Soil resources and the profitability and sustainability of farms: Theory and Evidence from a soil quality investment model

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Abstract

Soil resources play a role in food security and climate change mitigation. Through their practices, farmers impact the physical, biological and chemical quality of their soil. However, farmers face a trade-off between the short-term objectives of production and profitability and the long-term objective of soil resource conservation. In this article, we investigate the conditions under which farmers have a private interest in preserving their soil quality. We use a simplified theoretical soil quality investment model, where farmers maximize their revenues under a soil quality dynamic constraint. Here, soil quality is an endogenous production factor. We show that the existence of an equilibrium depends on the cooperation between soil quality and productive inputs. The results are confronted to a statistical illustration in the Grand Ouest of France.In this case, nitrogen fertilizers are not cooperating with soil organic carbon. Public incentives to reduce nitrogen fertilizers would not trigger feedback effect.

Key words: optimal control, soil quality, endogenous production factor

1 Introduction

The importance of soil resources is increasingly recognized, as both a production and regulation factor (Lal, 2004), and is at the center of the "4/1000 Initiative: Soil for Food Security and Climate", an international, multi-stakeholder voluntary action plan presented at the 21st Session of the Conference of the Parties to the United Nations Framework Convention on Climate Change (COP21) in Paris on December 1, 2015.

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Through their practices, farmers affect the physical, chemical and biological quality of soil. These impacts can be positive or negative, depending on how farming practices are implemented with respect to the location of the land, the climate, and the initial soil quality. However, some farming practices are more likely to be beneficial to soil quality than others (Chitrit and Gautronneau, 2011). For instance, agroecology practices related to soil resources aim at maintaining or increasing soil quality. The concept of agroecology used here refers to the intensified use of natural processes and resources, including soil resources.

In this context, there are internal and external pressures to reduce agricultural support, while energy, fertilizer and crop production costs are expected to increase. However, farming practices that aim at maintaining or increasing soil quality imply more complex agricultural practices than those of conventional agriculture and require farmers to adopt an innovation and research logic (Ghali et al, 2014). Hence, there can be a trade-off between the short-term production and profitability objectives and the long-term soil conservation objective.

Here, the focus is on the farmer's decision-making process. To translate economically these questions, we use a model where the farmer seeks to maximize his farm profitability, while considering long-term soil quality dynamics. It consists in estimating the optimal level of soil quality when the farmer controls this resource. Actually, in the economics literature, optimal control models have been used to explain farmers' motives to invest in conservation practices (Saliba, 1985; Barbier, 1998), since there can be conflicts between profitability and sustainability objectives (Segarra and Taylor, 1987; Barbier, 1990; Quang, Schreinemachers and Berger, 2014).

Our work is based on the theoretical optimal control models of McConnell (1983), Saliba (1985), Barbier (1990) and Hediger (2003). In these models, although soil quality is mentioned, the dynamics considered are those related to soil erosion (soil loss) and soil depth (McConnell, 1983; Saliba, 1985; Barbier 1990; Hediger, 2003). Considering only one aspect of soil quality is quite reductive with regard to the many characteristics of soil that can impact soil productivity, which can be physical, chemical or biological. In addition, the production function and the cooperation between the productive inputs and the soil quality parameter considered are not discussed in these articles. McConnell (1983) and Saliba (1985) do not mention the cross-effect of productive inputs and soil. Barbier (1990) considers that soil depth and productive inputs are coopering, while Hediger (2003) do not specify the cooperation relationship between productive inputs and soil depth in terms of crop production. However, the cooperation relationship between soil quality and productive inputs is critical to establish the existence of an equilibrium and to design public policies. These are the missing elements to which this article aims to contribute.

The objective of this article is to determine the conditions under which farmers have a private interest in maintaining or increasing soil quality when they maximize their revenue under a soil quality dynamic constraint. A particular attention is given to the cooperation relationship between productive inputs and soil quality.

To do so, we propose a soil quality investment model wherein soil quality dynamics are considered and applied to a crop production system. From this model, the equilibrium and the dynamics of this equilibrium are discussed with respect to soil quality and productive inputs cooperation. Soil quality is considered here with respect to the physical, chemical and biological attributes that are affected by farming practices. The results obtained are illustrated by a statistical analysis conducted for the Grand Ouest of France [1].

Relationships between soil quality, agricultural practices and crop yield are presented in section 2. In section 3, we analyze our optimal soil quality investment model, with respect to the cooperation relationship between soil quality and productive inputs. Section 4 is devoted to the statistical analysis of the main crops production functions in the Grand Ouest of France. In section 5, our statistical results are discussed with respect to our theoretical results.

2 Soil quality, agricultural practices and crop production

In the soil quality investment model proposed, two types of practices are identified: (1) productive inputs m and (2) conservation practices u (or investments in soil quality). Investments in soil quality correspond to extra costs induced by implementing conservation practices and by a more complex management of the system.

There are two production factors, soil quality and productive inputs. The crop production function is represented by $y = \phi(q, m)$. Soil quality q is composed of endogenous attributes s and exogenous attributes a. Exogenous attributes, such as soil type or other site-specific attributes, are fixed. Endogenous attributes are affected by farming practices. Crop production per hectare $y(t)$ depends on soil quality q and productive input intensity m. t denotes time. The production function is $C^{(2)}$ (twice continuously differentiable). Since soil quality exogenous attributes a are fixed, the crop production function can be written as[2]:

$$y(t) = \phi(q(s(t), a), m(t)) = f(s(t), m(t)) \tag{1}$$

$$f_s > 0, f_m > 0, f_{ss} < 0, f_{mm} < 0, \tag{2}$$

$$f_{sm} = f_{ms} \gtreqless 0, f_{ss}f_{mm} - (f_{ms})^2 > 0 \tag{3}$$

It is assumed that crop production f increases with soil quality ($f_s > 0$) and productive inputs ($f_m > 0$). However, the higher the soil quality, the slower the increase in production observed ($f_{ss} < 0$). Besides, the more productive inputs are intensively used, the smaller their positive impact ($f_{mm} < 0$). We consider the two situations of cooperation between inputs ($f_{sm} \gtreqless 0$). Cooperating inputs can be considered as inputs that work as a team (Alchian and Demsetz, 1972). The output is yielded by this team. When production factors are cooperating, their marginal cross effect on crop yield is positive. However, when their marginal cross effect on crop yield is negative, then the production factors are considered as non cooperating. In some cases, application of chemical inputs and soil quality are cooperating, when the latter is low or

[1] Here, the Grand Ouest of France is composed of four French administrative regions: Brittany, Normandy, Pays de la Loire and Poitou-Charentes (former region, now part of the Grande Aquitaine).

[2] For simplicity, soil quality endogenous attributes will be referred to as soil quality in the rest of this article.

in transition from conventional to conservation practices ($f_{sm} > 0$) (Smith et al, 2000; Mekuria and Waddington, 2002). In other cases, they can be non cooperating, for instance in cases where an additional unit of fertilizer decreases the cross marginal effect of soil quality and fertilizer on crop yield. This can also correspond to a situation where soil quality and chemical inputs are substitutes if the marginal productivity of chemical inputs decreases with higher soil quality ($f_{sm} < 0$).

