Research incentives and tradeoff for improving productivity of different crops

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Abstract

This paper analyzes the crop diversification chosen by farmers in a context where they have to buy one input from a monopoly seed supplier. The analysis is restricted to a case with two crops, one being more productive compared to the other. A congestion effect is included on the demand side: the interest for one crop decreases as the proportion of this crop increases. If this effect is high enough, the first best is to grow both crops with a higher proportion of the most productive one. In such a case, the pricing strategy of the seed supplier leads to an equilibrium that is qualitatively different. Depending on the parameters, the equilibrium is such that we have either one crop or two crops with a larger proportion of the less productive one. This dichotomy depends more drastically on the congestion effect when the research incentives of the monopoly are taken into account.

1 Introduction

Crop diversification is at the core of agricultural economics issues. It is expected that a more productive crop should be more often used in crop rotations and consequently in cropping-plans. However this higher frequency may be curbed, in particular because it generally leads to more frequent pest problems as well as to decreasing output prices. There is consequently an interest for having crops with relatively similar productivity levels in order to maintain a minimum level of crop diversification (Meynard et al. 2013). In this paper we focus more particularly on crop genetic improvement, which is an important determinant of crop productivity. We more particularly analyze the pricing strategy and research incentives for various crops.

This economic issue is related to the economic literature on the drivers of innovation. The survey by Cohen (2010) shows that these drivers are related to industry structure, appropriability (e.g., Intellectual Property Rights), demand (e.g., market size) and technological opportunities. In this paper, we are more particularly interested in market size. Empirical analysis suggests that research investment should be modeled as a fixed cost, so that firms want to invest in large markets to better cover this cost. Recent empirical analysis applied to the pharmaceutical industry shows a positive relationship between product innovation (new drugs) and the market size associated with different type of diseases, or drug classes (Acemoglu et Linn 2000, Dubois et al. 2011). In agriculture, and more particularly seed supply, Charlot et al. (2015) show that the market size for different cash crops has a positive effect on the number of new products (seed varieties) introduced each year (they measure market size by crop acreage and a dummy related to crop with hybrid seed). Hence, as one crop becomes more frequently used by farmers, we can expect seed companies to invest more in research for this crop. And this leads consequently to more unbalanced productivity levels between the crops. The lack of R&D in crops with relatively small market size is thus likely to increase the productivity difference between seeds, possibly creating orphan markets, i.e.: markets where very few innovations occur.

In this paper, we model a situation with a representative farmer allocating land across two crops. This allocation is determined by the seed productivity and price of each crop. Hence, the allocation of the farmer determines a demand system for seeds. Seeds are assumed to be supplied by a monopoly who sequentially chooses research investment (that determines seed productivity) and seed prices. The two crops are substitutes. However, an important assumption is that, as one crop becomes more frequently used, the farmer faces more important crop protection problems for this crop (leading to yield damages or spendings on pesticide to decrease this damage). This is actually a *congestion effect*. As for a golf club (Hart, 1996), the larger the number of members using the green, the lower the value of the club for a new member. The latter would prefer another club if this one was saturated by players. This issue can be solved by increasing the membership price to deter the entrance of new members. Here, the structure of our model is different, in the sense that the congestion effect does not depend on the number of a good's users, but on the good's market share. Total market size (i.e. the land available to the farmer) is indeed fixed, so that the market is shared between the two crops. Standard literature on public goods, such as Atkinson and Stiglitz (1987), presents the congestion effect as a decrease in a good's value when its users become too numerous, which generates a negative externality on each user. This congestion effect, by creating rivalry, can also transform an (excludable) "club-good" into a "private good", or a (non excludable) "public good" into a "common pool resource".

Our analysis shows different important results. First, we show that that, if the congestion effect is important enough, the first best is to grow both crops, with a larger proportion of the most productive one. The monopoly can decide prices that lead to this first best proportion, but this pricing is not optimal for the monopoly because it leaves important rent to the farmer. If the the yield potential of the two crops are given, we show that the two crops remain on the market only if the difference between the yield potential of the two crops is small or if the congestion effect is high. Conversely, a weak congestion effect would create an orphan market to the benefit of the most productive crop. In this latter case, it is not profitable for a farmer to buy the less productive crop anymore. With a large enough congestion effect, the seed supplier changes drastically its strategy and prices the most productive seed at a high price, leading to a lower proportion of this seed compared to the less productive one. In both case, the rent let to the farmer is very low or even equal to 0. Hence the monopoly pricing strategy is qualitatively different from the first best. If the potential productivity is endogenous and related to a research investment chosen by the monopoly, we show that the zone of parameters leading to one crop or two crops are different. More precisely, the limit between the two zones depends more drastically on the congestion effect.

The structure of the paper is as follows. The general model is set out in section 2. The first best and the equilibrium of the game without R&D is derived in section 3. Section 4 analyzes the research incentives and their effect on the proportion of crops at the equilibrium. Section 5 concludes the paper.

2 The model

The problem examined in this paper is modeled as a three-stage game. In the very first stage, a monopoly determines the optimal level of R&D to allocate to two seeds, say A and B, in order to improve their productivity. He then chooses the seed prices that maximize its profit. In the last stage, a representative farmer chooses the optimal partition between the two crops, given that each seed is an input to produce each crop, and that the partition affects each crop's final productivity. We solve the game by backward induction, starting with the last stage of the game. When analyzing the second (pricing) stage, we suppose, without loss of generality, that the crop A is more productive than the crop B.

The farmer can produce two outputs, corresponding to the two crops A or B. For this purpose he needs to use one input, a seed with price w_i, $i \in \{A; B\}$ for each crop. The maximum, potential, revenue generated by one crop is y_i. This is the revenue one would obtain if there was no congestion effect. In the rest of this paper y_i will be called the *potential productivity.*[1] Crop A is more productive than crop B (under equal market shares), so $y_A \geq y_B$. We also assume that the aggregate market size is normalized to 1 and that the market is fully covered with the two crops. We denote $\theta \in [0; 1]$ the share of the market covered by crop A ($1 - \theta$ is the shared covered by B).

The core assumption of our model is that there is a "congestion" effect: for each crop, a fraction of the potential productivity y_i is lost when this crop is increasingly used. More precisely, we define a damage function $k_i \in [A; B]$, which is increasing with the proportion of the crop i. We assume that this increase is linear and proportional to y_i. The farmer thus faces a damage function equal to $k_A = y_A \alpha \theta$ for crop A and $k_B = y_B \alpha (1 - \theta)$ for crop B (with $\alpha \in [0; 1]$). This is a fundamental assumption of our model: the larger the adoption of a specific crop, the lower its productivity. This effect is frequently observed in agricultural economics, especially when a lack of crop rotation leads to pest adaptation[2], and to productivity decrease. When $\alpha = 1$, each crop faces a full damage function (if $\theta = 1$ the farmer looses all revenues, i.e, the congestion effect is total), while when $\alpha = 0$, we observe no effect of the proportion of

[1] Here y_i is an aggregate measure of a revenue generated by the sale of one crop, gross of the congestion effect. It takes into account the performance of the seed, as well as the output price of the crop. An assumption in our model is that the increase in productivity for one crop has no impact on output prices.

[2] For a more precise and detailed analysis of this issue, see the Meynard (2013).

the crop. It is important to note that the damage reduces multiplicatively the revenues generated by the crop. Yield loss in agriculture are frequently defined in relative term (such disease generated a $x\%$ yield loss). Indeed, if a disease appears in a field where the yield potential is, say, 1 ton/ha, the yield loss will be much lower compared to a field where the yield potential is twice as large (i.e. 2 tons/ha).

The profit of the farmer is thus given by:

$$\pi^F = \theta \cdot (y_A(1-\alpha\theta) - w_A) + (1-\theta) \cdot (y_B(1-\alpha(1-\theta)) - w_B) \tag{1}$$

Hence, the only decision taken by the farmer, seed prices being given, is to choose the proportion of crop A, θ, to maximize profits.

Consider now the seed supplier, who acts as a monopoly on the whole market. The demand for seed A is θ, while that for B is $1-\theta$. We assume that the marginal cost of production can be normalized to 0 for both seeds. Thus, the monopolistic supplier's (gross) profit is:

$$\pi^M = \theta w_A + (1-\theta) w_B \tag{2}$$

At the second stage (y_A and y_B being exogenous) the monopoly have to find the optimal prices (w_A, w_B) that maximize its gross profit. The monopoly faces participation constraints for the farmer, as we assume that he has to make a positive profit for each of the two crops. One important assumption here is that that monopoly is not allowed to sell a bundle of both seed with given proportion at one price. This assumption is ralistic with reference to the seed market where seed for different crop are sold separately.[3].

Figure 1 illustrates the gain of both the farmer and monopoly as well as total welfare. The rectangle areas located at the top are the farmer's profit on each crop, while the rectangle areas located at the bottom are the monopoly's profit. As we will see, the monopoly pricing will have to maximize the sum of the two downer areas, taking into account the partition θ chosen by the farmer, as well as the participation constraints.

3 Results with exogenous potential productivity

This section presents the results obtained for given seeds' potential productivity. The next section is devoted to the equilibrium when this potential are chosen in the very first stage by the monopoly, *via* R&D efforts.

