Only the firm 1 shares her knowledge,
 - For ω superior to \overline{W}_3
 - Both firms share their knowledge.