Soil quality changes over time depend on a soil natural degradation factor δ and on a soil natural formation factor g. The detrimental effects of productive inputs and the positive impacts of conservation practices on soil quality changes are both considered. The soil degradation factor δ depends on the productive inputs m, considered to be soil quality–degrading practices. For instance, pesticides can have non-desirable, detrimental effects on auxiliaries, and fertilizers can increase soil acidity (Verhulst et al, 2010), thus decreasing soil productivity.

The farmer can invest in his soil quality through conservation practices u, which impact the soil regeneration factor g. Soil quality dynamics function is $C^{(2)}$ and such that:

$$\dot{s}(t) = -\delta(m(t))s(t) + g(u(t)) \tag{4}$$

$$\delta_m > 0, \delta_{mm} > 0, g_u > 0, g_{uu} < 0 \tag{5}$$

It is assumed that the natural soil formation factor g positively depends on conservation practices u, which increase soil quality ($g_u > 0$). For instance, leaving crop residues on the soil surface decreases erosion (Cutforth and McConkey, 1997; Malhi and Lemke, 2007) and increases the number of auxiliaries. The more conservation practices are implemented, the lower their positive impact on soil quality ($g_{uu} < 0$). Soil quality is all the more degraded as productive inputs are used ($\delta_m > 0$). Moreover, the detrimental impact of productive inputs on soil quality is increasing in the use of productive inputs ($\delta_{mm} > 0$).

3 Optimal soil quality investments

The farmer, owner of his land, maximizes his profits. Hence, it is assumed that the farmer has a long-term objective of land capitalization, through his soil quality. Soil dynamics processes evolve in a continuous manner, so our model is continuous. Besides, soil dynamics is slow, and for changes to be significant, has to be considered in large time horizon. Since solutions between an infinite-horizon problem and a large but finite-horizon problem do not differ significantly (Léonard and Van Long, 2002), for mathematical convenience, it is also assumed that the farmer maximizes his profits through an infinite time horizon.

Profits depend on crop yield, crop prices and practices costs.The constant marginal cost of productive input use m is denoted c_1, and the constant marginal cost associated with conservation practices u is denoted c_2. Marginal costs encompass labour and energy costs associated with each activity. The crop price p is constant. The farmer's profits can be written as:

$$\pi(t) = pf(s(t), m(t)) - c_1 m(t) - c_2 u(t) \tag{6}$$

Since the farmer considers the dynamics of his soil [3], he has the following optimization problem:

$$\underset{m,u}{\text{Max}} \int_0^{T\to\infty} e^{-rt}[pf(s(t), m(t)) - c_1 m(t) - c_2 u(t)]dt \tag{7}$$

$$\text{subject to:} \quad \dot{s}(t) \tag{8}$$

The current value Hamiltonian of this problem can be written as:

$$\tilde{H}(m, u, s, \mu) = pf(s(t), m(t)) - c_1 m(t) - c_2 u(t) + \mu \dot{s} \tag{9}$$

According to the maximum principle, the optimal paths of m, u, s and μ satisfy:

$$\tilde{H}_m = pf_m - c_1 - \mu\delta_m s = 0 \tag{10}$$

$$\tilde{H}_u = -c_2 + \mu g_u = 0 \tag{11}$$

$$\dot{\mu} - r\mu = -\tilde{H}_s \Leftrightarrow \dot{\mu} = r\mu - pf_s + \delta(m)\mu = \mu(r + \delta(m)) - pf_s \tag{12}$$

Condition (10) states that the marginal revenues obtained from using more productive inputs must be balanced by their marginal damages to soil quality, expressed as the marginal value of soil quality. Condition (11) states that conservation practices u should be implemented such that the costs of conservation inputs c_2 are equal to the additional benefits generated in terms of the marginal value of soil quality. The co-state variable μ, which is the implicit value of soil quality, has a rate of change that depends on the interest rate r, the degradation rate δ, the current soil quality implicit value μ, the crop price p and the the influence of soil quality on crop yield f_s (condition (12)). The implicit value of soil quality grows at a rate of discount and degradation minus the contribution of soil quality to current profits. $\dot{\mu}$ also depends on productive inputs through their aggravating impact on the soil degradation rate.

Management intensity and soil quality investment can be represented as implicit functions of soil quality s and marginal soil rent μ:

$$\frac{\partial m}{\partial s} = -\frac{\tilde{H}_{ms}}{\tilde{H}_{mm}} = -\frac{pf_{ms} - \mu\delta_m}{pf_{mm} - \mu\delta_{mm}s} \gtreqless 0 \tag{13}$$

$$\frac{\partial m}{\partial \mu} = -\frac{\tilde{H}_{m\mu}}{\tilde{H}_{mm}} = -\frac{-\delta_m s}{pf_{mm} - \mu\delta_{mm}s} < 0 \tag{14}$$

$$\frac{\partial u}{\partial s} = -\frac{\tilde{H}_{us}}{\tilde{H}_{uu}} = -\frac{0}{\mu g_{uu}} = 0 \tag{15}$$

$$\frac{\partial u}{\partial \mu} = -\frac{\tilde{H}_{u\mu}}{\tilde{H}_{uu}} = -\frac{g_u}{\mu g_{uu}} > 0 \tag{16}$$

Since productive inputs negatively impact soil quality, productive input use decreases with the

[3] Appendix A considers the particular case where the farmer does not consider soil quality dynamics in their decision-making process, leading to an unsustainable long-term degradation of his soil.

marginal soil rent ((14)). According to (16), soil conservation practices implementation increases with the marginal soil rent. However, a change in soil quality does not trigger a change in soil conservation practices ((15)).

When the production factors are not cooperating, (13) is negative, and the productive inputs use decreases with soil quality. However, when production factors are cooperating, the sign of (13) and more specifically the sign of H_{ms} is ambiguous: while productive inputs and soil quality are cooperating production factors, the use of productive inputs deteriorates soil quality.

When production factors are cooperating, two cases can be distinguished:
(1) The case where $H_{ms} > 0$, which can also be written as $pf_{ms} > \mu\delta_m$. This is the case where the use of productive inputs gives more benefits in terms of revenues than losses in terms of the marginal value of soil quality.
(2) The case where $H_{ms} < 0$, which can also be written as $pf_{ms} < \mu\delta_m$. This is the opposite case. It corresponds to a situation where the marginal damages on soil quality caused by productive inputs are higher than the marginal benefits in terms of productivity.

In addition, H_{ms} can be rewritten using condition (10), such that:

$$pf_{ms} - \mu\delta_m \gtreqless 0 \Leftrightarrow \frac{\partial \Pi_m}{\partial s} \gtreqless \frac{\Pi_m}{s} \tag{17}$$

where Π_m/s is the marginal profit of productive inputs m per unit of soil quality, and $\partial\Pi_m/\partial s$ is the marginal profit of productive inputs m for one additional unit of soil quality.

There can be a threshold value of soil quality $s_{\#}$ below which soil quality is sufficiently low that the cooperating marginal productivity of m and s exceeds the marginal damages of m. However, above this threshold, the marginal damages are more important than the marginal cooperating productivity. In this case, the shadow values of soil quality μ are higher than below the threshold $s_{\#}$. (see Figure 1).