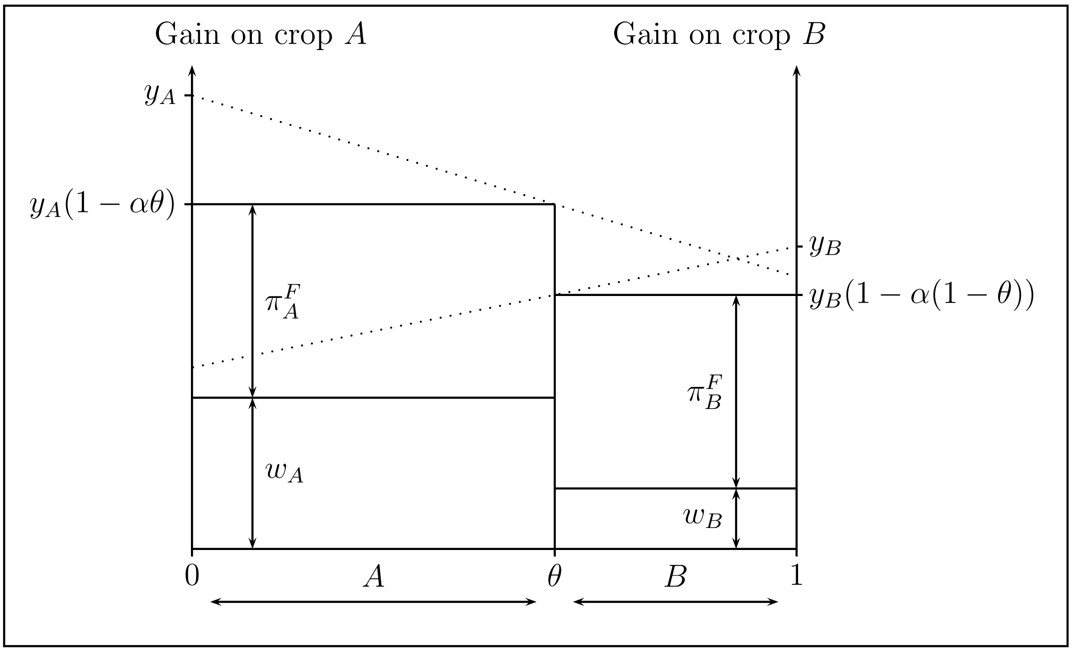
3.1 First best

For given potential productivities y_A and y_B, total welfare depends only on θ:

$$W = \theta y_A(1-\alpha\theta) + (1-\theta) y_B (1-\alpha(1-\theta))$$

[3]If the monopoly could sell a bundle of both seed for which he decides both the proportion and the price, then he can capture the whole welfare. His strategy then leads to the first best

Figure 1: Illustration of farmer and monopoly gains on each crop



Total welfare is maximum for $\theta = \theta^{FB}$ where:

$$\theta^{FB} = \frac{y_B}{y_A + y_B} + \frac{y_A - y_B}{2\alpha(y_A + y_B)} \tag{3}$$

Notice that $\theta^{FB} < 1$ only if $\alpha > (y_A - y_B)/(2y_A)$. In other words, with a very small congestion effect, the first best is to have only crop A. Conversely, with a strong enough congestion effect, the first best corresponds to an interior solution with both crop A and crop B. Notice also that θ^{FB} can be rewritten as follows:

$$\theta^{FB} = \frac{1}{2} + \frac{1-\alpha}{2\alpha} \cdot \frac{y_A - y_B}{y_A + y_B}$$

Hence $\theta^{FB} \geq 1/2$ which means that, despite the congestion effect, the collective interest is to grow a larger share the most productive crop (A).

It should be observed also that, at the first best, the per unit welfare is greater for crop A than for crop B.

$$y_A(1 - \alpha \cdot \theta^{FB}) > y_B(1 - \alpha \cdot (1 - \theta^{FB}))$$

This result can be established by direct computation.[4]

[4] $y_A(1 - \alpha \cdot \theta^{FB}) - y_B(1 - \alpha \cdot (1 - \theta^{FB})) = (y_A - y_B)/2$.

To show that differently, suppose that the farmer chooses a value of θ such that the per unit welfare generated is identical for the two crops:

$$y_A(1-\alpha\cdot\theta) = y_B(1-\alpha\cdot(1-\theta)) \qquad \Leftrightarrow \qquad \theta = \frac{1}{2} + \frac{2-\alpha}{2\alpha}\cdot\frac{y_A-y_B}{y_A+y_B}$$

It can be shown that welfare can then be increased by decreasing θ. To do so we can compute:

$$\begin{aligned}\frac{dW}{d\theta} &= \frac{d}{d\theta}(\theta y_A(1-\alpha\cdot\theta)) + \frac{d}{d\theta}((1-\theta)y_B(1-\alpha\cdot(1-\theta)))\\ &= y_A(1-\alpha\cdot\theta) - \theta\alpha y_A - y_B(1-\alpha\cdot(1-\theta)) + (1-\theta)\alpha y_B\\ &= -\theta\alpha y_A + (1-\theta)\alpha y_B < 0\end{aligned}$$

We indeed have $\theta\alpha y_A > (1-\theta)\alpha y_B$ because $\theta > 1/2$ and $y_A > y_B$. Finally, if we decrease θ welfare increases because (i) we have a positive per unit gain on crop A that is greater than the per unit loss on crop B; and (ii) the (per unit) gain on crop A applies to a larger number of units compared to B (because $\theta > 1/2$).

This result can be summarized as follows:

Lemma 1. *At the first best allocation, the per unit welfare is higher for the crop with the highest potential productivity.*

3.2 The farmer's allocation of land across crops

In the last stage of the game, the farmer chooses the profit-maximising crop partition $\tilde{\theta}$. The farmer's profit is given by (1), which is concave in θ (so that the second-order conditions are met[5]). The first order condition yields:

$$\tilde{\theta} = \frac{y_B}{y_A+y_B} + \frac{(y_A-w_A)-(y_B-w_B)}{2\alpha(y_A+y_B)}$$

This value corresponds to an interior solution (i.e. $\tilde{\theta} \in [0,1]$) if $w_A - w_B \in [y_A - y_B - 2\alpha y_A, y_A - y_B - 2\alpha y_B]$. Otherwise, the farmer only produces crop A (i.e. $\theta = 1$) if $w_A - w_B < y_A - y_B - 2\alpha y_A$ and, conversely, only produces crop B (i.e. $\theta = 0$) if $w_A - w_B < y_A - y_B + 2\alpha y_B$. This result is illustrated in figure 2. Notice that the higher is α the larger is the price interval leading to an interior solution. In other words, a weaker congestion effect favors a bang-bang solution with either $\theta = 0$ or $\theta = 1$.

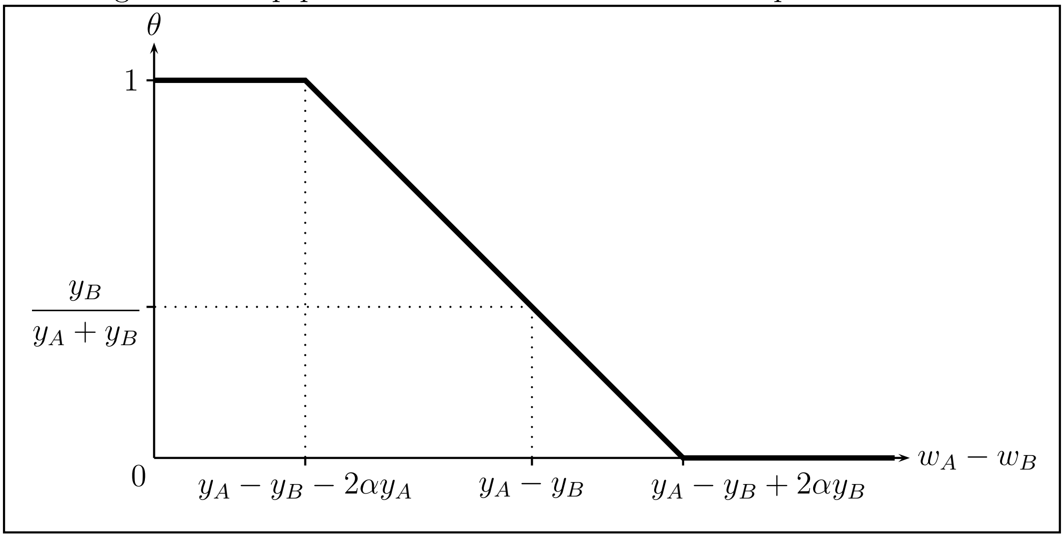
We now describe some properties of $\tilde{\theta}$ when it corresponds to an interior solution.