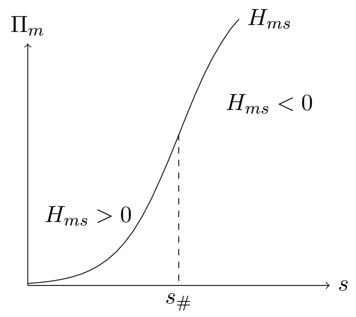

Figure 1: Soil quality threshold and the marginal productivity of productive inputs.

When $H_{ms} < 0$, the existence of the equilibrium is not ensured. This corresponds to two situations: (1) the situation wherein soil quality and productive inputs are not cooperating; (2) the situation wherein production factors are cooperating, but with benefits in terms of marginal

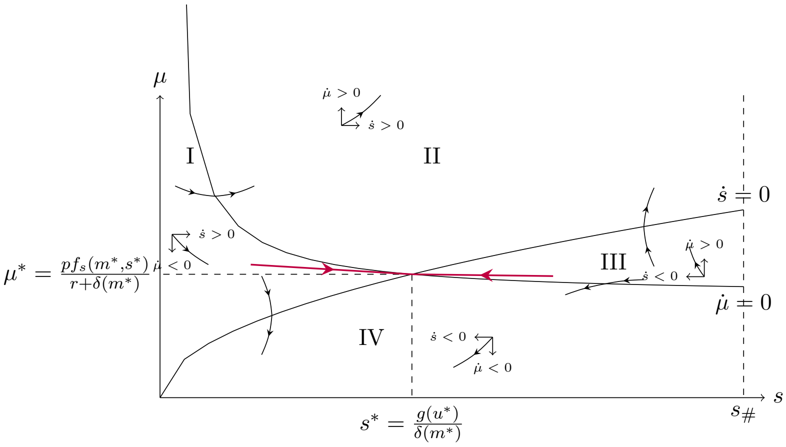

Figure 2: Phase diagram: long term equilibrium when $H_{ms} > 0$.

crop production lower than the marginal damages caused by productive inputs on soil quality.

When $H_{ms} > 0$, the long-term equilibrium can be represented in a phase diagram (see Figure 2). It is a steady state, attained through a stable transition path, or optimal strategy, departing from an initial state s_0 towards (s^*, μ^*). Stability properties and determination of the long-term equilibrium are described in Appendix B. The phase diagram corresponds to the situation where soil quality is below a soil quality threshold $s_{\#}$ and where the damages caused by the use of productive inputs are more than offset by its cooperating benefits for soil quality in terms of revenue (see Figure 1).

There are two optimal strategies leading to the steady state equilibrium. All strategies differing from these two optimal strategies move away from the steady state equilibrium.

For instance, hen the initial soil quality is low ($s_0 < s^* < s_{\#}$), the optimal trajectory is located in region I. This is a situation where the soil is of low productivity. To improve this situation, investments in soil conservation are made, which diminish as soil quality increases while soil quality value decreases ((16)). Indeed, as soil quality increases, less investment is required ((11)). Since here productive inputs and soil quality are cooperating production factors, the farmer adjusts his productive inputs to the level of soil quality ((13)).

The next section proposes an illustration of our problem through a statistical analysis of the crop production functions in the Grand Ouest of France by statistically examining the relationship between soil quality and productive inputs.

4 Empirical relationships between soil quality, crop yield and farming practices: evidence from the Grand Ouest of France

This statistical analysis is performed in the Grand Ouest of France, since it is a relative homogenous area in terms of production, climate and topography while ensuring a minimum amount of data to conduct the analysis. The analysis are peformed for the two main crops grown in the area: soft wheat and maize grain. The objective is to examine statistically the cooperating relationship between productive inputs an soil quality.

To do so, we have specified a quadratic production function that satisfies the conditions of our theoretical model. Due to data limitation, the productive inputs considered here are nitrogen (N) fertilizers inputs and phosphorus (P) fertilizers inputs. Soil organic carbon (SOC) is used as a proxy of soil quality. SOC pool is a reliable indicator of soil quality changes (Lal, 2015). We also consider the cumulative amount of rainfall during crop growth (RAIN), and the amount of soil clay, silt and sand. The preceding crops are considered (PC). Actually, for each region, we consider the percentage of surfaces for which the preceding crop is the same than the current one. It is assumed that this value is the same for the "départements" of the same region.

The proportions of soil clay, silt and sand are considered as constant. Data relative to nitrogen and phosphorus fertilizers are available for years 2001 and 2006, but not for year 2011. Since it is not enough to perform a time series analysis, we have put together data for year 2001 and 2006.

Crop yield and cumulated rainfall data are also available for years 2001 and 2006. These two years have not been particularly good in terms of yields for soft wheat and maize grain crops. The cumulated amounts of rainfall during crop growth have been obtained using data from the the French website *Infoclimat* which delivers the monthly climate in various observatories. Weather data have been collected at a "département" scale. For each crop, the cumulated amount of growth rainfall is computed by adding up the monthly cumulated rainfall of the months where rain is considered as a critical growth factor. For soft wheat, it corresponds to the months of March, April and May, and for maize, to May, June and July. For maize grain, irrigation is also considered. To do so, we used data from farming practices survey relative to the quantity of irrigation in mm and the proportion of surfaces irrigated (*Enquête sur les pratiques culturales, 2001, 2006*). Data relative to soil quality parameters are obtained from the BDAT (*Base de Données d'Analyse de la Terre*). It is a network of soil analysis measures, provided voluntary by laboratories of soil analysis. Data is available at the cantonal scale, and by five years period. Crop yields data are from the Annual Agricultural Statistics surveys for 2001 and 2006. The crop yield data are available at the "département" scale.

Farming practices data (N fertilization, P fertilization, preceding crop and irrigation) are only available at the regional scale, from surveys conducted in 2001 and 2006, by the French Ministry of Agriculture and Alimentation (DISAR plaform website) From these surveys, we have the average amount of N and P fertilizers applied on parcels that have been treated, and the percentage of surface fertilized respectively with N and P. Sine the average level of crop yield encompasses both treated and non treated parcels, we have adjusted the amount of fertilizers

	Regressions	
	Soft wheat	**Maize grain**
Non-constant variance test	$p = 0.6915$	$p = 0.2799$
Breusch-Godfrey test	$p = 0.7376$	$p = 0.3972$

Table 1: Tests on the crop production regressions: non-constant variance test and Breusch-Godfrey test.

with the ratio of parcels treated, such that:

$$NFERTI(CROP)_t = NFERTIAPPLIED(CROP)_t * \frac{FERTISURF(CROP)_t}{100} \tag{18}$$

$$PFERTI(CROP)_t = PFERTIAPPLIED(CROP)_t * \frac{FERTISURF(CROP)_t}{100} \tag{19}$$

To estimate our crop production regressions, we performed a multiple linear regression using the software R. These regressions are done for the two main crops grown in the Grand Ouest of France: soft wheat and maize grain.

For crop production regression, the regression is such that:

$$\begin{aligned} YIELD(CROP)_t = \beta_0 &+ \beta_1 CLAY + \beta_2 SILT + \beta_3 SAND + \beta_4 COOH_t \\ &+ \beta_5 NFERTI(CROP)_t + \beta_6 PFERTI(CROP)_t \\ &+ \beta_7 RAIN(CROP)_t + \beta_8 NFERTI(CROP)_t * PFERTI(CROP)_t \\ &+ \beta_9 SOC_t * NFERTI(CROP)_t + \beta_{10} SOC_t * PFERTI(CROP)_t \\ &+ \beta_{12} PC(CROP)_t \end{aligned} \tag{20}$$

SOC is expected to have a positive marginal effect on crop production. The second order effect is expected to be negative.It illustrates threshold effects, especially since a given soil has a finite storage capacity. N and P fertilizers are expected to have a positive marginal impact on crop production. The second order effect is expected to be negative: the more N or P fertilizers are applied, the lower their positive marginal effect on production. The cooperating relationship between fertilizers and SOC is the one to be closely examined here.

We have first performed a multiple linear regression, using the program “lm” of the statistical software R. We performed two tests on these regressions, to ensure that they verify the homoscedasticity condition and that there are no correlation between residues (see Table 1). According to our tests, our regressions respect the homoscedasticity condition, and they do not exhibit auto-correlation between residues. The results of our regressions are displayed in Table 2. Due to the relatively small amount of observations for each regressions, egressions are performed with the variables that explain soft wheat or maize grain yields the most (with respect to the R-squared value). This is why different explaining variables may be used for soft wheat yield and maize grain yield regressions.

As expected, soil organic carbon has a significant positive impact on crop yield for soft wheat. For maize grain yield, the impact is not statistically significant. N fertilizers are only

Explaining variables	Explained variables			
	Soft wheat yield		Maize grain yield	
	Estimate	p-value	Estimate	p-value
Intercept	42.19446	0.45087	-24.61	0.7409
Clay content in soil	-0.12235	0.06260 .	0.1666	0.1031
Silt content in soil	-0.12003	0.03649 ∗	0.1347	0.852 .
Sand content in soil	-0.15240	0.00974 ∗∗	0.05729	0.3864
Soil organic carbon (SOC)	6.95754	0.02296 ∗	-1.287	0.2108
N fertilizers inputs	1.48444	0.00966 ∗∗	0.7159	0.0102 ∗
P fertilizers inputs	-2.23053	0.02183 ∗	-1.583	0.091 .
Rain fall during growing season + irrigation	-	-	0.0945	0.0260 ∗
Percentage of surfaces where the preceding crop is the same than the current one	-0.41006	0.15677	-	-
Cross impact of N and P fertilizers inputs	-	-	0.000004654	0.9983
Cross impact of SOC and N fertilizers inputs	-0.06426	0.03089 ∗	-0.04114	0.0167 ∗
Cross impact of SOC and P fertilizers inputs	0.08291	0.10541	0.091	0.0397 ∗
Number of observations	36		26	
Multiple R-squared	0.76		0.71	
Adjusted R-squared	0.68		0.51	
Signif. codes : $0.001^{***}, 0.01^{**}, 0.05^{*}, 0.1^{\cdot}$				

Table 2: Crop production regressions results.

statistically significant positive impact on soft wheat and maize grain yields. This is consistent with our assumptions. However, P fertilizers seem to negatively impact soft wheat production. The cumulated amount of rainfall during crop growth has a positive impact on maize grain yield, which is consistent with the nature of this crop. The cooperation effect between N fertilizers and soil organic carbon is statistically significant and negative for both crops. As for the cooperation between P fertilizers and soil organic carbon, it exhibits a positive significant sign for maize grain.

One potential bias of our analysis is that farmers are likely to choose to grow crops in their high quality soils. This could explain why crop yields are positively correlated to soil quality. However, we are using crop yield data for two different years. Since farmers have an interest to practice crop rotations, even short ones, we are likely to observe also crops allocation that reflect that phenomenon, thus reducing this bias. Another potential bias is the impact of farmers' practices and choices on SOC - which is the hypothesis of our theoretical models. This could mean that soil organic carbon is endogenous in our regressions. However, here, we use soil quality parameters data respectively from 1995 to 1999 and from 2000 to 2004, to be regressed on crop yield data for respectively years 2001 and 2006, furthermore at larger scale. Hence soil quality data are not directly impacted by the farming practices or crop allocation of the years considered. As such, we can consider here that the soil quality parameters used in our regressions are not endogenous. Another bias that is not addressed here is related to the spatial autocorrelation. Neighboring cantons may have functional relationships between one another, due to a particular spatial organization of activities. For instance, a canton with a high proportion of maize grain is likely to also present a high proportion of cattle, with a relative high amount of spreading that can impact neighboring parcels located in a different cantons.

5 Cooperating inputs and non-cooperating inputs: what does it change?

In the illustrative statistical analysis proposed in the previous section, there are two different situations. N fertilizers inputs and SOC are non-cooperating inputs, and P fertilizers and SOC are cooperating inputs. Let us compare two situations: one where the cooperating inputs P and SOC leads to an equilibrium where the benefits of the cooperating relationship are higher than the damages caused to SOC; and one where the non-cooperating inputs N and SOC also lead to an equilibrium. Both situation are plausible, and the second one does not violate our mathematical conditions, though such situation cannot be demonstrated. These two situations are interesting to compare with respect to their comparative static analysis.

A comparative static analysis of this problem allows us to determine how the endogenous variables in our model would differ from the steady state equilibrium with different values of the exogenous parameters (Léonard and Van Long, 2002). In our case, the endogenous variables that characterize the optimal steady state are productive inputs m, conservation practices u, soil quality s and the soil quality implicit value μ.

The comparative static analysis that corresponds to the P fertilizers and SOC cooperation when the damages caused by the use of productive inputs are more than offset by its cooperating benefits with soil quality in terms of revenue ($H_{ms} > 0$), yields the following results (see Appendix B for the computation details):

$$m = m(\overset{-}{c_1}, \overset{-}{c_2}, \overset{+}{p}, \overset{-}{r}) \tag{21}$$

$$u = u(\overset{-}{c_1}, \overset{-}{c_2}, \overset{+}{p}, \overset{-}{r}) \tag{22}$$

$$\mu = \mu(\overset{-}{c_1}, \overset{+}{c_2}, \overset{+}{p}, \overset{-}{r}) \tag{23}$$

$$s = s(\overset{-}{c_1}, \overset{-}{c_2}, \overset{+}{p}, \overset{-}{r}) \tag{24}$$

The comparative statics with respect to N fertilizers and SOC are presented below. For this case, the impacts of parameter changes in conservation practices u and soil quality implicit value μ, are of undetermined sign.

$$m = m(\overset{-}{c_1}, c_2, \overset{+}{p}, \overset{+}{r}) \tag{25}$$

$$u = u(c_1, c_2, p, r) \tag{26}$$

$$\mu = \mu(c_1, c_2, p, r) \tag{27}$$

$$s = s(\overset{+}{c_1}, c_2, \overset{+}{p}, \overset{-}{r}) \tag{28}$$

An increase in the cost associated with productive inputs c_1 leads to an expected decrease in