From the expression of $\tilde{\theta}$, it is clear that the two products are substitutes. Indeed, by computing the cross-price elasticity one can observe, for instance with a variation of the price of seed B, that

$$e_{w_B} = \frac{\partial\tilde{\theta}}{\partial w_B} \times \frac{w_B}{\tilde{\theta}} = \frac{w_B}{(y_A-w_A)-(y_B-w_B)+2\alpha y_B} > 0,$$

[5]We have : $\frac{\partial^2\pi^F}{\partial\theta^2} = -2\alpha y_A - 2\alpha y_B < 0$

Figure 2: Crop partition as a function of the seed price difference



since the latter quantity is positive. The same reasoning can be made on the price of seed A, and we find:

$$e_{w_A} = \frac{\partial(1-\tilde{\theta})}{\partial w_A} \times \frac{w_A}{1-\tilde{\theta}} = \frac{w_A}{(y_A - w_A) - (y_B - w_B) + 2\alpha y_A} > 0.$$

Thus, raising the price of seed A would increase the demand for product B, and *vice-versa*.

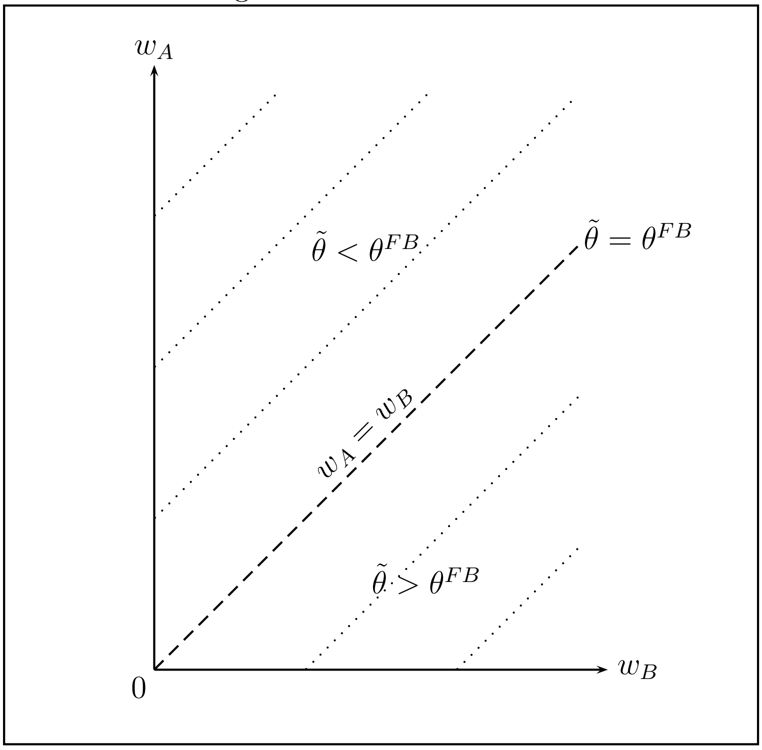
Note also that any pricing decision (w_A, w_B) such that $w_A = w_B + \Delta$ (with any positive or negative value Δ) leads to the same value of $\tilde{\theta}$ and consequently the same welfare level. In other words, partition $\tilde{\theta}$ and welfare depend only on $\Delta = w_A - w_B$. We can see that $\tilde{\theta} = \theta^{FB}$ if $w_A = w_B$, $\tilde{\theta} < \theta^{FB}$ if $w_A > w_B$ and $\tilde{\theta} > \theta^{FB}$ if $w_A < w_B$.

These results are illustrated in figure 3. All points leading to a given value of $\tilde{\theta}$ and of total welfare level are on the same (iso-welfare) line. The first best θ^{FB} corresponds the specific line such that $w_A = w_B$. Note also that the points on the same iso-welfare line differ by the sharing of the surplus between the monopoly and the farmer. As we move toward the north-est part of the graph, prices increase, leading to a higher profit for the monopoly and lower profit for the farmer.

3.3 The seeds pricing decision of the monopoly

The monopoly's pricing decision can lead the farmer to grow either two crops ($\theta \in]0, 1[$) or only one crop ($\theta = 0$ or 1). Hence, several equilibrium candidates are possible, each of them corresponding to an optimization by the monopoly under some participation constraint(s) of the farmer (i.e. non negative profit with each crop). Two equilibrium candidates are possible when the two crops are cultivated; they will be denoted $2C$ and

Figure 3: Iso-welfare lines



$2I$. One equilibrium candidate is possible when only one crop is cultivated, and will be denoted $1A$.

Consider first the cases with cultivation of the two crops. The participation constraints of the farmer, that are met for both crops, translate into constraints on the monopoly prices.

$$y_A(1-\alpha\tilde{\theta}) \geq 0 \quad \Leftrightarrow \quad w_A \leq w_A^{max}$$
$$\text{with:} \quad w_A^{max} = \frac{y_A(y_A+(3-2\alpha)y_B)}{y_A+2y_B} - \frac{y_A}{y_A+2y_B} \cdot w_B$$

$$y_B(1-\alpha(1-\tilde{\theta})) \geq 0 \quad \Leftrightarrow \quad w_B \leq w_B^{max}$$
$$\text{with:} \quad w_B^{max} = \frac{y_B(y_B+(3-2\alpha)y_A)}{y_B+2y_A} - \frac{y_B}{y_B+2y_A} \cdot w_A$$

Figure 4 illustrates these maximum values w_A^{max} and w_B^{max}. Note that, for any difference $\Delta = w_A - w_B$ (i.e. for any $\tilde{\theta}$), the monopoly increases its profit by increasing both w_A and w_B by the same amount. This price change does not affect total welfare ($\tilde{\theta}$ is constant) and only affects its sharing between the monopoly and the farmer. Hence, the monopoly is better off moving north-est along the line $w_A = w_B + \Delta$ until he binds one of the two participation constraints, $w_A \leq w_A^{max}$ or $w_B \leq w_B^{max}$ (or both of them).

Equilibrium candidate $2C$. One equilibrium candidate for the optimal monopoly price is the point where the two constraints $w_A \leq w_A^{max}$ and $w_B \leq w_B^{max}$ are binding (see figure 5). The corresponding pricing is:

$$w_A^{2C} = y_A - \alpha \cdot \frac{y_A y_B}{y_A+y_B} \qquad \text{and} \qquad w_B^{2C} = y_B - \alpha \cdot \frac{y_A y_B}{y_A+y_B}$$

With these prices, the farmer gains no profit so that the monopoly captures all the welfare. For the monopoly, this equilibrium candidate attempts at maximizing its profit by minimizing the surplus left to the farmer.

The proportion of crop A is then:

$$\tilde{\theta}^{2C} = \frac{y_B}{y_A+y_B}$$

We have $\tilde{\theta}^{2C} \in [0,1]$. Note that $w_A^{2C} - w_B^{2C} = y_A - y_B > 0$, so that $\tilde{\theta}^{2C} < \theta^{FB}$. Hence, although the monopoly appropriates it, total welfare is not maximum with this candidate $2C$.

Equilibrium candidate $2I$. This second candidate is one in which the monopoly binds one of the two constraints by choosing the highest input price for that crop (i.e. either $w_A = w_A^{max}$ or $w_B = w_B^{max}$) and chooses the price for the second crop in order to maximize its profit. The price of the second crop corresponds to an interior solution,

Figure 4: Participation constraints if the farmer grows two crops

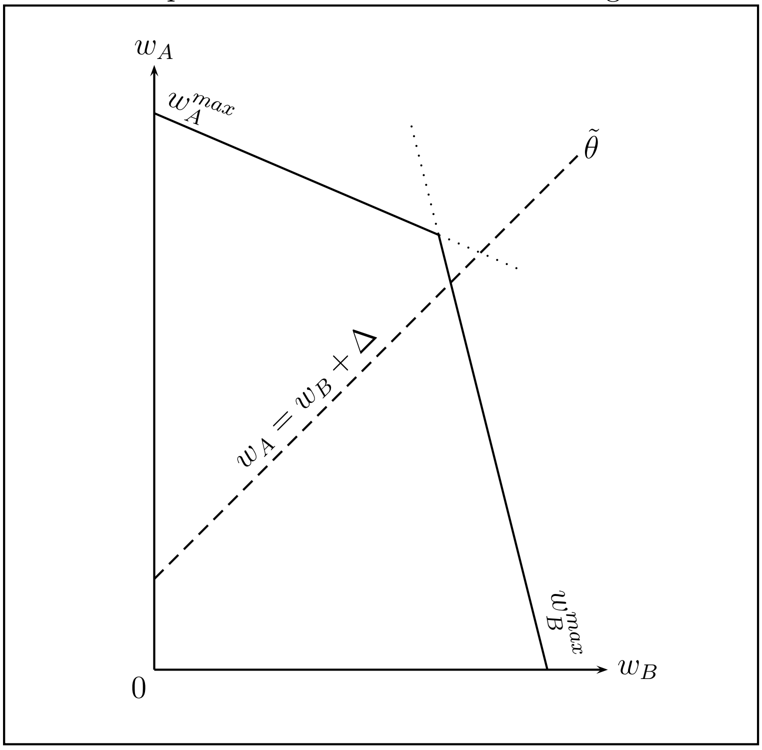
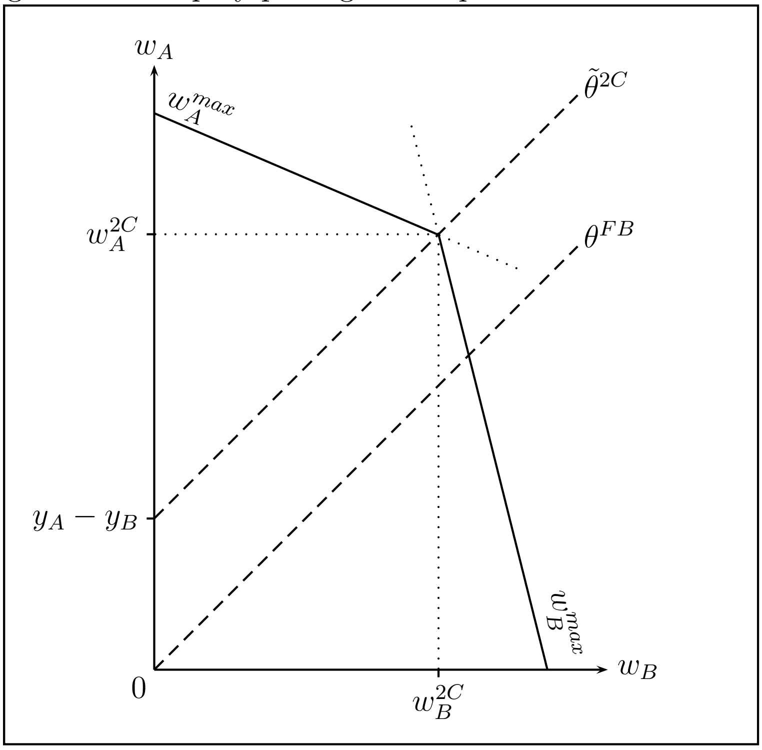