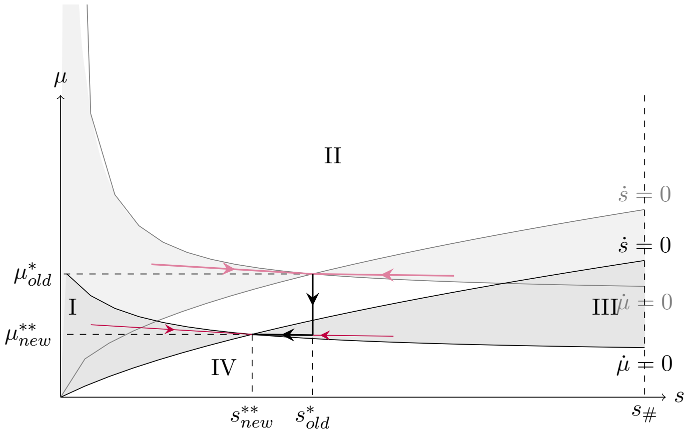

Figure 3: Phase diagram and comparative statics: impact of an increase in the cost of P fertilizers, in the case where $H_{ms} > 0$.

productive inputs and to a decrease in the equilibrium soil quality and the marginal value of soil, when the two production factors are cooperating (see Figure 3). Since the value attributed to soil quality is lower, smaller investments are made in conservation practices. However, when production factors are not cooperating, the expected decrease in productive inputs, here N fertilizers, is compensated by an increase in soil quality.

When productive inputs are cooperating, *an increase in the cost associated with soil conservation and non-productive practices* c_2 decreases investment in soil conservation. As a consequence, the optimum soil quality is lower, and the associated marginal value increases. Since productive inputs and soil quality are cooperating, the use of productive inputs associated with lower soil quality is smaller than in our original equilibrium.

An increase in the crop price p leads to an increase in soil quality and productive inputs. Indeed, the farmer faces the possibility of increasing production to attain an equilibrium where the marginal benefits of using more productive inputs equal the costs of these practices.When there is cooperation between these two variables, soil quality in equilibrium also increases. To maintain this level of soil quality, higher investment in soil conservation techniques is needed in this equilibrium. With a higher price and a higher productivity of soil quality at this optimum, the marginal soil quality is also higher. Even when production factors are not cooperating, the increase in crop price also leads to an increase in both production factors.

An increase in the discount rate r can correspond to a higher preference for the present: the farmer values present revenue more than future revenue. As a consequence, soil quality will be either more depleted or less restored by the farmer, who will be less willing to invest in soil conservation measures since the marginal value attributed to soil quality has decreased in this equilibrium. When production factors are cooperating, the level of productive inputs also decreases: the loss in soil productivity seems to be offset by lower productive inputs expenses. However, when production factors are not cooperating, productive inputs use increases, while soil quality decreases.

6 Conclusion

This article examines whether farmers have a private interest in maintaining or increasing soil quality. A particular attention is given to the cooperating relationship between productive inputs and soil quality. It explores and discusses the different optimal strategies to achieving long-term equilibrium. In addition, the dynamic elements of soil resource management problems have been characterized. An illustrative statistical analysis is proposed for the case of the Grand Ouest of France, to establish which sort of cooperating relationship can be observed at a regional level between respectively P fertilizers, N fertilizers and SOC.

The investment model proposed highlights some favorable situations for the maintenance and enhancement of soil quality. The model shows the importance of cooperation between the two production factors (soil quality and productive inputs). When production factors are co-operating *and* when the marginal cooperating productivity is higher than the marginal damages of productive inputs to soil quality, there exists a long-term optimal equilibrium with strategies that can be followed by the farmer to reach the optimum. However, when production factors are nor cooperating *or* when the marginal productivity of the cooperating inputs is lower than the marginal and detrimental impact of productive inputs on soil quality, one cannot draw conclusions about the existence of an equilibrium.

These ambiguities show that we are facing empirical issues that depend on technical interactions that are difficult to discover and control. Actually, in our statistical analysis shows that both situations can be observed at the same time. It has consequences in terms of policy. In a context where there is a public interest in preserving soil quality through carbon sequestration, public policies can be designed to favor an increase in soil quality. From our comparative statics, when production inputs are not cooperating, it is relevant to tax productive inputs costs. No feedback would be observed, it could even trigger a positive snowball effect. However, such strategy applied in P fertilizers in our example would have a negative feedback effect. Farmers might reduce their use of P fertilizers, but they would not invest in soil quality, leading to a decrease in soil quality.

A better knowledge of the cooperating relationship between soil quality parameters and productive inputs, together with a better understanding of soil quality dynamics is critical to an effective design of public policies and avoid undesirable side effects.

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Appendix A: To consider or not consider soil quality dynamics

When farmers do not consider soil quality dynamics, considering both cases, the first-order conditions of our problem can be rewritten as:

$$H_m = pf_m - c_1 = 0 \tag{29}$$

$$H_u = -c_2 = 0 \tag{30}$$

Since the farmer does not consider soil quality dynamics or the detrimental impact of productive inputs on soil quality, he does not internalize the additional cost of using productive inputs in terms of the marginal value of soil quality.
However, according to condition (30), the optimal use of conservation practices is such that the investment is equal to zero at any point in time. That is, when the dynamics of soil quality are not considered, no soil conservation investments are made. Hence, we are always in a situation of underinvestment in soil quality.

One can still expected that soil quality will attain a long-term equilibrium (Smith et al, 2000) such that:

$$\dot{s} = -\delta(m)s + g(u) = 0 \Leftrightarrow s^S = \frac{g(0)}{\delta(m^S)} \tag{31}$$

When comparing the long-term soil quality equilibrium (s^S) and not considering soil quality dynamics and the optimum soil quality level (s^*) when considering soil quality dynamics, one obtains:

$$s^S = \frac{g(0)}{\delta(m^S)} \quad \text{and} \quad s^* = \frac{g(u^*)}{\delta(m^*)} \tag{32}$$

In addition, from conditions (10) and (29) of the two optimization problems, at any point of time, and in particular, for the bundles (m^*, s^*) and (m^S, s^S), we have:

$$pf_m(m^*, s^*) - c_1 - \mu\delta_m(m^*)s^* = 0 \quad \text{and} \quad pf_m(m^S, s^S) - c_1 = 0 \tag{33}$$

$$\Leftrightarrow pf_m(m^*, s^*) - c_1 - \mu\delta_m(m^*)s^* = pf_m(m^S, s^S) - c_1 \tag{34}$$

$$\Leftrightarrow f_m(m^*, s^*) - \frac{\mu}{p}\delta_m(m^*)s^* = f_m(m^S, s^S) \tag{35}$$

due to the more complex relationship between productive inputs m and soil quality s. In addition to the cooperation effect in terms of production, the detrimental impact of productive inputs on soil quality dynamics is considered. In this second case, several situations are plausible depending on the initial soil quality.

- $m^* < m^S$ and $s^* > s^S$
  From the assumptions of our model:

$$m^* < m^S \Rightarrow \delta(m^*) < \delta(m^S) \tag{36}$$

$$g(u^*) > g(0) \Rightarrow \frac{g(u^*)}{\delta(m^*)} > \frac{g(0)}{\delta(s^S)} \tag{37}$$

$$s^* > s^S \tag{38}$$

- $m^* = m^S$ and $s^* > s^S$
  From the assumptions of our model:

$$m^* = m^S \Rightarrow \delta(m^*) = \delta(m^S) \tag{39}$$

$$g(u^*) > g(0) \Rightarrow \frac{g(u^*)}{\delta(m^*)} > \frac{g(0)}{\delta(s^S)} \tag{40}$$

$$s^* > s^S \tag{41}$$

These two cases are consistent with (35). Their interpretation is fairly intuitive: these are situations do when not consider soil quality dynamics; thus, the farmer uses productive inputs without compensating for the reduction in soil quality, which degrades his soil quality below the optimum. There is overuse of productive inputs and underuse of soil quality investments.

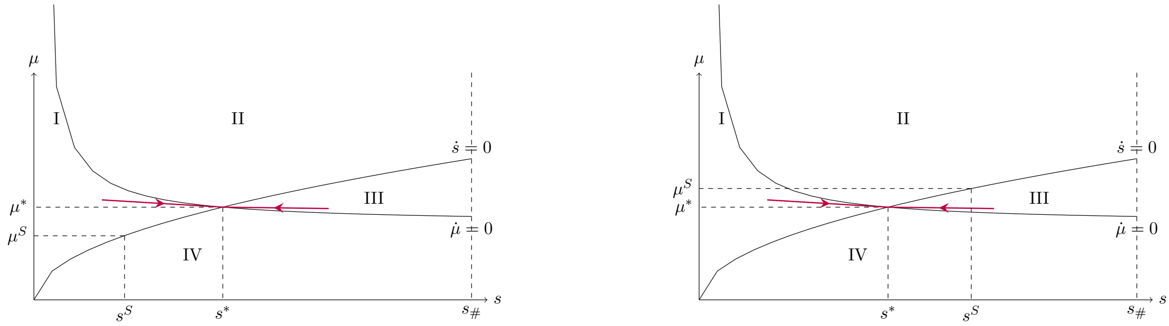
- $m^* > m^S$

$$m^* > m^S \Rightarrow \delta(m^*) > \delta(m^S) \tag{42}$$

$$g(u^*) > g(0) \Rightarrow \frac{g(u^*)}{\delta(m^*)} \gtreqless \frac{g(0)}{\delta(s^S)} \tag{43}$$

$$s^* \gtreqless s^S \tag{44}$$

This can correspond to different situations. One is where initial soil quality is above the optimum and sufficiently high for long-term soil quality to stabilize above the optimum, even when the farmer does not compensate for the impact of productive inputs on his soil. This is possible if



Phase diagram: Not considering soil quality dynamics: case 2

the farmer also use fewer productive inputs than optimal, thus causing less damage. In the other situation, initial soil quality is not sufficiently high for the reduced use of productive inputs to compensate for the lack of investment in soil quality.

In most cases, not considering soil quality dynamics leads to a long-term equilibrium level of soil quality that is lower than optimal. This can be observed when the farmer overuses or underuses productive inputs compared to the cases where the farmer considers soil quality. Indeed, in all cases, no investments are made in soil quality. The damage, whether natural or caused by the use of productive inputs, is not compensated for. In one of the situations described, a sufficiently high initial soil quality level can still lead to a long-term equilibrium of soil quality that is higher than optimal. This case corresponds to a situation where the cooperation relationship between soil quality and productive inputs is underused.

The problem is that in all cases, the long-term equilibrium of soil quality is not stable: these are situations that cross $\dot{s} = 0$, so the strategies followed by the farmer remain non-optimal, with underinvestment in soil quality leading to depletion of the resource.

Appendix B: Computation of the soil investment model

Phase diagram and stability properties of our problem: ambiguity due to the prevalence of the cooperating benefits and the marginal damages to soil quality

The long-run or steady state equilibrium of the optimal control problem is determined by the intersection of the ($\dot{\mu} = 0$) and ($\dot{s} = 0$) demarcation curves such that:

$$A(s, \mu) = \dot{\mu} \gtreqless 0$$
$$\text{if} \;\; \mu(r + \delta(m(s, \mu))) - pf_s(m(s, \mu), s) \gtreqless 0 \tag{45}$$
$$B(s, \mu) = \dot{s} \gtreqless 0$$
$$\text{if} \;\; -\delta(m(s, \mu))s + g(u(s, \mu)) \gtreqless 0 \tag{46}$$

The slopes of the stationary loci are given by:

$$\left.\frac{d\mu}{ds}\right|_{B=\dot{s}=0} = -\frac{\partial H_\mu/\partial s}{\partial H_\mu/\partial \mu} = -\frac{-\delta_m m_s s - \delta(m) + g_u u_s}{-\delta_m m_\mu s + g_u u_\mu} = -\frac{-\delta_m m_s s - \delta(m)}{-\delta_m m_\mu s + g_u u_\mu} \tag{47}$$

$$\begin{aligned}\left.\frac{d\mu}{ds}\right|_{A=\dot{\mu}=0} &= -\frac{\partial(\mu r - H_s)/\partial s}{\partial(\mu r - H_s)/\partial \mu} \\ &= -\frac{\partial(\mu(r+\delta(m(s,\mu))) - pf_s(s,m(s,\mu)))/\partial s}{\partial(\mu(r+\delta(m(s,\mu))) - pf_s(s,m(s,\mu)))/\partial \mu} = -\frac{\delta_m m_s \mu - pf_{ss} - pf_{sm} m_s}{r + \delta_m m_\mu \mu + \delta(m) - pf_{sm} m_\mu}\end{aligned} \tag{48}$$

To determine the stability properties of our problem, i.e., whether all solutions converge toward the steady state, one can evaluate the Jacobian matrix:

$$J = \begin{bmatrix} \partial\dot{s}/\partial s & \partial\dot{s}/\partial\mu \\ \partial\dot{\mu}/\partial s & \partial\dot{\mu}/\partial\mu \end{bmatrix} = \begin{bmatrix} H_{\mu s} & H_{\mu\mu} \\ -H_{ss} & r - H_{s\mu} \end{bmatrix} = \begin{bmatrix} \underset{(?)}{-\delta_m m_s s - \delta} & \underset{(+)}{-\delta_m m_\mu s + g_u u_\mu} \\ \underset{(?)}{m_s(-H_{ms}) - pf_{ss}} & \underset{(?)}{r + m_\mu(-H_{ms}) + \delta} \end{bmatrix} \tag{49}$$

at the steady sate (s^*, μ^*). Computing the trace of the Jacobian matrix, it appears that:

$$tr[J] = -\delta_m m_s s - \delta + r + m_\mu(-H_{ms}) + \delta = -m_s(\delta_m s - \delta_m s) + r = r > 0 \tag{50}$$

Since the eigenvalues of the Jacobian matrix equal its trace, at least one eigenvalue is positive, which implies that the fixed point (here, the intersection of the ($\dot{\mu} = 0$) and ($\dot{s} = 0$) demarcation curves) is not locally asymptotically stable (Caputo, 2005). If the determinant of the Jacobian matrix is negative, then the steady state is a local saddle point (Hediger, 2003; Narain and Fisher, 2006). Otherwise, if the determinant of the Jacobian matrix is positive, the steady state is an unstable node or at the center of an unstable spiral (Caputo, 2005) such that the system is not converging toward a steady state.