Figure 5: Monopoly pricing with equilibrium candidate $2C$



lower than the maximum level, so that the farmer earns some profit on this crop. The monopoly does not appropriate the whole surplus.

We can show first that the interest of the monopoly is to bind the constraint on crop B (i.e. $w_B^{2I} = w_B^{max}$). More precisely, we can show that for any pricing that binds the constraint on crop A (i.e. such that $w_A = w_B^{max}$), the monopoly's profit is lower compared to the candidate $2C$. This result can be understood graphically from figure 5. If only the constraint on crop A is binding, then we have $w_A - w_B > y_A - y_B$ and $\tilde{\theta} < \tilde{\theta}^{2C}$. Hence (i) total welfare is lower compared to that under candidate $2C$ (i.e. $\tilde{\theta}$ is further away from θ^{FB} compared to $\tilde{\theta}^{2C}$) and (ii) some profit is left to the farmer because only one constraint is binding. Hence the monopoly profit is lower than with candidate $2C$ because the total welfare is lower and the profit left to the farmer is higher.

The equilibrium candidate is therefore such that $w_B = w_B^{max}$. After introducing this constraint in the monopoly profit, we can maximize the monopoly profit with respect to w_A. Profit maximization leads to:

$$w_A^{2I} = \frac{2y_A^2 + (5-2\alpha)y_A y_B + (1+\alpha)y_B^2}{4(y_A + y_B)}$$

When introducing w_A^{2I} in w_B^{max}, we get:

$$w_B^{2I} = \frac{(5-4\alpha)y_A y_B + (3-\alpha)y_B^2}{4(y_A + y_B)}$$

With such pricing, the proportion of crop A is:

$$\tilde{\theta}^{2I} = \frac{3}{4} \cdot \frac{y_B}{y_A + y_B} + \frac{y_A - y_B}{4\alpha(y_A + y_B)}$$

This candidate requires that $\tilde{\theta}^{2I} \in [\tilde{\theta}^{2C}, 1]$.

$$\tilde{\theta}^{2I} > \tilde{\theta}^{2C} \quad \Leftrightarrow \quad \alpha < \alpha_{max}^{2I} \quad \text{with:} \quad \alpha_{max}^{2I} = \frac{y_A - y_B}{y_B}$$

and

$$\tilde{\theta}^{2I} < 1 \quad \Leftrightarrow \quad \alpha > \alpha_{min}^{2I} \quad \text{with:} \quad \alpha_{min}^{2I} = \frac{y_A - y_B}{4y_A + y_B}$$

Hence, this equilibrium candidate requires that $\alpha \in [\alpha_{min}^{2I}, \alpha_{max}^{2I}]$.

Equilibrium candidate $1A$. Consider now the equilibrium candidate that leads the farmer to grow only one crop. It is straightforward that this crop should be the one with the highest potential productivity (i.e. A). The farmer gain is $\pi^F = y_A(1-\alpha) - w_A$. The monopoly defines a price that binds the participation constraint of the farmer ($w_A = y_A(1-\alpha)$). Seed price w_B should be high enough that the farmer grows only crop A. With reference to figure 2, we can see that we should have $w_A - w_B < y_A - y_B - 2\alpha y_A$ which is equivalent to $w_B > y_B + \alpha y_A$. In summary, the candidate $1A$ is such that $w_A^{1A} = y_A(1-\alpha)$ and $w_B^{1A} > y_B + \alpha y_A$.

Table 1: Synthesis of equilibrium candidates with monopoly pricing

	$2C$	$2I$	$1A$
$\alpha < \alpha^{2I}_{min}$	X	$\emptyset$	X
$\alpha^{2I}_{min} < \alpha < \alpha^{2I}_{max}$	X	X	X
$\alpha > \alpha^{2I}_{max}$	X	$\emptyset$	X

Comparison of equilibrium candidates. Table 1 provides a synthesis of the conditions for applying the three equilibrium candidates. We start by comparing them by pairs, and then synthesize to define the equilibrium.

First, whenever candidate $2I$ is feasible (i.e. $\alpha \in [\alpha^{2I}_{min}, \alpha^{2I}_{max}]$), it leads to a higher profit compared to candidate $2C$.[6]

Second, if we compare the candidates $2I$ and $1A$, we have:

$$\pi^M(w_A^{2I}, w_B^{2I}) - \pi^M(w_A^{1A}, w_B^{1A}) = \frac{(8y_A^2 + y_B^2)\alpha^2 - 2(y_A - y_B)(4y_A + y_B)\alpha + (y_A - y_B)^2}{8\alpha(y_A + y_B)}$$

This difference is positive if the numerator is positive. This numerator is quadratic and convex in α. The lower root is lower than α^{2I}_{min}, and the higher root is

$$\alpha^{12} = \frac{y_A - y_B}{8y_A^2 + y_B^2} \cdot \left(4y_A + y_B + 2\sqrt{2y_A(y_A + y_B)}\right)$$

This higher root is between α^{2I}_{min} and α^{2I}_{max}. Hence if α is between α^{2I}_{min} and α^{12} then the profit difference is negative, so that the profit is higher with candidate $1A$. Conversely, if α is between α^{12} and α^{2I}_{max} then the profit difference is positive, so that the profit is higher with candidate $2I$.

Last, let us compare candidates $2C$ and $1A$. We have:

$$\pi^M(w_A^{2C}, w_B^{2C}) - \pi^M(w_A^{1A}, w_B^{1A}) = \frac{y_A}{y_A + y_B} \cdot (y_A(\alpha - 1) + y_B)$$

This difference is positive if $\alpha > (y_A - y_B)/y_A$.

Figure 6 synthesizes the pairwise comparisons and defines the equilibrium for each value of α. Figure 7 defines the parameters region leading to the different types of equilibrium. Finally, the equilibrium monopoly pricing decision can be summarized as follows:

Lemma 2. *Under monopoly supply of seed, the equilibrium pricing is :*

- (w_A^{1A}, w_B^{1A}) *if* $\alpha < \alpha^{12}$ *— this lead to a full coverage of the market by crop A and no profit for the farmer.*

[6] We indeed have : $\pi^M(w_A^{2I}, \hat{w}_B^{2I}) - \pi^M(w_A^{2I}, w_B^{2I}) = \dfrac{(y_A - y_B(1+\alpha)^2}{8\alpha(yA + y_B)} > 0$

Figure 6: Comparison of the three equilibrium candidates

	0	α^{2I}_{min}	α^{12}	$\frac{y_A - y_B}{y_A}$	α^{2I}_{max} $\rightarrow \alpha$
$2C$ *vs* $2I$	$2C \succ 2I$	$2I \succ 2C$	$2I \succ 2C$	$2I \succ 2C$	$2C \succ 2I$
$1A$ *vs* $2I$	$1A \succ 2I$	$1A \succ 2I$	$2I \succ 1A$	$2I \succ 1A$	$2I \succ 1A$
$1A$ *vs* $2C$	$1A \succ 2C$	$1A \succ 2C$	$1A \succ 2C$	$2C \succ 1A$	$2C \succ 1A$
Equilibrium	$1A$	$1A$	$2I$	$2I$	$2C$

- (w_A^{2I}, w_B^{2I}) *if* $\alpha \in [\alpha^{12}, \alpha^{2I}_{max}]$ *— this leads to a sharing of the market among the two crops and positive profit for the farmer on crop* A.
- (w_A^{2C}, w_B^{2C}) *if* $\alpha > \alpha^{2I}_{max}$ *— this lead to a sharing of the market among the two crops and no profit for the farmer.*

The candidates $2C$ and $2I$ on the one hand and $1A$ on the other hand correspond to two opposite strategies. This is illustrated in figure 8 with the strategies $2C$ and $1A$, the strategy $2I$ being close to $2C$ as will be seen later. Let us first assume that the monopoly wants to reach the first best. He needs to set $w_A = w_B$. As indicated in lemma 1, per unit welfare is higher with crop A. Hence the best pricing for the monopoly is to define prices equal to the per unit welfare generated by crop B ($w_A = w_B = y_B(1-\alpha(1-\theta^{FB}))$. With this pricing the monopoly's and farmer's profit are:

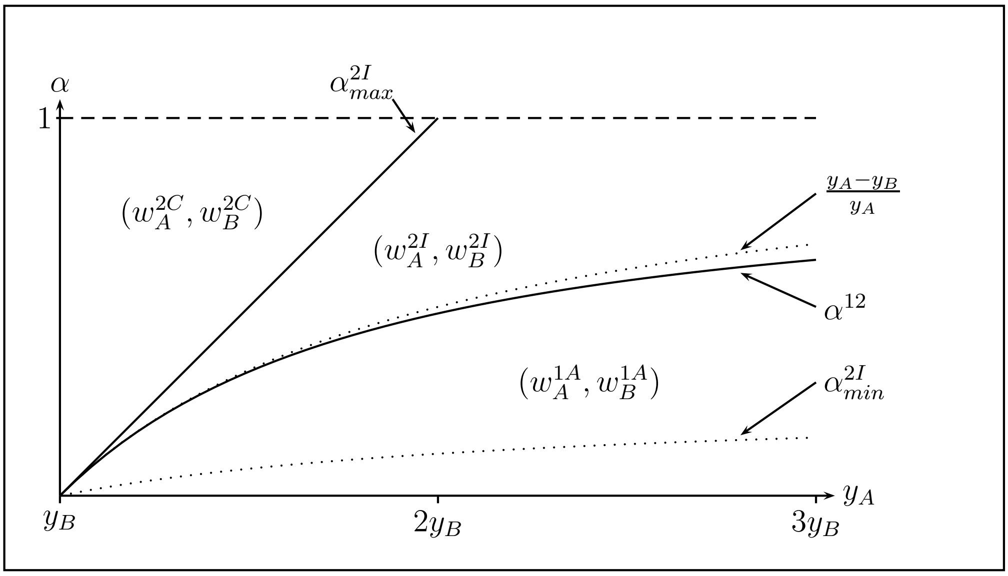
$$\pi^M = y_B(1-\alpha(1-\theta^{FB})) \quad \text{and} \quad \pi^F = \theta^{FB} \cdot \big(y_A(1-\alpha\theta^{FB}) - y_B(1-\alpha(1-\theta^{FB}))\big)$$

This strategy leads to the highest welfare but a significant part of it is left to the farmer, so that the monopoly profit is not maximal. Starting from this point, the monopoly can adopt two opposite strategies that reduce the farmer surplus to 0. With the example given in figure 8, these two strategies are beneficial, compared to the original situation leading to θ^{FB}. One first strategy is to increase w_A in order to capture a larger part of the per unit welfare generated by crop A. Increasing w_A leads to a decrease of θ and a decrease of the per unit welfare on crop B, so that w_B has to be decreased. However, as can be seen on figure 8, the gain on crop A outweighs the loss on crop B.[7] The

[7] Formally we have:

$$\tilde{\theta}^{2C} \cdot \Big(y_A(1-\alpha\tilde{\theta}^{2C}) - y_B(1-\alpha(1-\theta^{FB}))\Big) >$$
$$(1-\tilde{\theta}^{2C}) \cdot \Big(y_B(1-\alpha(1-\theta^{FB}) - y_B(1-\alpha(1-\tilde{\theta}^{2C}))\Big)$$

Figure 7: Equilibrium with monopoly supply



alternative strategy is to fully cover the market with crop A and define a price equal to the per unit welfare generated by crop A. The congestion on crop A is maximum, leading to a decrease of the per unit welfare on this crop. However, with the specific value taken for figure 8, this level is higher compared to the per unit welfare with crop B when $\theta = \theta^{FB}$, so that this strategy is also beneficial.

In summary, figure 8 illustrates three strategies that have pros and cons for the monopoly. FB leads to the highest welfare but leaves significant profit to the farmer. The profit of the farmer is reduced to 0 with $2C$ and $1A$, but the most productive crop is underused with strategy $2C$ and overused with strategy $1A$.

Table 2.a provides two numerical examples that complement figure 8. In both examples, we can check that the farmer surplus is large if $\theta = \theta^{FB}$. Candidate $2I$, that is not illustrated in figure 8, leads to a situation very close to $2C$. Both strategies correspond to the same mechanism of price discrimination and lead to similar proportions of crops A and B.

These two examples illustrate the switch from one type of strategy to the other. The equilibrium is $1A$ in the first example ($\alpha = 0.30$), and $2I$ in the second example, which corresponds to a small increase in the congestion effect ($\alpha = 0.35$). In the two examples, the profit levels of the monopoly with the three candidates are similar with a given set of parameters. Hence, a small change in one parameter leads to a switch in the ranking of candidates. This switch has a drastic effect on the proportion of the two crops: we jump from $\tilde{\theta}^{2I} = 0.467$ for $\alpha = 0.35$ to the extreme value $\tilde{\theta}^{1A} = 1$ for $\alpha = 0.30$.

Figure 8: Contrasting strategies $2C$ and $1A$

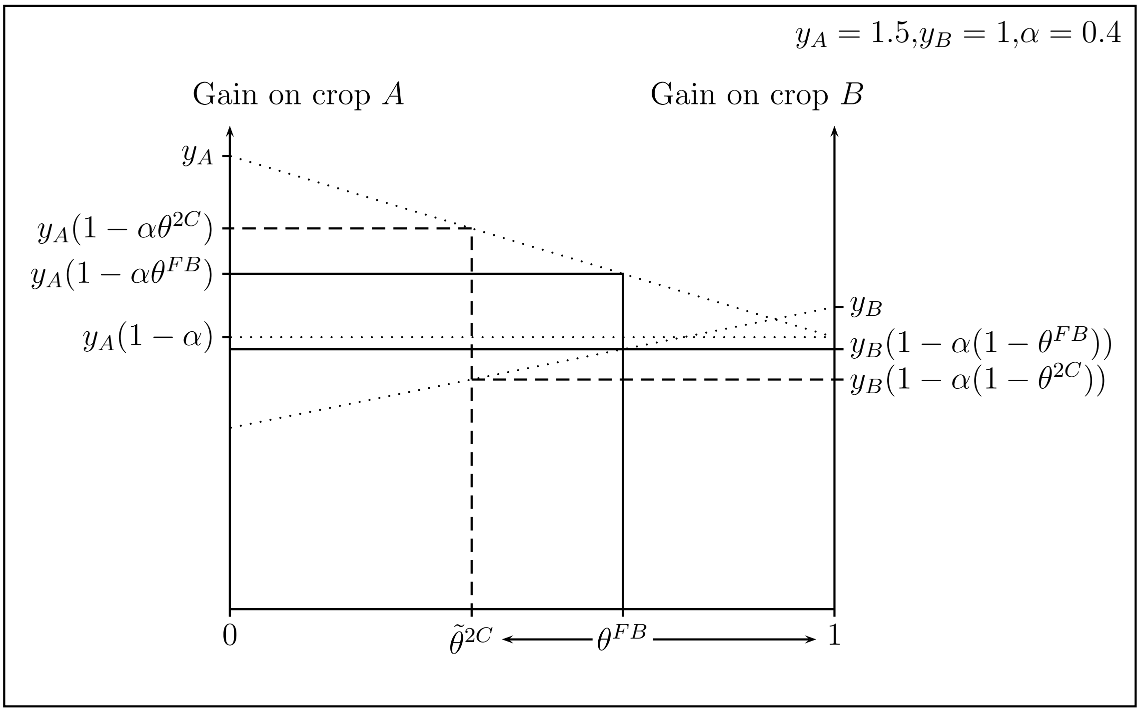


Table 2: Characteristics of the first best and equilibrium candidates: some examples

Table 2.a. Two values for α with $y_A = 1.5$, $y_B = 1$

	$\alpha = 0.30$				$\alpha = 0.35$			
	FB	$2C$	$2I$	$1A$	FB	$2C$	$2I$	$1A$
w_A	0.920	1.320	1.240	1.050	0.890	1.290	1.230	0.975
w_B	0.920	0.820	0.840	> 1.450	0.890	0.790	0.805	> 1.525
$\tilde{\theta}$	0.733	0.400	0.467	1	0.686	0.400	0.443	1
W	1.103	1.020	1.050	1.050	1.061	0.990	1.010	0.975
W^F	0.183	0	0.022	0	0.171	0	0.017	0
W^M	0.920	1.020	1.037	1.050	0.890	0.990	0.993	0.975