With a general form of the problem, that is, without specifying the functional forms of the different functions considered, the existence of an equilibrium can be found in the case where $H_{ms} > 0$. However, no conclusion can be made in the case where $H_{ms} < 0$.

When the marginal cooperating benefits are higher than the marginal damages to soil quality: Phase diagram and stability properties of our problem

In the case where $H_{ms} > 0$, which corresponds to the case where the marginal benefits of using productive inputs in terms of revenues is higher than the damages in terms of the marginal value of soil quality, there is a steady state equilibrium, since the Jacobian matrix is such that:

$$\begin{aligned}
\det J &= \begin{vmatrix} H_{\mu s} & H_{\mu\mu} \\ -H_{ss} & r - H_{s\mu} \end{vmatrix} = H_{\mu s}(r - H_{s\mu}) - H_{\mu\mu}(-H_{ss}) \\
&= (-\delta_m m_s s - \delta(m) + u_s)(r + \delta_m m_\mu \mu + \delta(m) - pf_{sm} m_\mu) \\
&\quad - (-\delta_m m_\mu s + u_\mu g_u)(\delta_m m_s \mu - pf_{ss} - pf_{sm} m_s) \\
&= (-\delta_m m_s s - \delta(m))(r + m_\mu(-H_{sm}) + \delta(m)) - (-\delta_m m_\mu s + u_\mu g_u)(m_s(-H_{ms}) - pf_{ss}) \\
&= (-\delta_m m_s s - \delta(m))(r + m_\mu(-H_{sm}) + \delta(m)) - (-\delta_m m_\mu s + u_\mu g_u)\left(\left(-\frac{H_{ms}}{H_{mm}}\right)(-H_{ms}) - pf_{ss}\right) \\
&= (-\delta_m m_s s - \delta(m))(r + m_\mu(-H_{sm}) + \delta(m)) - (-\delta_m m_\mu s + u_\mu g_u)\left(\frac{H_{ms}^2 - pf_{ss}H_{mm}}{H_{mm}}\right) \\
&= (-\delta_m m_s s - \delta(m))(r + m_\mu(-H_{sm}) + \delta(m)) \\
&\quad - (-\delta_m m_\mu s + u_\mu g_u)\left(\frac{p^2(f_{ms}^2 - f_{ss}f_{mm}) + \mu\delta_m(\mu\delta_m - 2pf_{sm}) + pf_{ss}\mu\delta_{mm}s}{H_{mm}}\right) \\
&< 0 \qquad (51)
\end{aligned}$$

From conditions (2), (3) and (5) and equations (13) to (16), given positive r and p and assuming that $H_{ms} > 0$, then $H_{\mu s} < 0$, $r - H_{s\mu} > 0$, $H_{\mu\mu} > 0$ and $H_{\mu\mu}(-H_{ss}) > 0$. From these results, the determinant of the Jacobian matrix is negative.

The slopes of the stationary loci are given by:

$$\begin{aligned}
\left.\frac{d\mu}{ds}\right|_{B=0} &= -\frac{\partial H_\mu/\partial s}{\partial H_\mu/\partial \mu} \\
&= -\frac{H_{\mu s}}{H_{\mu\mu}} > 0 \qquad (52) \\
\left.\frac{d\mu}{ds}\right|_{A=0} &= -\frac{\partial(\mu r - H_s)/\partial s}{\partial(\mu r - H_s)/\partial \mu} \\
&= -\frac{-H_{ss}}{r - H_{s\mu}} < 0 \qquad (53)
\end{aligned}$$

From conditions (2), (3) and (5) and equations (13) to (16), given positive r and p and assuming that $H_{ms} > 0$, the gradient of the ($\dot{s} = 0$) curve is positive. Given these conditions, the gradient of the ($\dot{\mu} = 0$) curve is negative.

In addition, the slopes of the trajectories in the (s, μ) space are such that:

$$\frac{d\mu}{ds} = \left(\frac{d\mu}{dt}\right) \cdot \left(\frac{dt}{ds}\right) = \frac{\dot{\mu}}{\dot{s}} \qquad (54)$$

Hence, when a trajectory goes through a locus where $\dot{\mu} = 0$, it has a slope zero, and when it goes through a locus where $\dot{s} = 0$, it has an infinite slope.

Furthermore, when $\dot{s} = 0$ and $\dot{\mu} = 0$ and in the case where the steady state is a local saddle point (which is the case when $H_{ms} > 0$), we have:

$$\left[\underbrace{\frac{\partial \dot{s}}{\partial s}}_{-} \underbrace{\frac{\partial \dot{\mu}}{\partial \mu}}_{+} - \underbrace{\frac{\partial \dot{\mu}}{\partial s}}_{+} \underbrace{\frac{\partial \dot{s}}{\partial \mu}}_{+} \right] < 0 \Leftrightarrow \frac{-\partial \dot{s}/\partial s}{\partial \dot{s}/\partial \mu} > \frac{-\partial \dot{\mu}/\partial s}{\partial \dot{\mu}/\partial \mu} \tag{55}$$

from which one can conclude that the slope of the $\dot{s} = 0$ isocline is greater than the slope of the $\dot{\mu} = 0$ isocline in the neighborhood of the steady state. This is true if and only if the steady state is a local saddle point (Caputo, 2005).

Comparative statics of case 2, when $H_{ms} > 0$

We aim to estimate the impact of a change in a given parameter; here, c_1, c_2, p and r are the costs associated with soil degrading practices m, soil quality investment (or conservation practices), crop prices and discount rates, respectively. When one parameter changes, all variables change. However, the other parameters remain fixed and have a zero differential. To study this change, we evaluate the total differentials at the original equilibrium, that is, the total differentials of the first-order conditions (FOCs) when $\dot{\mu} = \dot{s} = 0$.

The FOCs at equilibrium are such that:

$$\tilde{H}_m = pf_m - c_1 - \mu\delta_m s = 0 \tag{56}$$

$$\tilde{H}_u = -c_2 + \mu g_u = 0 \tag{57}$$

$$\tilde{H}_\mu = -\delta(m)s + g(u) = 0 \tag{58}$$

$$\dot{\mu} - r\mu = -\tilde{H}_s \Leftrightarrow \dot{\mu} = r\mu - pf_s + \delta(m)\mu = \mu(r + \delta(m)) - pf_s = 0 \tag{59}$$

The total differentials of the system are such that:

$$(pf_{mm} - \mu\delta_{mm}s)dm + 0du - \delta_m s d\mu + (pf_{ms} - \mu\delta_m)ds + f_m dp - dc_1 + 0dc_2 + 0dr = 0 \tag{60}$$

$$0dm + \mu g_{uu}du + g_u d\mu + 0ds + 0dp + 0dc_1 - dc_2 + 0dr = 0 \tag{61}$$

$$-\delta_m s dm + g_u du + 0d\mu - \delta(m)ds + 0dp + 0dc_1 + 0dc_2 + 0dr = 0 \tag{62}$$

$$(\mu\delta_m - pf_{sm})dm + 0du + (r + \delta(m))d\mu - pf_{ss}ds - f_s dp + 0dc_1 + 0dc_2 + \mu dr = 0 \tag{63}$$