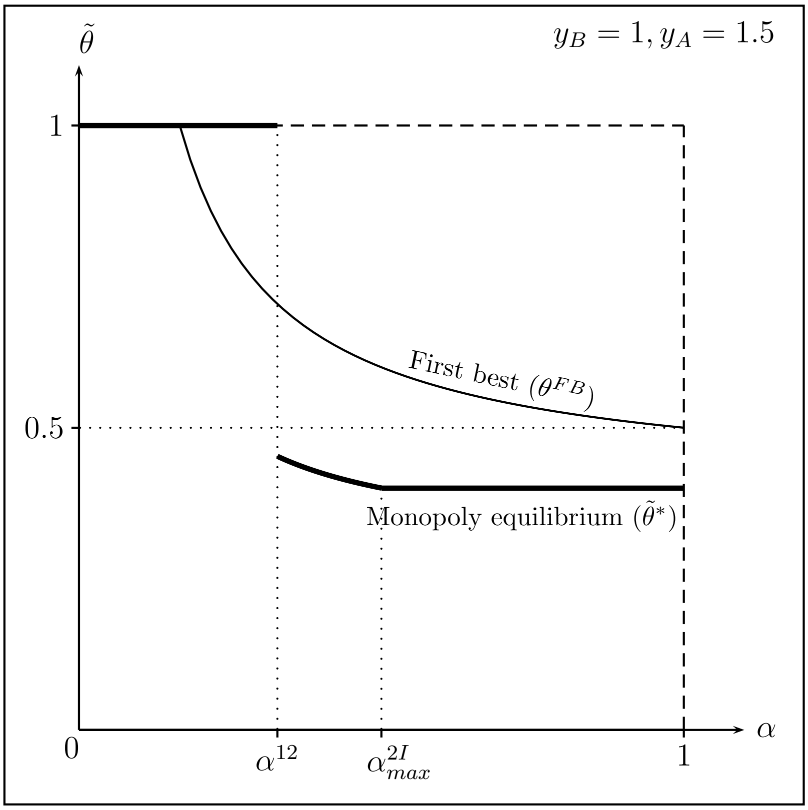
Table 2.b. Two values for y_A with $y_A = 1.5$, $\alpha = 0.4$

	$y_A = 1.6$				$y_A = 1.8$			
	FB	$2C$	$2I$	$1A$	FB	$2C$	$2I$	$1A$
w_A	0.87	1.35	1.27	0.96	0.89	1.54	1.38	1.08
w_B	0.87	0.75	0.77	> 1.64	0.89	0.74	0.78	> 1.72
$\tilde{\theta}$	0.67	0.38	0.43	1	0.71	0.36	0.45	1
W	1.07	0.98	1.01	1.00	1.17	1.03	1.09	1.08
W^F	0.20	0	0.02	0	0.28	0	0.04	0
W^M	0.87	0.98	0.99	0.96	0.89	1.03	1.05	1.08

The discontinuity in the equilibria can also be seen when the maximum potential productivity of crop A increases (table 2.b). In this case, the equilibrium is $2C$ for $y_A = 1.6$, and $1A$ for a small increase in y_A up to $y_A = 1.8$. This switch entails a jump in the crops' proportion from $\tilde{\theta}^{2I} = 0.43$ (for $y_A = 1.6$) to $\tilde{\theta}^{1A} = 1$ (for $y_A = 1.8$). An interesting implication is that a small increase in the potential productivity of one crop leads to the other crop no longer being cultivated.

Figure 9 illustrates more the discontinuity in the equilibrium value of $\tilde{\theta}$ at the threshold value α^{12}, when one switches from strategy $1A$ to strategy $2I$. More precisely, if $\alpha < \alpha^{12}$, crop B is no longer cultivated ($\tilde{\theta}^{1A} = 1$). Conversely, if $\alpha > \alpha^{12}$ we can

Figure 9: Equilibrium and first best value of θ as a function of α



show[8] that we always have $\tilde{\theta} < 0.5$. Figure 10 also illustrates this discontinuity for various as y_A increases over y_B and for various valies of α.[9]

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[8]Both $\tilde{\theta}^{2C}$ and $\tilde{\theta}^{2I}$ are lower than 0.5 when $\alpha > \alpha^{12}$. We have $\tilde{\theta}^{2C} = y_B/(y_A + y_B) < 0.5$. We also have $\tilde{\theta}^{2I} < 0.5$ because $\tilde{\theta}^{2I}$ is decreasing in α and lower than 0.5 if $\alpha = \alpha^{12}$.

$$\tilde{\theta}^{2I}\Big|_{\alpha=\alpha^{12}} = \frac{1}{2} - \frac{(2y_A - y_B)\sqrt{2y_A(y_A + y_B)} - y_B(y_A + y_B)}{2(y_A + y_B)(4y_A + y_B + 2\sqrt{2y_A(y_A + y_B)})}$$

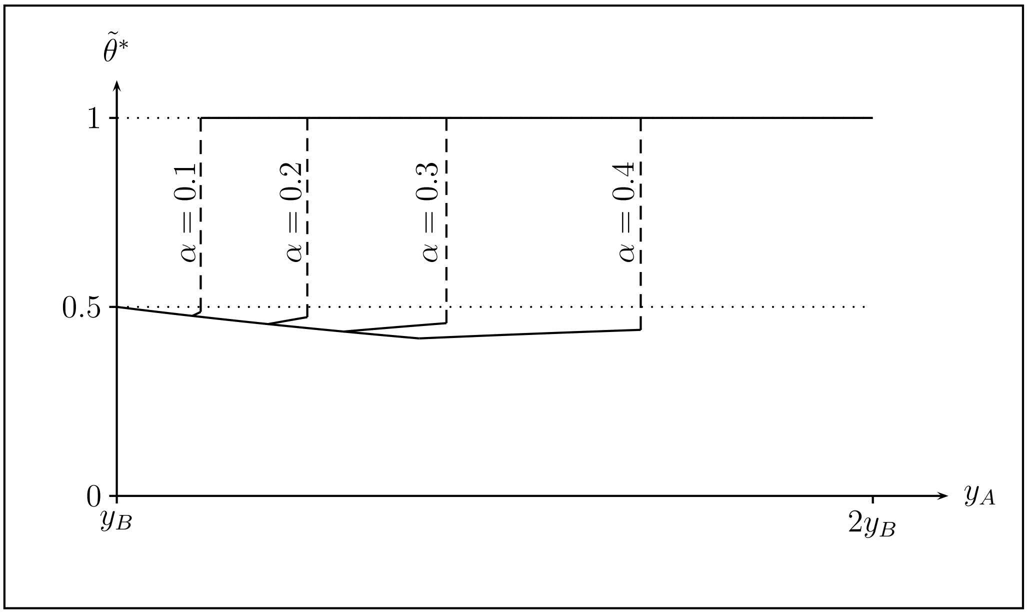
This value is lower than 1/2 because

$$\begin{aligned}(2y_A - y_B)\sqrt{2y_A(y_A + y_B)} - y_B(y_A + y_B) = \\ y_B^2 \cdot \Big(2y_A/y_B - 1\sqrt{2y_A/y_B(y_A/y_B + 1)} - (y_A/y_B + 1)\Big) \geq 0 \quad \text{for} \quad y_A/y_B \geq 1\end{aligned}$$

[9]Note that the threshold value α^{12} between $2I$ and $1A$ is equivalent to a threshold value on y_A:

$$\alpha > \alpha_{12} \quad \Leftrightarrow \quad \alpha > 0.85 \text{ or } y_A < y_A^{12} \quad \text{with} \quad y_A^{12} = \frac{1 - 3\alpha - 2\alpha\sqrt{2(2 - \alpha - \alpha^2)}}{1 - 8\alpha - 8\alpha^2} \cdot y_B$$

Figure 10: Crop proportion at the equilibrium ($\tilde{\theta}^*$) as a function of y_A

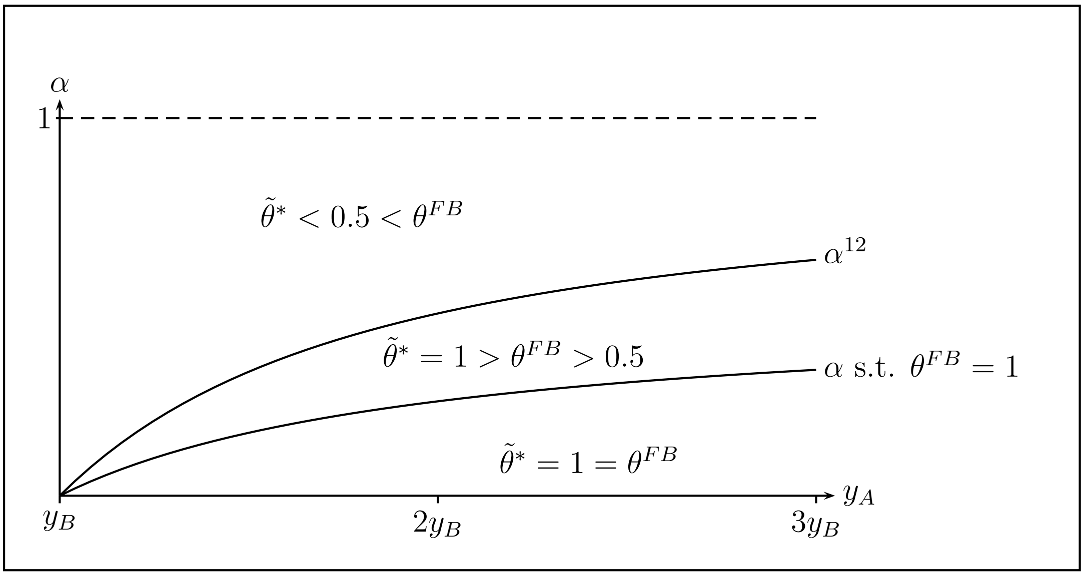


Last, figure 11 synthesizes the properties of the equilibrium value of $\tilde{\theta}$ for any parameter value. This result is summarized in the following proposition.

Proposition 1. *When the congestion effect is weak, the first best is to grow only the most productive crop, and this is implemented in equilibrium.*

When the congestion effect is larger, the first best is to grow both crops with a higher share of the one with the hightest potential productivity. The equilibrium never leads to the first best: it leads to a full (and excessive) coverage by the most productive crop for intermediate values of the congestion effect; and to a minority (and insufficient) coverage by the most productive crop for high values of the congestion effect.

Figure 11: Crop proportion at the equilibrium ($\tilde{\theta}^*$) and the first best (θ^{FB})



4 R&D investment by the monopoly

We assume now that the monopoly has the opportunity to increase the potential productivity by investing in research and development. We suppose that the research investment on crop i leads to an improvement of the yield from y_i to $y_i + \delta_i$, and costs $\delta_i^2/2$. There is no economies of scope between the research on the two crops so that the research costs for the two crops are separable and additive. In the symmetric case where the yield before the research investment are identical ($y_A = y_B$), then the research investment on the two crops are identical. We examine here the situation where the initial situation is asymmetric, and keep the assumption that $y_A > y_B$. Does research investment leads to increase or to attenuate this asymmetry?

The net profit of the monopoly is

$$\Pi^M = \pi^M \left(y_A + \delta_A, y_B + \delta_B\right) - \frac{\delta_A^2}{2} - \frac{\delta_B^2}{2}$$

The research investment is chosen on the basis of the marginal effect on the profit earned *ex post*, after the research investment. Not that the pricing equilibrium correspond now to a subgame equilibrium. The different types of pricing equilibrium presented before ($2C$, $2I$, $1A$) leads to different level of research investment. We first characterize the optimal research decision knowing the equilibrium research *after* the investment. We then analyze the optimal research decision for any initial situation (y_A, y_B) *before* the investment. For this second part, the analysis is restricted to the case where $y_B = 1$ because the the solution for the optimal investment are not tractable in some cases.

Lemma 3. *If the optimal research investment leads to a subgame equilibrium where the two crops are grown, then we can conclude that the investment on the most productive crop (ex post) has been lower compared to the other crop. Conversely, if the optimal research investment leads to a subgame equilibrium where only one crop is grown, then we can conclude that the investment has been focused only on this crop.*

Consider first the subgames $2C$ and $2I$ where the two crops are grown. The result can be established by analyzing the impact of a marginal variation of y_A or y_B on the gross profit. More precisely, we need to show that, at the *ex post* position (y'_A, y'_B), we have $\partial\pi^M/\partial y'_A < \partial\pi^M/\partial y'_B$.

This is indeed checked with $2C$ because:

$$\frac{\partial\pi^M_{2C}}{\partial y'_A} - \frac{\partial\pi^M_{2C}}{\partial y'_B} = -\frac{(2-\alpha)(y'_A - y'_B)}{y'_A + y'_B} < 0$$

With $2I$ we have:

$$\frac{\partial\pi^M_{2I}}{\partial y'_A} - \frac{\partial\pi^M_{2I}}{\partial y'_B} = \frac{(4y'_A - 5y'_B)\alpha^2 - (7y'_A - 5y'_B)\alpha + 2(y'_A - y'_B)}{4(y'_A + y'_B)}$$

The numerator is quadratic in α and it can be shown that it is always negative for $\alpha > \alpha^{12}$, which is a condition for being in situation $2I$.[10]

[10] The numerator is negative because: if $y'_A < 4y'_B/5$, it is concave in α and the higher root is lower than α^{12}, and alternatively, if $y'_A < 4y'_B/5$, it is convex in α and the interval $[\alpha^{12}, 1]$ is included between the two roots.

Consider now the subgame $1A$ where only one crop is grown. Then the most simplest way to prove lemma 3 is to compile directly the optimal level of investment:

$$\frac{\partial \Pi^M_{1A}}{\partial \delta_A} = 0 \quad \Leftrightarrow \quad \delta_A = \delta_A^{1A} \quad \text{with} \quad \delta_A^{1A} = 1 - \alpha$$

and

$$\frac{\partial \Pi^M_{1A}}{\partial \delta_B} = 0 \quad \Leftrightarrow \quad \delta_B = \delta_B^{1A} \quad \text{with} \quad \delta_B^{1A} = 0$$

In the rest of this section we denote:

$$\left(\delta_A^X, \delta_B^X\right) = \text{argmax}_{\left(\delta_A^X, \delta_B^X\right)} \Pi^M_X \quad \text{with} \quad X \in \{2C, 2I, 1A\}$$

$\left(\delta_A^{1A}, \delta_B^{1A}\right)$ is defined above. $\left(\delta_A^{2A}, \delta_B^{2A}\right)$ and $\left(\delta_A^{2A}, \delta_B^{2A}\right)$ are not tractable but they can be compiled numerically for any value of y_A, y_B and α.

The properties of lemma 3 are defined with respect to the value of y_A and y_B *after* the R&D. We now analyze the equilibrium for any value of y_A and y_B (such that $y_A > y_B$) *before* the R&D. One important observation is that, for a given initial (i.e. before R&D) position, different investment strategies leading to different types of subgame (pricing) equilibrium can be adopted.

This is illustrated with figure 12. Note that this figures requires to translate the condition on α leading $2C$, $2I$ or $1A$ (cf. lemma 2 and figure 7) as conditions on y_A.[11] The initial position ($\circ$) is such that, without research investment, the monopoly adopt a $2C$ pricing strategy. We then consider the best research investment with each type of pricing strategy ($2C$, $2I$ and $1A$) and check whether the strategy that has been considered can indeed be implemented in the final position after R&D. This is indeed the case for $1A$ and $2C$. More precisely, the pricing strategy in the position $(y_A^\circ + \delta_A^{1A}, y_B^\circ + \delta_B^{1A})$ is indeed $1A$, and the pricing strategy in the position $(y_A^\circ + \delta_A^{2C}, y_B^\circ + \delta_B^{2C})$ is indeed $2C$. Conversely, this is not the case with $2I$ because $(y_A^\circ + \delta_A^{2I}, y_B^\circ + \delta_B^{2I})$ is in the region where the pricing strategy is $2C$. The optimal research strategy requires to compare the net profit with $(y_A^\circ + \delta_A^{1A}, y_B^\circ + \delta_B^{1A})$ and $(y_A^\circ + \delta_A^{2C}, y_B^\circ + \delta_B^{2C})$. The results are presented in table 3 and we observe that the net profit is higher if the monopoly chooses $(\delta_A^{1A}, \delta_B^{1A})$. Hence, staring at (y_A°, y_B°), the pricing strategy without research investment is $2C$, leading the farmer to grow two crops, while the optimal research investment leads to a pricing strategy $1A$ and the farmer to grow only one crop. In table 3, we also consider the alternative case where $y_A = 1.1$ (instead of $y_A = 1.2$ in the previous example). In this case, the research investment can also lead to $(y_A^\circ + \delta_A^{1A}, y_B^\circ + \delta_B^{1A})$ and $(y_A^\circ + \delta_A^{2C}, y_B^\circ + \delta_B^{2C})$, but the profit is higher in the last case.

[11] $\alpha = \alpha^{2I}_{max}$ is equivalent to $y_A = y_A^{IC}$ with $y_A^{IC} = (1+\alpha) \cdot y_B$. Also

$$\alpha = \alpha^{12} \quad \Leftrightarrow \quad y_A = y_A^{12} \quad \text{with} \quad y_A^{12} = \frac{1 - 3\alpha - 2\alpha\sqrt{2(2-\alpha-\alpha^2)}}{1 - 8\alpha - 8\alpha^2} \cdot y_B$$

The subgame pricing equilibrium is $2C$ if $y_A < y_A^{IC}$, $2I$ if $y_A \in [y_A^{IC}, y_A^{12}]$, and $1A$ if $y_A > y_A^{12}$.

Table 3: Some examples of comparison of research strategies ($\alpha = 0.25$)

y_A	1.1			1.2		
Pricing strat. after R&D	$2C$	$2I$	$1A$	$2C$	$2I$	$1A$
δ_A	0.419	0.271	0.750	0.402	0.291	0.750
δ_B	0.456	0.671	0	0.475	0.599	0
$(y_A + \delta_A)/(y_B + \delta_B)$	1.043	0.848	1.850	1.086	0.932	1.950
Π^M	1.109	1.142	1.106	1.150	1.170	1.181

with $\alpha = 0.25$, $y_A^{IC}/y_B = 1.25$ and $y_A^{12}/y_B = 1.33$

Table 4 illustrates an alternative case with higher congestion effect ($\alpha = 0.5$ instead of $\alpha = 0.25$ in figure 12 and table 3). With the two initial positions ($y_A = 2.2$ or 2.3) the optimal pricing strategy without research is $1A$. Research strategy can lead either to as pricing strategy $2I$ and $1A$ and, with $y_A = 2.2$, the net profit is higher if the final position leads to a $2I$ pricing. In this particular case, the pricing strategy leads the farmer to grow two crops with research and only one crop without research.