The determinant of the matrix of the system, denoted B is positive:

$$|B| = \begin{vmatrix} pf_{mm} - \mu\delta_{mm}s & 0 & -\delta_m s & pf_{ms} - \mu\delta_m \\ 0 & \mu g_{uu} & g_u & 0 \\ -\delta_m s & g_u & 0 & -\delta \\ \mu\delta_m - pf_{sm} & 0 & r+\delta & -pf_{ss} \end{vmatrix} = \mu g_{uu} \begin{vmatrix} H_{mm} & -\delta_m s & H_{ms} \\ -\delta_m s & 0 & -\delta \\ -H_{ms} & r+\delta & -pf_{ss} \end{vmatrix}$$

$$- g_u \begin{vmatrix} H_{mm} & -\delta_m s & H_{ms} \\ 0 & g_u & 0 \\ -H_{ms} & r+\delta & -pf_{ss} \end{vmatrix}$$

$$= \mu g_{uu} \left(\delta_m s \begin{vmatrix} -\delta_m s & -\delta \\ -H_{ms} & -pf_{ss} \end{vmatrix} - (r+\delta) \begin{vmatrix} H_{mm} & H_{ms} \\ -\delta_m s & -\delta \end{vmatrix} \right) - g_u \left(g_u \begin{vmatrix} H_{mm} & H_{ms} \\ -H_{ms} & -pf_{ss} \end{vmatrix} \right)$$

$$= \mu g_{uu}(\delta m_s(\delta_m s p f_{ss} - H_{ms}\delta) - (r+\delta)(-H_{mm}\delta + H_{ms}\delta_m s)) - g_u^2(-H_{mm}H_{ss} + H_{ms}^2)$$

$$= \mu g_{uu}((\delta_m s)^2 H_{ss} - \delta_m s \delta H_{ms} + (r+\delta)H_{mm}\delta - (r+\delta)H_{ms}\delta_m s) - g_u^2(-H_{mm}H_{ss} + H_{ms}^2) > 0 \tag{64}$$

Applying Cramer's rule, we obtain the following comparative statics for the case where the damages caused by the use of productive inputs are more than offset by their cooperating benefits with soil quality in terms of revenue ($H_{ms} > 0$):

$$m = m(\overset{-}{c_1}, \overset{-}{c_2}, \overset{+}{p}, \overset{?}{r}) \tag{65}$$

$$u = u(\overset{-}{c_1}, \overset{-}{c_2}, \overset{+}{p}, \overset{-}{r}) \tag{66}$$

$$\mu = \mu(\overset{-}{c_1}, \overset{+}{c_2}, \overset{+}{p}, \overset{-}{r}) \tag{67}$$

$$s = s(\overset{-}{c_1}, \overset{-}{c_2}, \overset{?}{p}, \overset{?}{r}) \tag{68}$$

Using this method, some impacts are ambiguously signed. Hence, an alternative methodology is used to determine the impacts of changes in the discount rate and in the crop price on the steady state. Indeed, it is not the FOCs that are taken into account but the ($\dot{s} = 0$) and ($\dot{\mu} = 0$) equations, while using the expressions of m and u as implicit functions of soil quality s and marginal soil quality μ.

Hence, we have the following set of equations:

$$\dot{s} = H_\mu = -\delta(m^*(s,\mu)) + g(u^*(s^*,\mu^*)) = 0 \tag{69}$$

$$\dot{\mu} = r\mu - H_s = \mu^*(r + \delta(m^*(s,\mu))) - pf_s(m^*(s,\mu), s) = 0 \tag{70}$$

Differentiating the system with respect to s, μ, p and r yields:

$$(-\delta_m m_s - \delta(m) + g_u u_s)ds + (g_u u_\mu - \delta_m m_\mu s)d\mu + 0dr + 0dp = 0 \tag{71}$$

$$(\mu\delta_m m_s - pf_{sm}m_s - pf_{ss})ds + (r + \delta(m) + \mu\delta_m m_\mu - pf_{sm}m_\mu)d\mu + \mu dr - f_s dp = 0 \tag{72}$$

Only considering changes in r gives the following system:

$$\begin{bmatrix} (-\delta_m m_s - \delta(m)) & (g_u u_\mu - \delta_m m_\mu s) \\ (-H_{ms} m_s - p f_{ss}) & (r + \delta - m_\mu H_{ms}) \end{bmatrix} \begin{bmatrix} ds/dr \\ d\mu/dr \end{bmatrix} = \begin{bmatrix} 0 \\ -\mu \end{bmatrix} \tag{73}$$

Applying Cramer's rule yields the following results:

$$\frac{ds}{dr} = \frac{\begin{vmatrix} 0 & (g_u u_\mu - \delta_m m_\mu s) \\ -\mu & (r + \delta(m) - m_\mu H_{ms}) \end{vmatrix}}{|J|} = \frac{\mu(g_u u_\mu - \delta_m m_\mu s)}{|J|} < 0 \tag{74}$$

$$\frac{d\mu}{dr} = \frac{\begin{vmatrix} (-\delta_m m_s - \delta(m)) & 0 \\ (-H_{ms} m_s - p f_{ss}) & -\mu \end{vmatrix}}{|J|} = \frac{\mu(\delta_m m_s s + \delta(m))}{|J|} < 0 \tag{75}$$

Similarly, only considering changes in p yields the following system:

$$\begin{bmatrix} -\delta_m m_s - \delta(m) & g_u u_\mu - \delta_m m_\mu s \\ -H_{ms} m_s - p f_{ss} & r + \delta - m_\mu H_{ms} \end{bmatrix} \begin{bmatrix} ds/dr \\ d\mu/dr \end{bmatrix} = \begin{bmatrix} 0 \\ f_s \end{bmatrix} \tag{76}$$

Applying Cramer's rule yields the following results:

$$\frac{ds}{dp} = \frac{\begin{vmatrix} 0 & g_u u_\mu - \delta_m m_\mu s \\ f_s & r + \delta(m) - m_\mu H_{ms} \end{vmatrix}}{|J|} = \frac{-f_s(g_u u_\mu - \delta_m m_\mu s)}{|J|} > 0 \tag{77}$$

$$\frac{d\mu}{dp} = \frac{\begin{vmatrix} -\delta_m m_s - \delta(m) & 0 \\ -H_{ms} m_s - p f_{ss} & f_s \end{vmatrix}}{|J|} = \frac{f_s(\delta_m m_s s + \delta(m))}{|J|} > 0 \tag{78}$$

The comparative statics for the case where the damage caused by the use of productive inputs is more than offset by its cooperating benefits with soil quality in terms of revenue ($H_{ms} > 0$) are the following:

$$m = m(\overset{-}{c_1}, \overset{-}{c_2}, \overset{+}{p}, \overset{-}{r}) \tag{79}$$

$$u = u(\overset{-}{c_1}, \overset{-}{c_2}, \overset{+}{p}, \overset{-}{r}) \tag{80}$$

$$\mu = \mu(\overset{-}{c_1}, \overset{+}{c_2}, \overset{+}{p}, \overset{-}{r}) \tag{81}$$

$$s = s(\overset{-}{c_1}, \overset{-}{c_2}, \overset{+}{p}, \overset{-}{r}) \tag{82}$$