The comparison made in table 3 and 4 can been extended to any value of α (between 0 and 1) and y_A between 1 and 3. The results are presented in figure 13. Remind that the optimal research investment leading to a pricing $2C$ and $2I$ are not tractable. Hence, we build figure 13 on the basis of numerical compilation for a large number of parameters value.[12] This figure enables to establish the following proposition.

Proposition 2. *Research investments affect the range of crops grown at the equilibrium. The set of parameters leading to an equilibrium where two crops are grown (rather than one) is reduced with low congestion effect and extended with large congestion effect.*

In other words, research investments attenuate the effect of the initial difference between y_A and y_B and increase the impact of the congestion effect α. Indeed, if we observe the threshold value on α between the equilibrium with one and two crops (figure 13) this value is less sensitive to y_A if the potential productivity is endogenous rather than exogenous.

5 Conclusion

This chapter presents a model where we analyze how some congestion on the demand side affects the pricing and research strategy of a monopoly. With high enough congestion effect, the first best is to grow a mix of two crops. However, the equilibrium is qualitatively different and the monopoly strategy leads either to an full coverage of the

[12]The compilation was made with a program in C. We compile the equilibrium for all combinations between 21 values of y_A between 1 and 3 (with a step of 0.1) and 1001 values of α between 0 and 1 (with a step of 0.001).

Figure 12: An example of position shift resulted from R&D

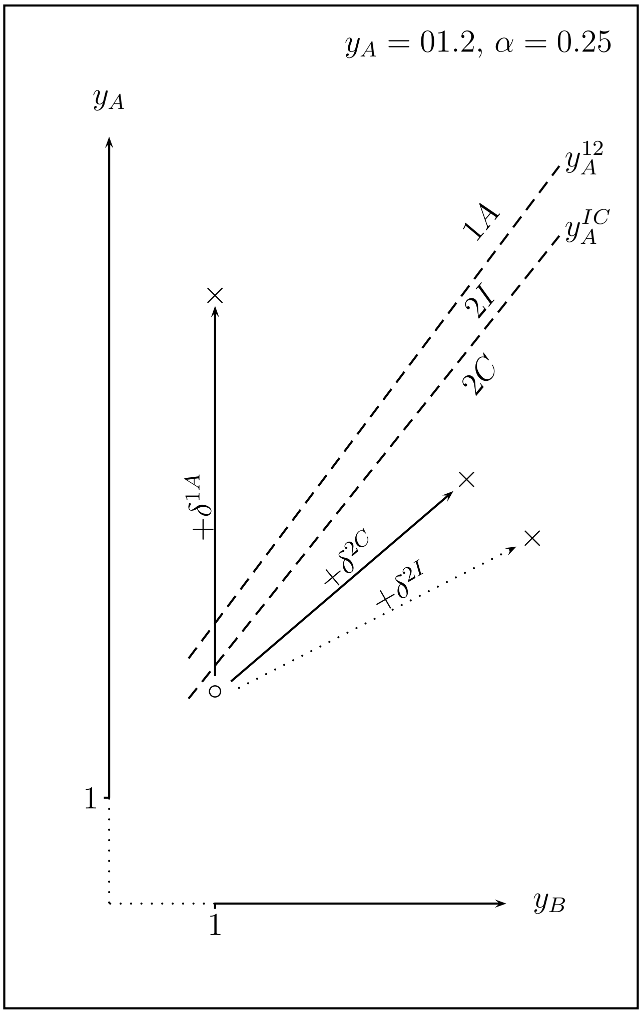
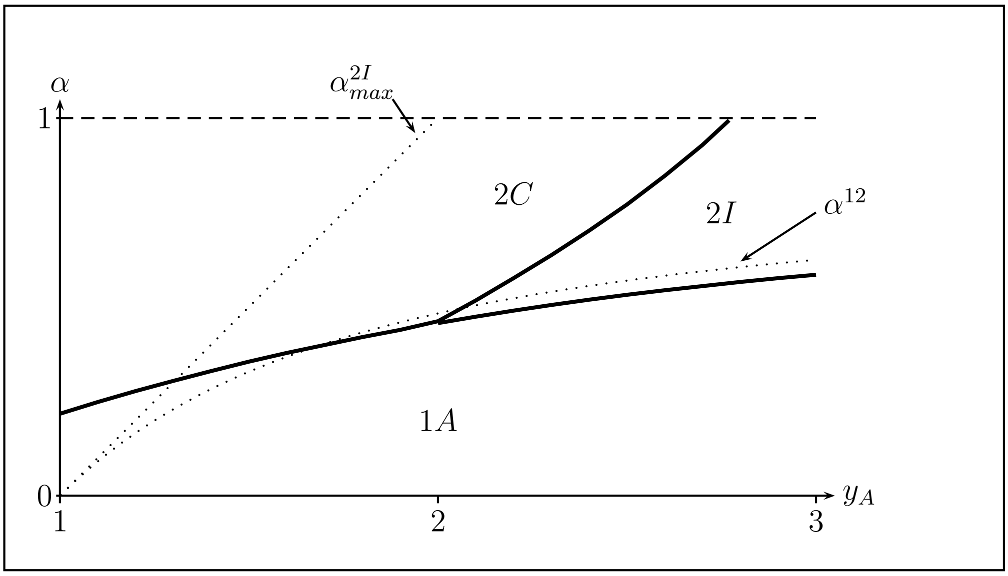


Table 4: Some examples of comparison of research strategies ($\alpha = 0.50$)

y_A	2.2			2.3		
Pricing strat. after R&D	$2C$	$2I$	$1A$	$2C$	$2I$	$1A$
δ_A	0.229	0.241	0.500	0.221	0.241	0.500
δ_B	0.558	0.539	0	0.570	0.538	0
$(y_A + \delta_A)/(y_B + \delta_B)$	1.559	1.586	2.700	1.606	1.652	2.800
Π^M	1.242	1.243	1.225	1.264	1.267	1.275

with $\alpha = 0.50$, $y_A^{IC}/y_B = 1.50$ and $y_A^{12}/y_B = 2.08$

Figure 13: Equilibrium with monopoly supply and endogenous R&D



market by the most productive crop or to an insufficient proportion of this crop. This result is still observed if research incentives are taken into account. Taking research incentives into account increases the impact of the congestion effect and attenuates the effect of the initial difference between the potential productivity of the two crops.

Several extension of this analysis are worth mentioning. First, the robustness of the results should be analyzed when some assumption are relaxed. What are the properties when some competition is introduced on the supply side (ex: duopoly between one supplier of seed A and one supplier of seed B)? We need also to consider the possibility for the farmer not to cover the whole market with crop A and B when the congestion effect is important. Secondly, more general setting should be considered. If we consider the pest problem as the reason for the congestion effect, the farmer can eliminate all or part of the congestion problem by using pesticide. How does this influence the seed supply equilibrium? Also, what would happen if the pesticide and one of the seed are supplied by the same company as it can be anticipated from the merger of Bayer and Monsanto? At last, we consider a setting without externality. Indeed, if market power is eliminated, the two prices are identical and the first best is reached. However, growing only one crop rather than two reduces the biodiversity and this may generate negative externality. Also, using technology like pesticide to eliminate congestion effect can also generate negative externality. What are the policy instrument that can be used to moderate such negative externalities?

References

Acemoglu D. and J. Linne.2004. Market Size in Innovation :Theory and Evidence from the Pharmaceutical Industry. *Quaterly Journal of Economics* 119, n 3 : 1049-90.

Atkinson B. and Stiglitz J. 1987. Lectures on public economics

Belleflamme P. and Peitz M. 2010. Industrial Organization. *Cambridge university press*, chapter 5 and 18.

Carpentier A., Bariber J.M., Bontems P., Lacroix A., Laplana R., Lemarié S., Turpin N. 2005. Pesticide, Agriculture et environnement, expertise scientifique collective, chapitre 5.

Charlot S., Dridi C. and Lemarié S. 2015. Market size and innovation : An application to the French seed market for large crops. AAEA Conference, San Fransisco, CA, july 26-28

Cohen W.M. 2010. Fifty years of empirical studies of innovative activity and performance. *Handbooks in Economics*, 1:129-213.

Dubois P., De Mouzon O., Scott Morton F. and Seabright P. 2011. Market Size and Pharmaceutical Innovation.

Ferguson S. 2015. Endogenous Product Differentiation, Market Size and Price, *Review of International Economics*

Hart O. and Moore J.1996. The governance of exchanges : member's cooperatives versus outside ownership. *Oxford Review of Economic Policy*, vol 12, n 4.

Maynard J.M, Messéan A., Charlier A., Charrier A., Fares M., Bail M., Magrini M. and Savini I. 2013. Freins et leviers à la diversification des cultures. Etude au niveau des exploitations agricoles et des filières. Synthèse du rapport d'étude INRA pour les ministères en charge de l'Agriculture et de l'Ecologie.

Scotchmer S. 2004. Innovation and incentives. *the MIT press.* chapter 9.

Tirole J. 1988. The Theory of Industrial Organization. *the MIT press.* Chapter 1, 3 and 10.